THE PUBLISHING HOUSE
OF THE ROMANIAN ACADEMY

GEONOMY
Research article

COMPARATIVE STUDY ON OBTAINING COORDINATES FOR FIXED POINTS USING TRAVERSE METHOD AND LEAST SQUARES METHOD

CORNEL PĂUNESCU[1], DANIELA IORDAN[2] and COSMIN CIUCULESCU[3]

[1] University of Bucharest, Faculty of Geology and Geophysics, Bucharest, Romania
[2]USAMV Bucharest, Bucharest, Romania
[3]Technical University of Civil Engineering, Bucharest, Romania
Corresponding author: Cornel PĂUNESCU, E-mail: cornelpaun@gmail.com

Received September 14, 2012

It is known that the position (coordinates) of the new points it is determined by several methods: intersections, double canceled, traverse, least squares method. The new points are used by specialists to perform topographic or cadastral plans. In recent years, most of them performed works for book registration of ownership on small areas. Given the development of GNSS technology they determine two support points with GNSS receivers and from these provides a traverse. As known from theory, traverses is not the best method. It is recommended the least squares method because account all measurements and provide an equation. This study demonstrates that.

Key words: least squares method, traverse, coordinate, radiate points.

INTRODUCTION

Topographic devices evolve dramatically in recent years. Since the advent of total stations and of the GPS technology later became GNSS, it is appeared elusive performance before. Thus we can speak today of accuracy of millimeters and millimeters errors. If we look at the data given us by reports after processing GNSS data we can say that the position is given with centimeter accuracy at least. In terms of the precision equipment used are credible, given the performance. But in terms of processing, are real accuracies, the coordinates are closest to the real value? Not always. Starting with GNSS receivers producing very good positions in WGS84 system, but must be transformed into plane stereographic 1970 system. Luckily today there's ROMPOS and TRANSDAT's not providing accurate to the millimeter, centimeter even, but to ensure uniformity. It is not the purpose of this articol to study how to get position using GNSS technology, so we will not deepen. We continue to traverse versus least squares method.

In general cadastre and surveying professionals is working on small-scale works, especially documentation of tabulation of land for smallholders. For surveying need a support network to radiate points of detail.

Network support is provided either by:

– Traverse from the old geodetic points existing in the area;

– Intersection points and form these perform a traverse to the area of interest;

– Determine two points with GNSS technology and form these perform a traverse in the area of interest.

As shown, the main method of transmission of coordinates is the closed traverse. It is, of course, a commonly used method for determining the coordinates, but does not offer solutions closest to the real value, as required by theory.

MATERIAL AND METHODS

Adjustment of the measurement using least squares method

The least squares method is well known and is not appropriate to mention here all formulas. Today, thanks to the developping of the computer system is no longer used Gauss scheme and conditional measurements but indirect measurements and matrix treatment. Each measurement gives an equation, each new point gives two unknowns (x-coordinate and y-coordinate). Number of measurements (equations) must be much larger than the number of unknowns to obtain the most probable value of the coordinate closer to the real value. Equations are two or three ways (directions, distances and GNSS measurements). In these study we had only two types of equations: directions and distance.

In generally the commonly used processing is the least squares method, indirect measurements, where each measurement (direction or distance) gives an equation. Equations are different for the two types of measurements, but have in common the unknowns, respectively x and y coordinates of the new points.

For both ways of obtaining the position (traverse and least squares method) is deemed correct distance was reduced to the stereographic1970 projection plan.

After we write the equations of corrections for directions, distances and differences of coordinates, matrix relations can be written:

$$v = Ax + l \tag{2.1}$$

$$N = A^T PA \tag{2.2}$$

$$x = -N^{-1}A^T Pl \tag{2.3}$$

where:

– $\boldsymbol{v}$ is the matrix corrections;
– $\boldsymbol{A}$ is the correction coefficients matrix;
– $\boldsymbol{x}$ is the unknowns matrix;
– $\boldsymbol{P}$ is the weight matrix.

The equations of correction for distance have the form:

$$v_{ij}^D = A_{ij}dx_j + B_{ij}dy_j - A_{ij}dx_i - B_{ij}dy_i + l_{ij}^D \tag{2.4}$$

The free term is calculated as follows:

$$l_{ij}^D = D_{ij}^0 - D_{ij}^M \tag{2.5}$$

and

$$A_{ij} = \frac{\Lambda x_{ij}}{D_{ij}} = \cos\Theta_{ij} \tag{2.6}$$

$$B_{ij} = \frac{\Lambda y_{ij}}{D_{ij}} = \sin\Theta_{ij} \tag{2.7}$$

The equations of corrections for directions are like:

$$v_{ij} = -dZ_i + a_{ij}dx_j + b_{ij}dy_j - a_{ij}dx_i - b_{ij}dy_i + l_{ij} \tag{2.8}$$

$$l_{ij} = \Theta_{ij}^0 - \alpha_{ij}^0 - Z_{ij}^0 \tag{2.9}$$

$$a_{ij} = -\rho^{00}\frac{\sin\Theta_{ij}^0}{D_{ij}^0} = -\rho^{00}\frac{\Lambda y_{ij}^0}{D_{ij}^0} \tag{2.10}$$

$$b_{ij} = -\rho^{00}\frac{\cos\Theta_{ij}^0}{D_{ij}^0} = -\rho^{00}\frac{\Lambda y_{ij}^0}{D_{ij}^0} \tag{2.11}$$

For each new point can be obtained the determined accuracies. The formulas is:

$$S_{xi} = S_0\sqrt{Q_{xixi}} \text{ , cu j=1,2 ......n} \tag{2.12}$$

Coefficients Qij are extracted from the main diagonal of the inverse matrix of the normal system (matrix N^{-1}).

Determination of coordinates using the traverse method

Traverse method is a method widely used in surveying and geodesy specialists for small stretching area. Basically it starts from a point with known coordinates that are stationary with total station and is aimed at a different point which is also known coordinates. From this point on aiming the next point traverse, linked with the above point and thus can determine the orientation of departure. It also measures the distance (from point with known coordinates to the new point and back again). Total station is then moved to the new point and on aiming the point we left and traverse next point. The operation is repeated until it reached the point of closing traverses, which is also known position and on aiming the before traverse point and another point with known coordinates for closed orientation.

Basically coordinates are determined step by step and finally closes on a known point. Coordinates transmitted by traverse should be close to those of inventory, given the precision of the old network and traverse accumulated errors.

General formulas for determination are:

$$\begin{aligned} x_B &= x_A + D_{0AB}\cos\theta_{AB} \\ y_B &= y_A + D_{0AB}\cos\theta_{AB} \end{aligned} \tag{3.1}$$

Coordinates determined step by step, finally reaching a point of known coordinates (point of closure). Between transmitted coordinates and coordinates existing difference occurs is called the traverse discrepaney in closing.

$$\begin{aligned} e_x &= x_B^{Tr} - x_B \\ e_y &= y_B^{Tr} - y_B \end{aligned} \tag{3.2}$$

Total error linear:

$$e_t = \sqrt{e_x^2 + e_y^2} \tag{3.3}$$

This error should not exceed the tolerance required. Tolerance is given by the machine precision with which they work.

If errors are within tolerance, the unitary corrections are calculated:

$$\begin{aligned} C_x &= \frac{\varepsilon_x}{D} \\ C_y &= \frac{\varepsilon_y}{D} \end{aligned} \tag{3.4}$$

Where $\boldsymbol{D}$ is the sum of small distances on the stereographic 1970 projection plane.

$$D = \sum D_0 \tag{3.5}$$

Unitary correction is applied to each Δx and Δy in function of the distance. After applying corrections, coordinates are transmit again step by step and $x_B^{Tr} = x_B$. Each point of the traverse will have new coordinates, corrected by traverse discrepaney in closing.

RESULTS AND DISCUSSION

We performed a framework of control points for a work whose beneficiary was Timis Water Basin Administration (Fig. 1). In the network were determined directions and distances. As points with known coordinates were 9 and new points were determined 10. Number of equations was 89, sufficient to apply the method of least squares, indirect measurements. The network was adjusted and have achieved accuracies shown in Table 1.

Since the network has a linear configuration of points that were stationed with total station, we considered it as a traverse supported at both ends (Fig. 2). We calculated coordinate with known formulas and obtained temporary positions. The traverses was adjusted and discrepaney in closing were -6.19 centimeters on ***x*** axis, +52.98 centimeters on ***y*** axis and respectively 53.34 centimeters discrepaney in closing total. The total length of the traverse was 5984.45 meters and the number of stations 12. On start from the point Gomila and closed on the point Nord Biled. Both points are part of the national geodetic network.

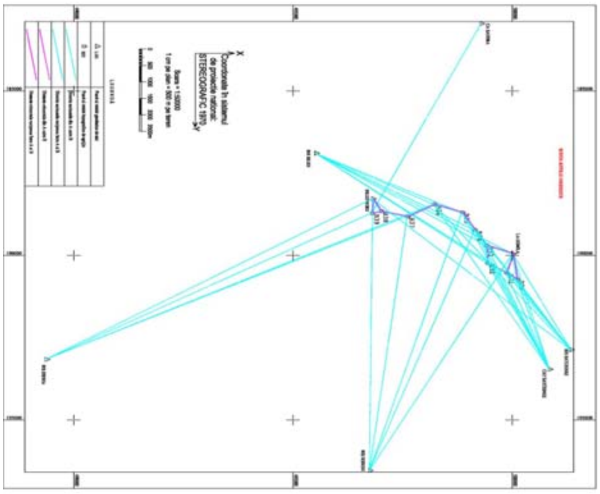

Fig. 1. Sketch of the geodetic network.

Table 1

Adjusted coordinates by the method of least squares and obtained accuracies

Point Name	X adjusted	accuracies x	Y adjusted	accuracies y	Total accuracies
	[m]	[mm]	[m]	[mm]	[mm]
BIS_SIRBEA	489381.005	0	193158.665	0	0
CA_SANDRA	499303.616	0	182951.287	0	0
BIS_BILED	495532.999	0	186890.594	0	0
BIS_SATCHINEZ	501354.193	0	192865.766	0	0
CAT_SATCHINEZ	500861.438	0	193459.266	0	0
LA_GOMILA	500021.708	0	189945.826	0	0
BIS_HODONI	496756.611	0	196534.092	0	0
BILED_NORD	496829.017	0	188278.994	0	0
BECIC	488937.319	0	193280.931	0	0
B33	499415.774	20	189689.407	23	30
B31	499875.812	20	190528.606	16	26
B30	500117.425	23	190729.652	18	29
B32	499445.179	22	190228.591	23	32
B34	499154.698	28	189222.829	30	41
B35	498864.445	32	188690.549	34	47
B36	498225.579	32	188442.562	37	49
B37	497616.381	27	188817.25	33	42
B38	497015.843	17	188654.847	15	22
B39	496808.699	18	188714.993	14	23
Average error for determining a point: 25 mm					

Table 2

Values and differences

Point Name	least squares method		Traverse method		dx [cm]	dy [cm]	d total [cm]
	x [m]	y[m]	x [m]	y [m]			
B39	496808.699	188714.993	496808.68	188715.035	1.8	-4.19	4.56
B38	497015.843	188654.847	497015.81	188654.906	2.72	-5.95	6.54
B37	497616.381	188817.25	497616.35	188817.239	2.33	1.127	2.59
B36	498225.579	188442.562	498225.48	188442.481	9.09	8.098	12.2
B35	498864.445	188690.549	498864.36	188690.427	7.91	12.2	14.5
B34	499154.698	189222.829	499154.65	189222.707	4.85	12.21	13.1
B33	499415.774	189689.407	499415.74	189689.287	2.64	11.97	12.3
B32	499445.179	190228.591	499445.16	190228.505	1.75	8.596	8.77
B31	499875.812	190528.606	499875.79	190528.539	1.57	6.723	6.9
B30	500117.425	190729.652	500117.41	190729.603	0.92	4.927	5.01

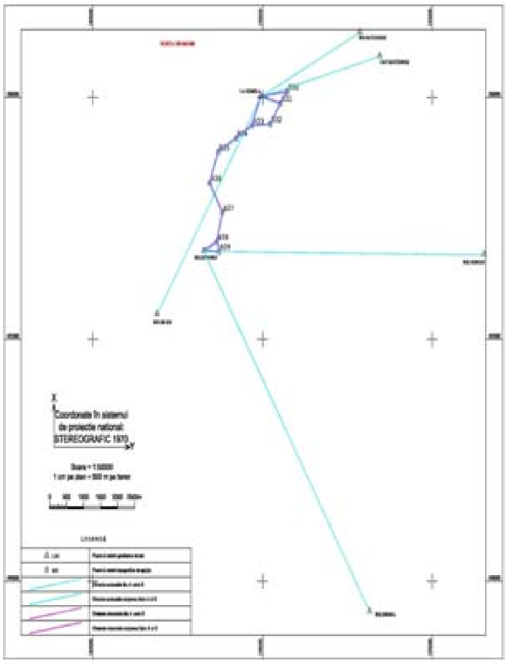

Fig. 2. Sketch of the traverse.

CONCLUSIONS AND FUTURE PROSPECTS

The case presented is an example that proves coordinates determined by traverse and coordinates determined by the method of least squares, indirect measurements are differences and that influence radiated coordinates. Radiated points are very often property corners and define contours. If two properties are measured by two different specialists and will get differently coordinates just using a different calculation method then the common point coordinates will differ substantially. Not to mention different techniques for measuring the ground, which can cause positioning errors.

In general the specialists with a small company use only the traverse method because is easier to calculate. It is wrong because at present, using advanced computers can process data very easily. Consider only excel program that comes along with purchasing your computer. This program allows to work with matrices, necessary for the method of least squares. We must know that usually do not develop a large network, but only for a property. More so, the calculations will be simpler.

From Table 2 it is observed that the coordinate differences are smaller to the end of the traverse and larger to middle, as was expected because the constraint is in the end. It can be seen that the total error in the middle than 13 centimeters.

In conclusion, it is necessary to obtained the final coordinates using the least squares method which gives a value much closer to the real value.

REFERENCES

1. Ghițău, Dumitru. Geodezie și Gravimetrie geodezică, Editura didactică și pedagogică, București 1983.
2. Moldoveanu, Constantin. Geodezie, Noțiuni de geodezie fizică și elipsoidală, poziționare, Editura MATRIXRom 2002.
3. Păunescu, Cornel, Curs de Geodezie - Topografie, Editura Universității din București, 2001.
4. Păunescu, Cornel, Sisteme Globale de Poziționare (GNSS), Editura Universității din București, 2012.