THE DIMENSIONAL ANALYSIS OF THE USLE - MUSLE SOIL EROSION MODEL

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Contemporary science, with a strong experimental supports use a large number of empirical or semi-empirical formulas, which contain dimensional defects. I tried to put these formulas in agreement with theory and suggest ways to deepen the scientific issues arising from these considerations. This paper presents a critical view of the soil erosion models, especially USLE and MUSLE model, and proposes several improvements to its formula. Critical issues and improvements are made in terms of dimensional analysis. Modified formulas were verified using experimental results obtained by methods of estimating soil erosion recently developed. Dimensional analysis offers a method for reducing complex physical problems to the simplest form prior to obtaining a quantitative answer. The formulas obtained are used to estimate soil erosion during rain events.

Key words: soil, erosion, MUSLE, dimensional, analysis.

INTRODUCTION

Mathematical model of water soil erosion of hill slope, USLE, is described, by Wischmeier and Smith[1], through the Universal Soil Loss Equation:

$$A = R \cdot K \cdot L \cdot S \cdot C \cdot P,\ C = C_1 C_2 \qquad (1)$$

where the meaning and dimension of the each factor is given in Table 1.

The FAO Soils Bulletin[2] has identified to the USLE model the next disadvantages:

– The model applies only to sheet erosion since the source of energy is the rain; so it never applies to linear or mass erosion;

– The type of countryside: the model has been tested and verified in peneplain and hilly country with 1–20% slopes, and excludes young mountains, especially slopes steeper than 40%;

– The type of rainfall: the relations between kinetic energy and rainfall intensity generally used in this model apply only to the American Great Plains and not to mountainous regions although different sub-models can be developed for the index of rainfall erosivity, R;

– A major limitation of the model is that it neglects certain interactions between factors in order to distinguish more easily the individual effect of each;

– The model applies only for average data over 20 years and is not valid for individual storms.

The full article respects the notation and writing of units for each author.

DIMENSIONAL ANALYSIS OF THE USLE FORMULA

Wischmeier and Smith[1], show that the long-term average annual soil loss, A is expressed in the units selected for K and for the period selected for R, but, in practice, there are usually so selected that they compute A in tones per acre per year. By this definition, the dimension of A is $ML^{-2}T^{-1}$, but there are other authors which use for A the dimension ML^{-2}, for example[3].

By[3], the rainfall erosivity, R is defined using the total storm energy, E, and the maximum 30-min intensity (I_{30}). These physical quantities have the next dimension: $[E] = MT^{-2}$, and $[I_{30}] = LT^{-1}$.

Table 1

Notations, meanings, and dimensions of the USLE model variables

Factor	Signification	Dimension
A	long-term average annual soil loss	$ML^{-2}T^{-1}$ $(ML^{-2})^{*}$
R	rainfall erosivity factor	MLT^{-4} $(MLT^{-3})^{*}$
K	the soil erodibility factor	$L^{-3}T^{3}$
L	topographic factor of length	$M^{0}L^{0}T^{0}$
S	topographic factor of slope	$M^{0}L^{0}T^{0}$
C_1	cropping management factor of vegetal cover	$M^{0}L^{0}T^{0}$
C_2	cropping management factor of tillage	$M^{0}L^{0}T^{0}$
P	conservation practices factor	$M^{0}L^{0}T^{0}$

*Different authors use different definitions for *A* and *R*.

Therefore the EI_{30} product dimension is MLT^{-3}. Taking into account the definition of energy *E*, it appears that this is actually a surface density of energy, as confirmed by the measurement unit of it. Consequently, the relationship (2), that connects these physical quantities, after[3] or[4] is not correct in terms of dimensional analysis:

$$E = 916 + 331\log_{10} I \tag{2}$$

where, by[3], *E* is kinetic energy (surface density of kinetic energy, dimension MT^{-2}), and *I* is rainfall intensity (dimension LT^{-1}). The equation (2) is not correct because, according to the dimensional analysis, the functions arguments must be dimensionless quantities. In (2), the argument of logarithm is *I*, which is not a dimensionless quantity. Checking dimensional relation (2) is impossible because to the logarithm of *I*, I cannot attribute any known physical dimension. This error is transmitted in the USLE model through the rainfall erosivity, *R*. Such dimensional errors in the calculation of *R* have continued to occur, for example [5]:

$$\log R = 1.93\log\sum\frac{p_i^2}{P} - 1.52\,, \tag{3}$$

where p_i is the monthly and *P* is the annual precipitation. The ratio between the square of the monthly precipitation and the annual precipitation have dimension LT^{-1}, also the sum which is the argument of the logarithm. Consequently equation (3) is not in accordance with the principles of dimensional analysis. Inconsistencies with the principles of dimensional analysis occur in the USLE model also in terms of soil erodibility, *K*. By [1] or [5], *K* is given by the next relation:

$$K = 2.8\cdot10^{-7}\cdot M^{1.14}\cdot\left(1.2 - a\right) + 4.3\cdot10^{-3}\cdot \cdot(b-2) + 3.3\left(c-3\right) \tag{4}$$

where *M* is the particle – size parameter (dimension L), *a* is percent organic matter, *b* is the soil – structure code used in soil classification, and *c* is the profile – permeability class. Considering the numerical constants and variables *a, b, c,* dimensionless, remains inconsistency with dimensional analysis principles of the term $M^{1.14}$, where *M* is an argument of a function and, according to the principles of dimensional analysis should be dimensionless. From relationship (4) is impossible the deduction of the dimension of the physical quantity *K*.

MUSLE SOIL EROSION MODEL

The modified USLE (MUSLE) replaced the rainfall erosivity factor, *R* with the product of rainfall amount and runoff amount in aim to predict soil erosion for a water erosion event[6].

By Randle *et al.*[7] the MUSLE soil erosion model is given by the next equation:

$$S = 95(Qp_p)^{0.56}KLSCP\,, \tag{5}$$

where *S* is the sediment yield for a single event in tons, *Q* is the total event runoff in ft^3, p_p is the event peak discharge, in $ft^3\ s^{-1}$, and *K, L, S C, P* are similar to the USLE equation (5).

By Blaszczynski[8], the MUSLE soil erosion model is given by the Eq. (6):

$$SY = RKLSCP\,. \tag{6}$$

This is similar to the USLE model, (1), but having modified rainfall erosivity according to the Eq. (7):

$$R = a(Q\cdot qp)b\,. \tag{7}$$

In the Eqs. (6) and (7), *SY* is the sediment yield per calculation unit, in tones, *a* and *b* are constant (not unspecified), *Q* is the volume of runoff, in acre-feet by[8], and qp is the peak flow in cubic feet per second. $K, L, S\ C, P$ is similar to the RUSLE soil erosion model.

Cinnirella *et al.*[9], present another version of the MUSLE model, given by the equation:

$$Y_j = R_{d,j} K \cdot LS \cdot C \cdot P \,, \tag{8}$$

where: Y_j is the sediment yield in t ha^{-1} for each event, K is the soil erodibility factor in t h kg^{-1} m^{-2}, LS is the topographic factor, C is the cover management factor, P is the support practices factor (all dimensionless) and $R_{d,j}$ in t ha^{-1} unit of K, is the runoff factor for each event, defined by the relation:

$$R_{d,j} = \frac{0.8776}{A_w}\left(q_{p,j} V_j\right)^{0.56}, \tag{9}$$

where $q_{p,j}$ is the peak flow rate of the flood event, in m^{3}s^{-1}, V_j is the runoff volume, in m^3 and A_w is the basin area in ha.

Another variant of the MUSLE model, is given by Pinto *et al.*[12]:

$$EP = R_{unoff} \cdot K\left(0.00984 \cdot L^{0.63} \cdot S^{1.18}\right), \tag{10}$$

where EP is the erosion potential, in t ha^{-1}, R_{unoff} is the runoff, in m^3 m^{3}s^{-1}, K is the soil erodibility, L is slope length factor and S is the slope steepness factor, last two quantities with unspecified dimension. This variant of MUSLE model is given by the equation of Williams[10] and adapted by Donzeli *et al.*[11], and by Pinto *et al.*[12]. The term R_{unoff} was used in Eq. (10) is specified by Donzeli in 1994 and Pinto, in 1996:

$$R_{unoff} = 89.6\left(Q \cdot qp\right)^{0.56}, \tag{11}$$

where Q is the surface flow volume, in m^3, and qp is the maximum flow of discharge, in m^{3}s^{-1}.

Loch *et al.*[13] used a model developed by Onstand and Foster in 1975:

$$A = W\ K\ LS\ C\ P\,, \tag{12}$$

where:

$$W = 0.5 EI_{30} + 0.349\ Q\ q_p^{0.333}, \tag{13}$$

is the combined rainfall/runoff erosivity term. In the Eq. (12), K is the soil erodibility, and LS, CP are the combined slope/length and cropping/ practice factors of the USLE (or RUSLE) model after[13]. In (13), EI_{30} is the product of storm energy and maximum 30 minute intensity (metric units), Q is the total runoff (mm), and q_p is the maximum runoff rate (mm h^{-1}).

Sadeghi[14] used a particular model MUSLE calibrated on experimental results:

$$S = 11.8\left(Q \cdot q_p\right)^{0.56} K \cdot LS \cdot C \cdot P\,, \tag{14}$$

where S is sediment yield in tones, Q is volume of runoff in m^3, q_p is peak flow rate in m^{3}s^{-1} and K, LS, C and P are respectively, the erodibility (in t·h·t^{-1}·m^{-1}·cm^{-1}), topography, crop management and soil erosion control practice factors (all dimensionless).

DIMENSIONAL ANALYSIS OF THE MUSLE SOIL EROSION MODEL

The simplest form of the MUSLE soil erosion model considered is described by Eqs. (6) and (7). This shows clearly that the transition from model USLE to the model MUSLE, is to replace rainfall erosivity, see[15], R. The dimensions of the parameters involved in (6) and (7) are:

$$[SY] = \mathrm{M},\ [Q] = \mathrm{L}^3,\ [qp] = \mathrm{L}^3\mathrm{T}^{-1}. \tag{15}$$

Adjustment constants must be dimensionless. Blaszczynski[8] does not specify whether a and b are dimensionless or not. For this reason, dimensional verification is impossible. Important in this example is the combination which replaces the runoff-rainfall erosivity R, and which appears in most USLE variants: Qqp. This combination dimension is $[Qqp] = \mathrm{L}^6\mathrm{T}^{-1}$. Many versions of the MUSLE model contain this factor to the power 0.56. Because the power is rational number (not integer) the basis must be a dimensionless physical quantity. If we accept this situation then we accept the physical quantity Qqp which have the dimension $\mathrm{L}^{3.36}\ \mathrm{T}^{-0.56}$. From the viewpoint of dimensional analysis, this factor is unacceptable. Dimensional error propagates in the formula MUSLE, so the sediment yield, will have a physical dimension in disagreement with dimensional analysis. The situation is similar for models defined by Eqs. (5), (8) and (9), (10) and (11) and (14). The model MUSLE defined by the equations (6) and (7) is described insufficiently precise, and the model MUSLE described by the Eq. (12) and (13), contain the term of q_p (maximum runoff rate) rise to power 0.333. If the constant 0.349, which is multiplied by the factor $Qq_p^{\ 0.333}$ has no physical dimension $\mathrm{ML}^{-2.999}\ \mathrm{T}^{-2.667}$, it is clear that formula (13) is dimensionally incorrect. All the three terms of the formula (14) must have same physical dimension, but this is impossible according to the principles of the dimensional analysis.

Many equations describing the mathematical models in the field of soil erosion and hydraulics contain such disagreements with dimensional analysis[16–19], which is the RUSLE2 documentation[20], which is the base for EUROSEM model.

Situations of this kind appear in many areas of science and mathematical relationships with dimensional defects are widely used in applications[21–23].

RESULTS AND DISCUSSION

Using π theorem of dimensional analysis, is possible to obtain interesting solutions to replace rainfall erosivity of the USLE model, R, with the physical quantities that better reflect the variation of impact energy on the ground during a rain erosion event. There are several possible and rational solutions:

$$R = \rho e^4 , \quad (16)$$

$$R = \frac{qe^3}{I^2}\rho , \quad (17)$$

$$R = \frac{q^2e^2}{I^4}\rho , \quad (18)$$

where Q is runoff volume, in m^3, q is flow rate in m^3s^{-1}, I is rainfall surface density, in m, e is rainfall intensity, in ms^{-1}, and ρ is rainfall water density. Now, a MUSLE formula can be written as Eq. (19):

$$Y = \alpha RKLSCP , \quad (19)$$

where R is given by one of the Eqs. (16), (17) or (18), α is a constant coefficient (dimensionless), and Y is the sediment yield, in kg $m^{-2}s^{-1}$. The coefficient α can be used to adjust the experimental data but can also depend on soil characteristics and be included in formulae for soil erodibility, K.

The three variants of the MUSLE soil erosion model were tested using experimental results. The experimental results were obtained in Valea Calugareasca vine area by [24]. Experimental and theoretical results are given in Table 2. The calibration of the three variants was done using the method of least squares. The coefficients of correlation between the experimental results and the prediction of the each variant MUSLE [variants given by the Eqs. (16), (17) and (18)], in the order they were stated, are: 0.873, 0.233, 0.196. In the same order, the optimal coefficient α was: $2.214\cdot10^{15}$, $2.689\cdot10^{9}$, $1.517\cdot10^{3}$. The best approximation for the experimental results is calculated using the formula (16) for the rainfall erosivity, R.

CONCLUSION

In the literature dedicated to the soil erosion, there are many formulas in disagreement with the principles of dimensional analysis.

Extensive use of formulas for estimating the risk of erosion, which are inconsistent with the principles of dimensional analysis, together with simultaneous use of multiple systems of units make use of cumbersome and slow, even for quick calculation programs.

All inconsistencies with the principles of dimensional analysis can be solved, but these should be accepted, completed and corrected by the users which determine the erosion risk maps in the world.

Table 2

Main parameters of the experiments carried out in Valea Calugareasca vineyard plantation.

Time total wetting, s	Time after beginning sediment flow, s	Surface watered plot, m^2	Average slope, %	K $m^{-3}\ s^3$	L	S	C	P	Sediment yield, $kgm^{-2}s^{-1}$	
									measured	computed
1464	480	84	7.1	0.033	0.795	0.716	0.460	1	$2.002\cdot10^{-5}$	$1.168\cdot10^{-5}$
8292	7320	84	7.1	0.033	0.795	0.716	0.216	1	$4.218\cdot10^{-6}$	$5.482\cdot10^{-6}$
14409	13980	54	10.13	0.033	0.638	1.189	0.135	1	$2.890\cdot10^{-5}$	$2.674\cdot10^{-5}$
3530	3120	54	10.13	0.033	0.638	1.189	0.135	1	$2.366\cdot10^{-5}$	$2.674\cdot10^{-5}$
23875	22920	78	7.4	0.033	0.766	0.758	0.279	1	$2.903\cdot10^{-6}$	$9.715\cdot10^{-6}$

Corrections of the dimensional analysis to the formulas for calculating soil loss by erosion, USLE and MUSLE, are essentially related to:

– constant coefficients for adjustment of the formulas with experimental data, must be dimensionless;

– argument functions must be dimensionless combinations of various physical quantities involved in the process of the soil erosion.

The adjustment constant coefficients may depend on parameters that are not caught in the model, and their consideration lead to widening of the model.

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