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NEIGHBORHOODS IN AND AMONG PHENOMENOLOGICAL CATEGORIES

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After the notion of neighborhood in the structural domain of sets is reminded one examines the neighborhoods problems for structural categories. In the structural domain there are metric and non-metric neighborhoods. In the phenomenological domains there are only non-metric neighborhoods. All the objects that have phenomenological morphisms with the given object define the neighborhood of an object in a phenomenological category. The neighborhood of a category is given by all the categories that have phenomenological functors with the given category. In the paper is also examined the notion of point in all the above cases. Some considerations are presented concerning the nature of phenomenological mathematical objects.

1. INTRODUCTION

The main types of phenomenological categories were examined in a previous paper [1]. By phenomenological [2], [3], [4] one understands:

- All processes and parts of reality that are not structural and have an informational character are phenomenological. Structures of only phenomenological elements are also phenomenological [5].
- With reference to the man, the phenomenological is the experiential and qualia. In general, it is a sensibility of matter, of a fundamental type of matter (informatter). This sensibility is a physical process, being at the same time a phenomenological information called also phenomenological sense. The contemporary meaning of phenomenology is that of a domain that investigates knowledge and practice of experience and phenomenological senses, in general [6]. See other commentaries in [7].

In Fig.1 it is shown how the phenomenological categories $C_{phe\ univ\ 1}$, $C_{phe\ univ\ 2}$, … of the universes in existence are objects of the Fundamental Category of Existence $C_{phe\ !1!}$. Inside the phenomenological category of a universe there are phenomenological categories of minds in that universe (for example, $C_{phe\ m\ 1}$, $C_{phe\ m\ 2}$, … in $C_{phe\ univ\ 1}$) and, of course, phenomenological categories corresponding only to the structures of that universe. One of the phenomenological categories in $C_{phe\ !1!}$ might be the phenomenological part of the Fundamental Consciousness of Existence [2]. Others could be free phenomenological categories [2].

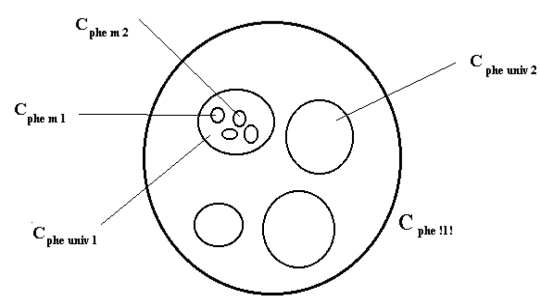

Fig.1

Fig.1 is only a scheme to order somewhat the understanding of processes at the phenomenological level, because the phenomenological reality is not a space with dimensions in the geometrical sense. The aim of this paper is to examine the neighboring properties of various phenomenological categories, inside and among them.

2. NEIGHBORHOODS IN THE STRUCTURAL REALITY. THE CASE OF SETS.

It is known that if one defines a topological space on a set X, the members of X are called points. A topological space, defined on a set, is < X, T > where X is the set and T, the topology, is a structure on the set X. Often, these notions, topological space and topology, are used as equivalent [8].

A space is made of the points x å X and of a defined structure on X. A metric space is < X, d > where d is the metrics (or the distance function, which is a function on XxX). The metrics d has the properties [9]:

1. d(x,y) = 0 if x = y ;
2. d(x,y) = d(y,x), property of symmetry;
3. d(x,z) d(x,y) + d(y,z), triangle inequality.

Consequently, d is named the distance between x and y.

If r å X , where r is a real positive number, the set

S(z,r) = {x åX | d(z,x)<r}

is named open sphere with center z and radius r. Open spheres are a basis for a topology on X..

A *topology* T on the set X is T P(X) where P(X) is the power-set (the set of all subsets of X, which contains X) - such that if Q T then Q T (T is closed under arbitrary unions) and, if A, B T then A B T (T is closed under the intersection of any finite number of its members).

A basis for a topology allows the construction of that topology, or a basis generates a topology. If B is a basis for a topology, B T, and the members of B are basic open sets [9].

In the case of a metric space, the open spheres defined above are a basis for a topology on X, a topology associated with the metric d. The metric space is a topological space.

In a metric space < X, d > one defines **a set Y containing all the points sufficiently near of a point** z Y if for some r>0 the open sphere S(z,r) is included in Y. Y contains all sufficiently points close to z if and only if [9] there is an open set Z such that z Z Y.

For a general space *(general topological space)* <X, T>, by definition, **Y is a neighborhood of a point** z, if for some open set Z the condition

z Z Y,

is satisfied.

It may be observed that there are two types of neighborhoods: metric and non-metric.

A point may have many neighborhoods. **The set of all neighborhoods of a point**, in a general space, is called a **filter** [9].

If two topological spaces share the same topological properties they are **homeomorphic**: 'The homeomorphism of topological spaces is clearly a particular instance of the isomorphism of general structures' [9].

If M and N are two sets and F : M N, and <M,T> and <N,T'> are topological spaces, for a point z M, F **is continuos at** z if for every x sufficiently close to z, F(x) is sufficiently close to F(z). F **is continuos** if it is continuos at every z M.

It may be observed [9] that for any neighborhood B of F(z) in T' there is a neighborhood A of z in T such as F(A) B.

These are the main notions concerning neighborhoods in the basic set theory and we followed above A. Levy [9] very closely.

Another way to define the points of a set is by using the map from a singleton set (a set with exactly one element) to an element of a set [10]. A point of a set X is a map from the **singleton set** (noted with **1**) to an element of the set: **1** X. This map is constituted of arrows from **1** to every element of the set X.

To proceed in such a manner to define points corresponds to the frame of thinking of category theory as explained by Lawvere and Schanuel in the following manner [10, p.225]: 'Our goal is to understand everything in terms of maps and their composition' (we might add, without mentioning members). And further [10, p.226]: 'The reason is that in other categorires, say dinamical systems or graphs, it is not clear what a member should be, but properties expressed in terms of maps and composition still make sense in any category'. Even for sets, 'everything that can be said about sets can be expressed in terms of maps and their composition' [10, p. 230].

3. NEIGHBORHOODS IN THE CASE OF STRUCTURAL CATEGORIES.

In the case of a structural category of sets C_S one define points *of objects* in the category. An object of C_S is a set. An element in a set X is a point of the set if there is a map **1** A where A X and **1** is a singleton set. An object in a category is not usually seen as a point.

In the case of structural categories, the objects of the category, in general, are not points.

A structural category C_S with objects that are abstract sets may have a special object (set) such that from every other object (set) there is exactly one map toward the special object (set). In such a case, the special object (set) is named **one point object 1**. This *is not a singleton set* as defined before, but a set inside the category, not external to the category. From every object B C_S there is exactly one map B **1.**

For *any* category C one defines [10], in a like manner, a ***terminal object*** S of C, if for *every* object D of C there is *exactly* one morphism (map) B S**.** The terminal object S is a special object of C and it is inside the category. The terminal object is the 'simplest universal mapping property' [10, p.213]. It is universal because it is in relation with all the objects B of the category C.

It is shown [10] that if there is a terminal object, it is unique: If S_1 and S_2 are both terminal objects in a category C, then there is exactly one map S_1 S_2, and that map is an isomorphism [10]. This unique terminal object is also noted with **1.**

If we look inside an object of a category, as it was shown, if the category is the category of abstract sets C_S, then every object being a set, it has points.

If the category is a general category of objects, then still may be defined points of its objects (but not points of the category). By definition [10], if the category C has an object terminal **1**, if D is any object of C then **1** D is named *a point of* D. The point of D is understood as a map, but also the reaching of an element (part) of D. Therefore, that part might be considered as a point. Once again, D is an object, which is not necessarily a set. Which is the significance of the *point* in such a case in which we can not speak about the points of a set? *It seems that we can speak about the elements of the object D, any nature they might have. Then, an element of D may be named a point. And an object has points.*

If one considers that in C every object has only one element, the category itself is a set.

For categories, the notion of point is used also in the case of the product of two objects of a category C. The product of objects D_1 and D_2 of a category C with a terminal object is an object P with two pair of maps p_1: P D_1 and p_2: P D_2 under some conditions [10, p.217] that will not be repeated here. The object D_1 has its points, the elements d_{1m}, the object D_2 has its points, the elements d_{2n}, the object P has the points < d_{1m}, d_{2n}> where m = 1,2, …, v and n = 1,2,…, w. Every point < d_{1m}, d_{2n}> , and there are v×w points, is not a simple element. It is a pair of two elements, each from one of the two objects D_1 and D_2.

What may be said about the neighborhoods for structural categories? There are three aspects:

Neighborhoods of the elements of an object in a category. If the object is a set, then the theory of neighborhoods in sets has to be applied, both for metric and non-metric spaces defined on them. If not, the problem is open for examination.

Neighborhoods among the objects of a category.

Neighborhoods among categories.

Considering the first aspect (neighborhoods of the elements of an object), in the case that the object is not a set, an element of the object may be treated as a point and perhaps the theory of neighborhoods in sets might be applied. In an intuitive way, some elements of the object may be related in some way to form a team, and as such they have a neighborhood. An element might have not only one, but many neighborhoods.

For the second aspect (neighborhoods among the objects of a category), it is necessary to change completely the perspective. What is important for neighborhoods is to have connections among objects, that is morphisms among them. Perhaps all the objects that are related by morphisms with the given object determine the neighborhood of an object. On the other side, an object in a structural category can not be seen as point.

For the neighborhoods among categories, following the ideas of the previous paragraph, only categories among which there are functors may be neighbors. A category may have strong neighborhoods with the categories with intense functors, or weak neighborhoods with categories having weak functors.

These structural notions for neighborhoods in and among categories may be not so important for the structural categories, but will become important for phenomenological categories.

4. PHENOMENOLOGICAL NEIGHBORHOODS.

In a phenomenological category [1] there are phenomenological objects and phenomenological sub-categories. We acknowledge the previous notions of neighborhoods for structural categories as being applicable also for phenomenological categories under the following formulations:

1. *Two phenomenological categories are neighbors if at least a functor is acting between them.*
2. *Two phenomenological objects in a category are neighbors if there are morphisms among them.*

The objects with which it has morphisms form the neighborhood of an object. It may happen that the entire phenomenological category to be a neighborhood for every of its objects.

If a phenomenological category together with a structural category is forming an object of reality (universe, human etc) they are not neighbors, they are *coupled.*

In a phenomenological category the elements of its objects may be treated as points as in the previous chapter for structural categories. *But there is no metrics in the phenomenological category*. Some phenomenological neighborhoods, that are always non-metric, may transform in metric neighborhoods in the equivalent structural category in a universe. But not all the phenomenological neighborhoods may know such a transformation. For instance, the phenomenological category of a mind, has phenomenological neighborhoods without corresponding metric neighborhoods in the brain.

All the universes in existence have phenomenological categories that are sub-categories of the Fundamental Phenomenological Category of Existence. All the phenomenological categories of universes are in the neighborhood of the fundamental monoid of existence [12]. Otherwise stated, all the phenomenological categories of the universes are neighbors with respect to the fundamental monoid of existence <1>.

Two phenomenological categories of two universes may be neighbors if there are functors between them. The phenomenological objects of a universe correspond to the structures of the universe and to the minds in that universe [1, where any living being is considered roughly a mind]. If a Universe is in a phenomenological neighborhood with another Universe, there is also a connection among the objects of the two phenomenological categories of these universes. The neighborhood between two phenomenological universes is transposed, by the functors between these, on the objects involved in functorial operations.

In particular, as every universe is in the neighborhood of <1>, any object of every universe is in the neighborhood of <1>.

A universe is born in the phenomenological category of the entire existence as a phenomenological category with an originally family of phenomenological objects and enters in a dynamics of coupling with orthoenergy to form the structural part of the universe. If this coupling does not take place, then the phenomenological universe is annihilated [13].

If the above mentioned coupling is done, the structural universe has metrics, and also an energetically metrics (that is values of energy, because the deep energy- the orthoenergy- does not have measure of energy). Concerning the metrics of the structural universe, one may ask if this metrics could be transposed on the phenomenological objects. The non-locality of elementary particles proved in quantum mechanics proves that this is not possible. In any case, not in totality. Two particles that are at a distance in the 3-dimensional space, they have no distance between them for some of their properties [14]. What is deep is not metric. But some neighborhoods in the deep phenomenological realm may transform into metrics in the structural domains.

A phenomenological sense determines some neighborhoods. Another phenomenological sense (of the same phenomenological object!) may determine other neighborhoods. Accordingly some phenomenological neighborhoods may determine a metrics in the structural realm, others may determine other metrics, or no metrics at all.

In the case of the phenomenological category of the mind [1], its phenomenological objects are in zones of neighborhood if they are connected by morphisms. These connections depend in part, for some objects, perhaps of their intentions to have connections with other phenomenological objects of the mind, because an intention is a phenomenological sense. As it was shown in [1] the phenomenological object of the phenomenological category of the mind, may belong to the subcategory related to the structural part of the brain (in general, of the living body) or to the subcategory related to the informational structural part of the brain (in general, living body). The dynamics of morphisms in the phenomenological category of the mind may create neighborhoods of some objects from the two subcategories mentioned above by phenomena of intention and will manifested as phenomenological senses and corresponding functors and morphisms.

Much more, the neighborhoods of an entire phenomenological category of a mind with the phenomenological categories of other minds in an universe might have the same plasticity, intentionality and tendency of neighborhood like that of a phenomenological object in a mind. Then, not only with other minds, but this might be possible also with the universe, and even with the Fundamental Consciousness of Existence. And inversely, a phenomenological mind may be approached by other phenomenological minds for establishing a neighborhood that may origin functors with various properties.

5. MAY WE RENOUNCE TO THE NOTION OF POINT?

Saunders Mac Lane is writing about the structural category theory:

'Since a category consists of arrows, our subject could also be described as learning how to live without elements, using arrows instead' [11, preface to the first edition, p.vii].

Arrows means maps (morphisms) and it may seem that elements (of objects) as points, and may be even objects, are to be overlooked. But this is not quite the case because even points may be understood as maps as it was shown in previous paragraphs. Of course, mathematically we can deal only with maps (arrows) and the question is if this has an important physical and informational sense for the phenomenological domains.

A phenomenological object may have, in the phenomenological realm, one element as a unique phenomenological sense or many elements if it has a number of phenomenological senses. These elements have an important physical and informational significance. They can not be overlooked from this point of view. Perhaps only for some mathematical treatments they might be overlooked but their presence has to be restored for the physical and informational interpretation of the results.

If elements, objects and categories are, perhaps, as essential, as morphisms and functors in the phenomenological domain, they have to be present as such in a mathematical treatment, at least in initial and final phases of this treatment.

The question is what type of mathematical object is a phenomenological category?

It is known that a mathematical object has a formal part and a mental part for the mind dealing with it. The mental part has a phenomenological component, a phenomenological category in itself for the respective mathematical object.

Treating the dynamics of the phenomenological categories as phenomenological mathematical objects, in general, not related to formal structural objects, this might become a fundamental problem. Will really be possible only to use the theory of categories and functors as a language for the phenomenological reality, or it will be possible to become a new chapter of mathematics extending the structural mathematics? Perhaps, the notion of point would not be convenient for such a 'phenomenological' mathematics. But the reality in totality is structural-phenomenological and an integrative mathematics [15] will open the best way of analyzing the structural-phenomenological categories as mathematical objects.

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