THE PUBLISHING HOUSE
OF THE ROMANIAN ACADEMY

PROCEEDINGS OF THE ROMANIAN ACADEMY, Series A,
Volume 6, Number 1/2005, pp. 000-000

TWO FUZZY ECONOMIC MODELS WITH NONLINEAR DYNAMICS

Horia-Nicolai TEODORESCU *[1,2], Marius ZBANCIOC [1,2]

[1] Technical University of Iasi
[2] Institute for Theoretical Informatics of the Romanian Academy
E-mail: hteodor@etc.tuiasi.ro, zmarius@iit.tuiasi.ro

Abstract. Successive steps of evaluation of the results and of the environment where the deciders act and successive decisions produce intricate dynamic evolutions that can be described, analyzed and predicted using adequate models. Complex human behaviors, like decision-making, often imply intricate feedback based rational and emotional processes. Fuzzy logic and rules may be used to model such decision-making processes and similar judgments. We propose new dynamic models for economic processes based on fuzzy logic rules involving feedback and iterative processes. Several analytical aspects of models and their dynamic behavior are investigated.

1. INTRODUCTION

Decision-making represents a complex process, involving rational, emotional, social and cultural factors [1], as well as evaluations of the results of the decision and evaluations of the environment where the decider acts. Most decision-making processes are dynamic, in the sense that iterative decisions are made, each decision being made taking into account results and situations that are time dependent and that have various time lags. When several decision-makers compete and make decisions based on their own results and on the actions of the competitors, the overall process may exhibit complex dynamics that are not easy to predict. Complex human behaviors, like decision-making, often imply intricate feedback based rational and emotional processes. This fact is well documented in the literature, see for example [1].

We use fuzzy logic and rules to model such decision-making processes and similar judgments. Here, we propose fuzzy models for time-dependent decision-making processes with several competing players in a market and we analyze the dynamics produced, demonstrating that the dynamics may be chaotic or oscillatory. The models are based on the principle that the players seek for profit maximization, while following Herbert Simon's theory of bounded rationality and opportunism [2]. The second model shows that sometimes, exacerbated opportunism and limited rationality may lead to a decrease in profits. We investigate analytical aspects of the models and their nonlinear dynamic behavior.

This paper continues the research in dynamic fuzzy economic models as introduced by the first author in [3-5], and the modeling and simulation of the dynamics of market models with fuzzy rules in decision making, as reported in [6-9].

The organization of this paper is as follows. The basic model is presented in the second section. The analysis of the model and the derivation of the characteristic function are presented in the third section. In the fourth section, several examples and selected results for specified cases are discussed. The last section is devoted to conclusions.

Member of the Romanian Academy

2. THE ECONOMIC MODELS AND THE ADAPTATION STRATEGIES

In a previous research aimed to evidence the dynamics associated to the expert systems that operate in decision or control loop, we have implemented two simple economic models [9]. The original models we have tested have been two-company models. The companies were competing in selling the same product on the market, using various strategies related to the selling price, while taking decisions on lowering or increasing the price based on the observation of the market tendencies [9].

In this paper, we present extensions of the basic models in [3-10]. The new models allow us to simulate N companies on the market performing in discrete time. Each company can use an increment for the price variation to be fixed or variable (with a fuzzy value). The increment depends on the profit the respective company has obtained at the previous steps, on the prices the other companies used at previous time moments, and possibly on the profits the other companies had – as estimated by the firm in focus. The two models differ by the strategy used by the players. It is assumed that all the actors have the same strategy in a model.

The models are intended to explain, under some simplified hypotheses, the dynamics of the prices on the market, when different vendors sell similar products. It is assumed that the vendors can monitor – with some delay – each other through the prices they practice on the market and possibly by learning the profit the competitors have.

The two strategies of adjusting the prices are subsequently named *comp-profit* and *max-profit*, respectively. According to the first strategy, the company tries to determine on the market the product selling-price used by the other companies at every time moment, and to guess the profit the other companies have had. Based on this information, each company using *comp-profit* strategy adjusts its product selling-price with the goal to increase the own profit with respect to the average profit. The overall model based on the first strategy is shown in Figure 1.

In the second model, all the players use the *max-profit* strategy. According to the *max-profit* strategy, only the prices practiced by the competitors are relevant and the only target is the maximization of the own profit. Both strategies would ideally conduct to the maximization of the profit, but the way of adjusting the prices differs. The *comp-profit* strategy reflects an “envious” thinking, while the *max-profit* reflects a more “objective” behavior. Notice that, in both strategies, there is a delay between the moment of learning the prices the competitors used and the estimated profits they had.

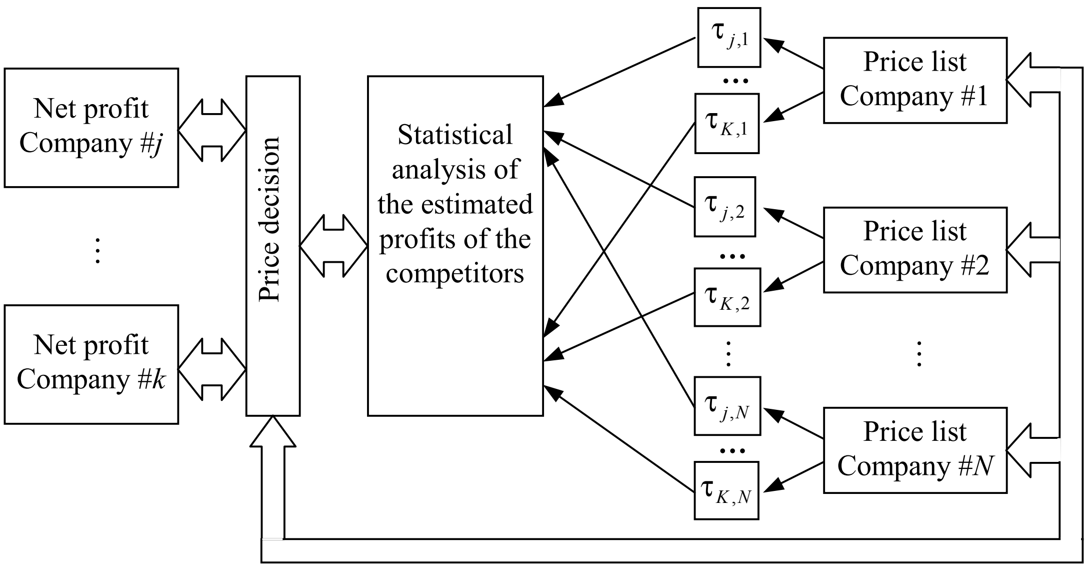

Fig. 1. Schematic diagram of the system with several companies playing according to the strategy *comp-profit*

The model input data are the initial prices, the matrix $[\tau_{ij}]$ of the delays (the company #*i* learns with a delay τ_{ij} the prices and the profits of the company #*j*), and the strategy chosen. As normal, the first diagonal of the matrix has null elements. The matrix may be asymmetrical, as the companies may have different abilities of learning the information on the other actors on the market. (The user of the software application we have developed for the modeling has to specify both the type of increment and the type of strategy. An implicit choice – *comp-profit* and *fixed increment* – is available.)

In the sequel, we use the following notations: the prices used by the company #*i* at time moment *k* are denoted by $p_i[k]$, while the initial prices are assumed equal for the first time moments, $p_N[0] = p_N[1] = ... = p_N[d_N]$.

At every time moment, each company determines its profit and, in the *comp-profit* model, evaluates the profits of the other players. According to the second model (comp-profit), every company compares its profit to the estimated profits of the other players and makes a decision based on these data.

3. MODEL WITH THE STRATEGY FOR PROFIT MAXIMIZATION

Fixed increment model

At every time moment *t*, the players determine their profit as a function of their current prices and of the most recent known prices of the competitors:

$$b_{k,1}[t] = f\left(p_k[t], p_1[t-\tau_{k,1}]\right),\ \ldots,\ b_{k,N}[t] = f\left(p_k[t], p_N[t-\tau_{k,N}]\right) \tag{1}$$

Then, the company estimates its profit as an average of the values obtained as above:

$$b_{med\ k}[t] = \frac{\sum_{i=1}^{N} b_{k,i}[t]}{N-1}, \quad i \neq k \tag{2}$$

These first equations are the "profit equations" in the model. The method of computation the profit may look oversimplified. However, human deciders, when submerged by too many data, have difficulties in estimation. They prefer to judge in simplified situations, like one-to-one competition and then to aggregate data. This is the rational for the above formulas, which show that the deciders estimate their profit based on a set of comparisons with other individual competitors, then they average the estimations.

To make a decision about the opportunity of price variation for its products, the company will evaluate its change in profit when a price change is made. We use two sub-models, related to the manner of price change at any time moment. In the first sub-model, the price change is fixed. In the second, the change is computed according to a fuzzy evaluation process, which will be explained latter. Disregarding how the increment is determined (fixed, or by a fuzzy procedure), the equations for the evaluation of the effect of price change at time moment *t* are:

- For the case the price is increased, assuming the other players maintain the same prices:

$${}^{+}b_{k,1}[t] = f\left(p_k[t] + incr, p_1[t-\tau_{k,1}]\right),\ \ldots\ ,\ {}^{+}b_{k,N}[t] = f\left(p_k[t] + incr, p_N[t-\tau_{k,N}]\right)$$

$${}^{+}b_{med\ k}[t] = \frac{\sum_{i=1}^{N} {}^{+}b_{k,i}[t]}{N-1}, \quad i \neq k\ .$$

where *incr* denotes the increment of the price change (positive, negative or zero increment).

- For the case the price is decreased, assuming the other players maintain the same prices:

$${}^{-}b_{k,1}[t] = f\left(p_k[t] - incr, p_1[t-\tau_{k,1}]\right),\ \ldots\ ,\ {}^{-}b_{k,N}[t] = f\left(p_k[t] - incr, p_N[t-\tau_{k,N}]\right)$$

$${}^{-}b_{med\ k}[t] = \frac{\sum_{i=1}^{N} {}^{-}b_{k,i}[t]}{N-1}, \quad i \neq k\ .$$

For the case of no price increase (zero increment), it is assumed that the profit at the current time moment, as computed with (1) and (2), is preserved. The prices are modified with the increment that maximizes the profit at the next step. These computations are made for all the players in the model, at every time moment. In this way, the dynamic of the system of *N* players is obtained for the desired duration.

In the implementation of the model, the subsequent algorithm has been used:

Initialize the lists of prices for the N companies. Initialize the fixed increment or chose a fuzzy increment. Initialize the number of time steps, *P*, $p \leftarrow P$;

(1) *while* ($p \geq 1$) *do*

(2) *for* k = 1 to *N*, sequentially select each of the *N* companies and compute the average profit of the company at the time moment *t*, as well as the estimation of the profits obtained after incrementing the price.

$$b_{med\ k}[t]\,, {}^{+}b_{med\ k}[t] \quad {}^{-}b_{med\ k}[t]$$

(3) Determine the best strategy, according to

$$\text{If } \max\left\{b_{med\ k}[t],\ {}^{+}b_{med\ k}[t],\ {}^{-}b_{med\ k}[t]\right\} = \ = \begin{cases} b_{med\ k}[t] & p_k[t+1] = p_k[t] \\ {}^{+}b_{med\ k}[t], \text{then} & p_k[t+1] = p_k[t] + incr \\ {}^{-}b_{med\ k}[t] & p_k[t+1] = p_k[t] - incr \end{cases}$$

(4) $p \leftarrow p-1$, *return to step 3*

The only rule used to modify the prices is asking for the profit maximization:

R: *Modify the price the company sells the product to maximize the profit.*

Fuzzy increment model

The fuzzy increment method relays on the estimation, for every company, of the average profit $b_{med\ k}[t]$ and of the averaged "delayed profit" of the concurrent firms, $b_{med\ delayed\ \bar{k}}[t],\ k = \overline{1,N}$. The "delayed profits" are computed as:

$$b_{1,k\ delayed}[t] = f\left(p_1[t-\tau_{k,1}], p_k[t]\right),\ \ldots,\ b_{N,k\,\text{delayed}}[t] = f\left(p_N[t-\tau_{k,N}], p_k[t]\right)$$

and the average of the "delayed profits" of the companies concurrent to the company #*k* is

$$b_{med\ delayed\ \bar{k}}[t] = \frac{\sum_{i=1}^{N} b_{i,k\ delayed}[t]}{N-1}, \quad i \neq k\ .$$

Notice that both models do not take into account the general theory of demand and supply. The model is suitable for small markets, namely for sets of resellers, with a speculative behavior, as encountered in developing countries. These are "local" models. Based on $b_{med\ k}[t]$ and $b_{med\ delayed\ \bar{k}}[t]$, the increment $incr_k[t]$ is determined based on the rules described in Table 1 (Mamdani-type fuzzy system) and using the membership functions illustrated in Fig. 2 and 3.

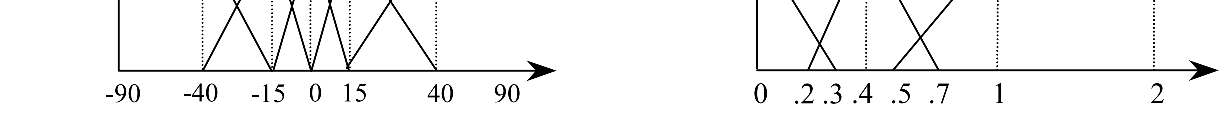

Fig. 2. Membership functions for the linguistic variables "*dif_profit*" and "*increment*"

Table 1 Rules for computing the fuzzy increment using price difference

dif profit	large negative	negative	zero	positive	large positive
increment	large	average	small	average	large

After the fuzzy increment is estimated, it is defuzzified and used in the procedure, like a fixed increment.

4. STRATEGY *"COMP-PROFIT"*

The behavior of the company, according to this strategy, is different and could be named "envy-guided behavior", as described in the first section.

The equations to compute the profit are the same as described for the first strategy. The major departure from the previous model consists in the way the price is modified. The price change is determined in this strategy based on the comparison of the own profit to the average estimated profit of the competitors, beyond the computation of the own profit. Precisely, the rules governing the strategy are:

R1: IF the own average profit is lower than the average "delayed profit" for the concurrent companies
AND the own selling price is lower than the average of the prices practiced by the other companies,
THEN increase the selling price.

R2: IF the own average profit is lower than the average "delayed profit" for the concurrent companies
AND the own selling price is higher than the average of the prices practiced by the other companies,
THEN decrease the selling price.

R3: IF the own average profit is higher than the average "delayed profit" for the concurrent companies
THEN evaluate the profits obtained by increasing or decreasing the selling price and choose the price change that maximizes the profit.

The average price for the concurrent firms, which is needed in the rules R1 and R3, is computed with some delay, based on available information, according to:

$$p_{med\ \bar{k}}[t] = \frac{\sum_{i=1}^{N} p_i[t-\tau_{k,i}]}{N-1}, \quad i \neq k .$$

The algorithm applied in the simulation of this model is:

(1) Initialize the lists of prices for the N companies. Initialize the fixed increment or chose a fuzzy increment. Initialize the number of time steps, *P*, $p \leftarrow P$;

(2) *while* ($p \geq 1$) *do*

(3) *for* k = 1 to *N*, sequentially select each of the N companies and determine the average profit, as well as the own profit at time moment *t*.

(4) Estimate, based on "delayed prices" (prices learned with delay), the profits obtained by the concurrent firms,

$b_{med\ k}[t]$ and $b_{med\ delayed\ \bar{k}}[t]$

(4) Modify the prices applying the strategy according to the rules R1, R2, R3, using the *Procedure_price*.

(5) $n \leftarrow n-1$, *return to step 3*

Procedure_price (company #*k*)

$p_k[t+1] = f\left(p_{med\ \bar{k}}[t]\ ,\ b_{med\ k}[t], b_{med\ delayed\ \bar{k}}[t]\right)$

if $b_{med\ k}[t] < b_{med\ delayed\ \bar{k}}[t]$ // profit lower than that of the concurrent firms

then if $p_k[t] \le p_{med\ \bar{k}}[t]$

then $p_k[t+1] = p_k[t] + incr$

else $p_k[t+1] = p_k[t] - incr$

if $b_{med\ k}[t] \ge b_{med\ delayed\ \bar{k}}[t]$ // profit higher than that of the concurrent firms

then compute the profits $b_{med\ k}[t+]$ and $b_{med\ k}[t-]$, and determine the price $p_k[t+1]$ according to $\max\{b_{med\ k}[t],\ b_{med\ k}[t+],\ b_{med\ k}[t-]\}$.

$$If\ \ \max\left\{b_{med\ k}[t],\ {}^{+}b_{med\ k}[t],\ {}^{-}b_{med\ k}[t]\right\} = \begin{cases} b_{med\ k}[t] & p_k[t+1] = p_k[t] \\ {}^{+}b_{med\ k}[t] & ,then\ \ p_k[t+1] = p_k[t] + incr \\ {}^{-}b_{med\ k}[t] & p_k[t+1] = p_k[t] - incr \end{cases}$$

end_procedure

The simulations of these models have been made using an application we developed in FuzzyCLIPS™6.1 (a language designed for rule-based fuzzy reasoning).

To compute the profits, Mamdani-type rules with two input linguistic variables have been used (see the Annex). The inputs to the rules, x_1 and x_2, are the current price used by the company under focus and the price used by a concurrent company (the later being known with some specified delay). The fuzzy output variable is the profit y of the company under discussion. Recall that in the computation of the fuzzy increment, single-input single-output rules are used.

5. RESULTS

The graphs in Figures 4 and 5 represent the evolution of the prices and of the profits, respectively, for a system of N = 5 companies. The initial prices are p_1=8, p_2=9, p_3=20, p_4=16, p_5=12, the matrix of delays is τ = {{0 2 3 1 2}; {3 0 2 2 1}; {3 2 0 2 3}; {2 1 2 0 3}; {1 3 1 3 0}} and the strategy is *comp-profit*.

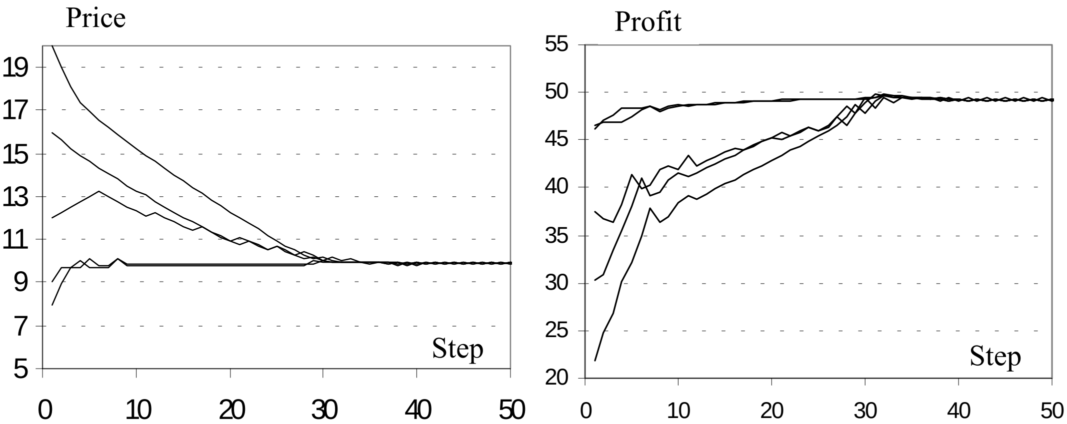

Fig. 3. Number of companies N=5; strategy: comp-profit, fuzzy increment (loop of period 2 obtained after 46 steps)

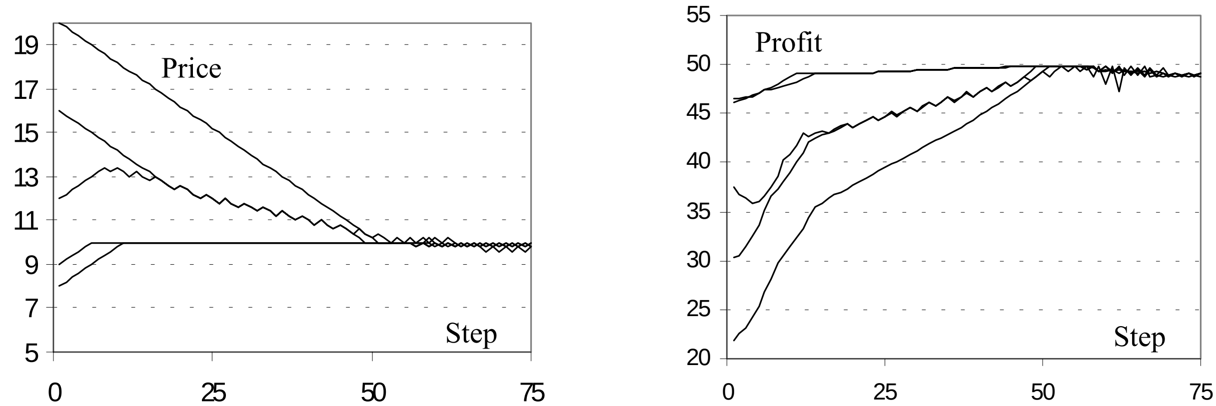

Fig. 4. Number of companies N=5; strategy: comp-profit, fixed increment (loop of period 2 obtained after 60 steps)

During simulations, we have noticed that the behavior tends to a stable set of prices, or tends to loops (cycles) of small periods (most frequently, period equal to 2). The stabilization is faster when using a fuzzy increment. The time spent until the stabilization (transitory regime) is longer for the fixed increment *incr* = 0.2. Small networks of players on the market tend to have loops with larger periods. While large periods may be obtained with networks with two companies, networks with 3 or 5 companies tend faster to stability or limit cycles – at least for reasonable delays, $\tau_{i.j} \in [0,4]$.

In Figures 6 and 7, the case of a loop of period 6 is illustrated, for a network with N=3 firms. The initial prices are p_1=8, p_2=20, $p3$=16, and the matrix of delays is τ = {{0 1 3}; {2 0 2}; {3 2 0}}. The strategy used in this example is *comp-profit*, with a fixed increment *incr* = 0.2.

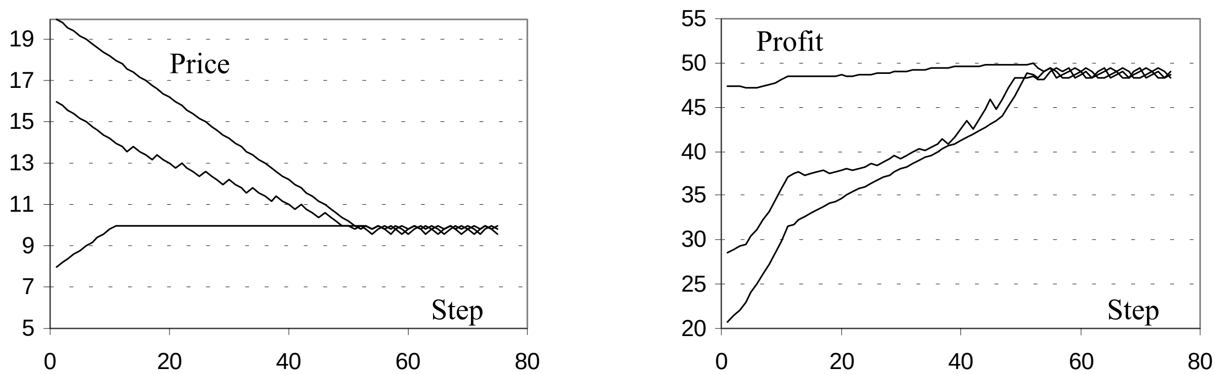

Fig. 5 Number of companies N=3; strategy: comp-profit, fixed increment fix (6^{th} loop after 55 steps)

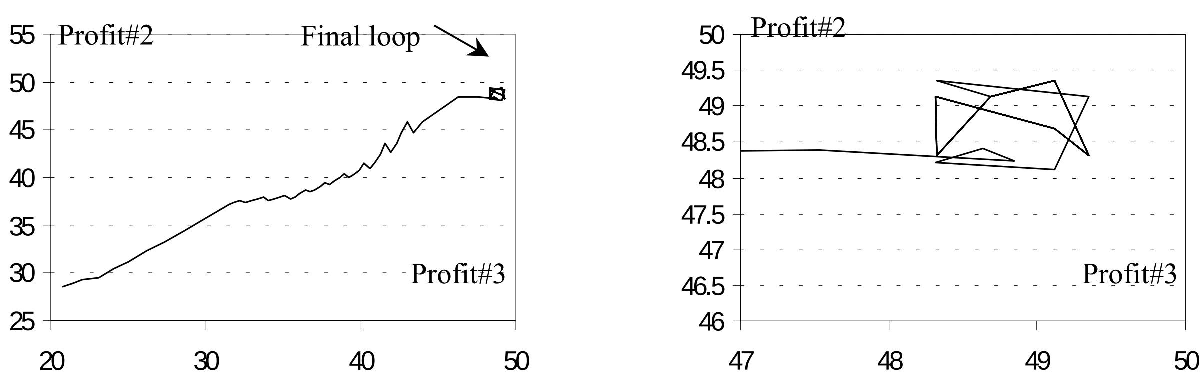

Fig. 6 Profit co-evolution of the Firm #2 and Firm #3 , with details of the region where the loop is produced

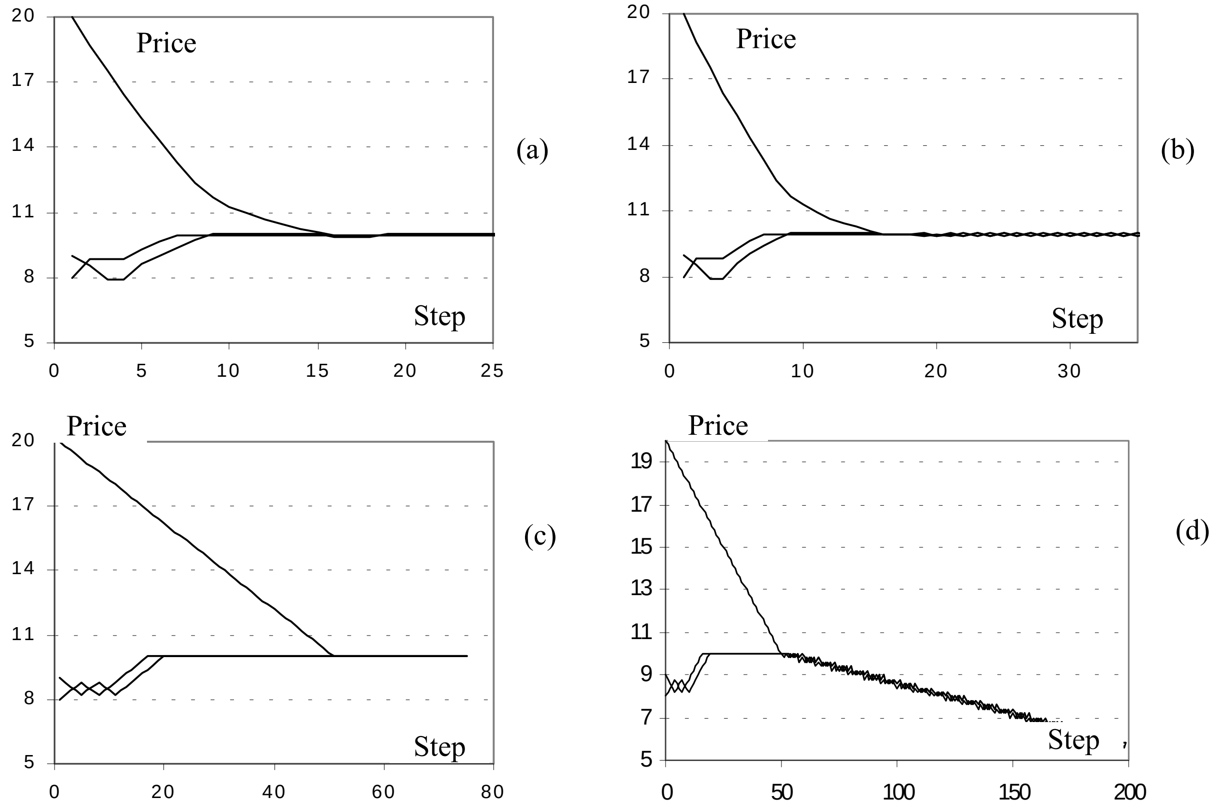

Fig. 7 Graph of the variation of the prices for N=3 firms. a) Strategy: *max-profit*, fuzzy increment (stable in p=10 after 22 steps). b) Strategy: *comp-profit*, fuzzy increment (period 2 loop reached after 35 steps). c) Strategy: *max-profit*, fixed increment =0.2 (stable in p=10 after 51 steps). d) Strategy: *comp-profit*, fixed increment = 0.2 (stable in p=4.4 after 246 steps)

The transitory regime generally lasts among 20 and 60 steps. In some simulated cases, however, at least 200 steps were necessary to reach a stable cycle or point.

For comparison, we illustrate in Figure 8 a network of three companies (N = 3), defined by the matrix of delays τ = {{0 2 2}; {2 0 2}; {3 3 0}}, and starting from the initial condition (initial prices) p_1=8, p_2=9, p_3=20. The four panels show the evolution of the three players for all the strategies and increment computation methods presented in this paper.

A striking conclusion that can be derived from this example is that, when companies adopt the most egocentric (envy-dominated) strategy, the result may be benefic for the customers and negative for the companies, because the prices will continuously fall up to a low value (panel d).

6. DISCUSSION

Our models differ in many respects from the ones appearing in the literature, yet the oscillatory dynamics has been revealed by many authors, e.g. [10]. The models, while based on quite simplifying assumptions, provide a useful insight on the process. They may help determining:

- how long it takes to players on the market to adapt to the best price, depending on the initial price they have proposed;
- what oscillations for the profit they may expect (with consequences on the fluctuations of the cash flow);
- what are good choices of the initial price when starting selling a product;
- how to smooth price variations;
- what is the best strategy they may adopt together with the other players, if agreement can be reached.

An essential part of the models is the fuzzy rules to determine the profit of the actors. This rule is presented in [9] and is recalled in the Annex. Notice that the rule does not take into account the price paid by the actors (companies in the model, which are assumed to be resellers). The price they pay is assumed the same for all, constant, and much lower than the price they use to resell. This indicates that our model apply to speculative sellers, like the ones in small cities in a developing market, like in those in Eastern Europe.

7. CONCLUSIONS

We have presented two models for players competing in the market and trying to increase their profit. The two models differ by the strategy used by the players. For each of the models, we discussed the effect of two manners for computing the change of the product selling-prices. The first method is based on a fixed increment variation, while the second method involves a fuzzy estimation of the best increment.

The models exhibit a nontrivial dynamic evolution of the market, with possible fluctuations lasting indefinitely. The simulation results for the two models demonstrate that the two models behave quite differently. This shows that strategy adopted by the market players has an essential role in the dynamics of the market.

An improvement to the models, to be performed in future research, is the use of both strategies in a single model, some companies adopting one strategy, while the others the other strategy.

ACKNOWLEDGMENTS.

A minor part of the research (simulation) for this paper has been partly supported by the Grant #155/2003-2004 of the Romanian Academy. The essential part of the research, including concepts and methodology, has been done by the first author independently of any other duty and it remains the intellectual property of the first author.

REFERENCES

[1]. CHRISTIAN LE BAS, *Expectations, interactions between agents and technological regimes. European J. of Economic and Social Systems* **15** N°1, 2001 77-87
[2]. *The Maximization Debates.* cepa.newschool.edu/het/essays/product/Maxim.htm
[3]. TEODORESCU, H.N., *Chaos in Fuzzy Expert Systems*, Proc. Fifth IFSA Congress (1993), Seoul, pp. 745-747
[4]. TEODORESCU, H.N., *Chaos in fuzzy systems and signals*, Proc. 2nd Int. Conf. on Fuzzy Logic and Neural Networks. Vol. 1., pp. 21-50, 1992, Iizuka, Japan
[5]. TEODORESCU, H.N. et al, *Analysis of Chaotic Trade Models and Improved Chaotic Trade Models*, Proceedings, 3rd International Conference on Fuzzy Logic, Neural Nets and Soft Computing, Iizuka, Japan, August 1-7, 1994, pp. 499-500
[6]. GIL LAFUENTE, A.M., GIL ALUJA, J., TEODORESCU, H.N., *Periodicity and chaos in economic fuzzy forecasting*, Vol. Fuzzy Systems. Proc. ISKIT'92, Iizuka, 1992. pp. 85-93
[7]. GIL ALUJA, J., TEODORESCU, H.N., GIL ALUJA, A.M., TACU, Al.P., *Chaotic Fuzzy Models in Economy*, Proc. 2nd Int. Conf. on Fuzzy Logic and Neural Networks. Vol. 1, pp. 153-156, 1992, Iizuka, Japan
[8]. GIL ALUJA, J., TEODORESCU, H.N., GIL LAFUENTE, A.M., BELOUSOV, V., *Chaos in recurrent economic control of enterprises*, Proc. First European Congress on Fuzzy & Intelligent Technologies. Aachen 1993. Verlag Augustinus Buchhandlung, Aachen, ISBN 3-86073-176-9. Vol. 1, pp. 982-986
[9]. TEODORESCU H.N., ZBANCIOC M., *The dynamics of fuzzy decision loops with application to models in economy.* Memoriile Secțiilor Științifice ale Academiei Române MAR Tome XXVI (2003) pp. 301-317
[10]. EMMANUELLE AURIOL and MICHEL BENAIM, *Convergence and oscillation in standardization games.* European J. of Economic and Social Systems **15** N° 1 (2001) 39-55

Received February 28, 2002

ANNEX. DETAILS ON FUZZY PROFIT COMPUTATION

The below definitions and rules have been used in the computation of the profit using fuzzy rules.

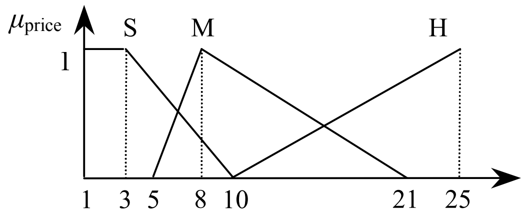

Fig. 8 Membership functions of the linguistic variable "*price*"

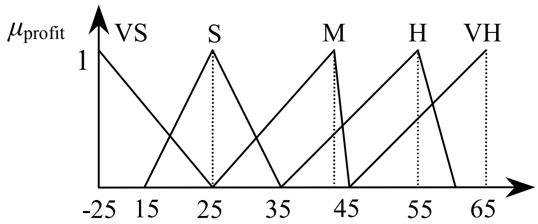

Fig. 9 Membership functions of the linguistic variable "*profit*"

Table 2 Rules for determining the profit as a function with variables the price P1 used by the focused company, and the price of the concurrent P2. The notations are: VS – very small, S – small, M – average, H– high, and VH – very high.

P1 \ P2	Small	Average	High
Small	M	More or less M	VH
Average	S	H	Somewhat H
High	VS	S	S