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ON DYNAMIC BEHAVIOUR OF THE ANTIVIBRATORY MATERIALS

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This paper present a nonlinear viscoelastic model for dynamic behaviour of the dissipative materials used for antivibratory isolation. Starting from the nonlinear integral form of the viscoelasticity law a nonlinear Kelvin-Voigt model with nonlinear stiffness and damping characteristics is obtained. Then, by using a global linearization criterion the equivalent linear characteristics corresponding to different excitation amplitudes result. This method leads to a good approximation of the nonlinear behaviour and it is convenient to use for current applications, especially in vibration control strategies.

1. INTRODUCTION

The dissipative materials used in antivibratory isolation systems exhibit a strong nonlinearity and a remarkable hysteretic damping capacity. Thus, a minimal mechanical model, which is able to describe these essential characteristics, is a dynamic nonlinear viscoelastic model with the acceptance of the phenomenological equivalence between viscous and hysteretic damping (reo-hysteretic hypothesis [4, 6].

A such nonviscous model was built in [5, 6] by using two dynamic nonlinear functions – one for material strength modeling and another including material damping, both in terms of strain level caused by external loading conditions. Because this model can be regard as an extension in the nonlinear domain for the non-viscous linear Kelvin-Voigt model [8], in the next we will use the denomination - the nonlinear Kelvin-Voigt model (NKV model).

The NKV model has a physical correspondent in the one-degree-of-freedom system of the resonant column (RC) apparatus [11]. The RC system can be considered as a one degree-of-freedom system that is made up of a single mass (the vibration device) supported by a spring and a damper represented by the specimen. But, both spring and damper have non-linear characteristics due to the mechanical properties of the specimen materials and thus the entire system is a non-linear one [5, 6] (fig.1.1).

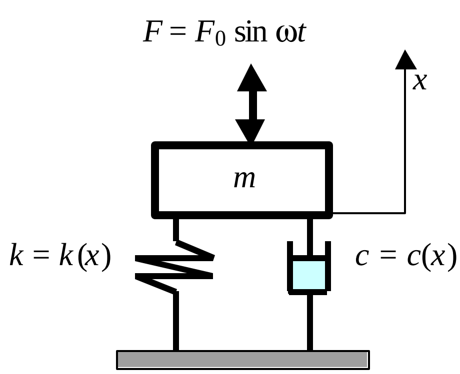

Fig. 1.1 NKV model

2. DYNAMIC MATERIAL FUNCTIONS

For the qualitative and quantitative evaluation of the dynamic material functions one can use the experimental data obtained from dynamic test performed in resonant column device [4, 6].

The resonant column apparatus was designed for laboratory determination of the dynamic response of soils by the means of the propagating steady-harmonic shear or longitudinal waves in a cylindrical soil

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specimen (***column***) under ***resonant*** frequency conditions [11]. The sample together with the vibration device is enclosed into a cell chambre where the confining conditions are supplied.

The same RC apparatus can be used not only with soil samples but also with any material samples. In the next, such determination will be illustrated using experimental resonant column data obtained from rubber sample tests.

In the Drnevich type apparatus, the bottom specimen end is fixed and at the top specimen base the vibration excitation device is attached (fixed-free end conditions). The response motion is picked up in terms of acceleration of the specimen-vibrator system. From the value of accelerometer output A, the impute current C and the resonant frequency f_r, as well as the sample geometry and end conditions one can obtains the amplitude of the longitudinal or shear strain invariants $(\varepsilon_0, \gamma_0)$; the velocity of longitudinal or torsional wave propagation (v_l, v_s); the rod E or shear modulus G and the longitudinal or torsional damping ratio (D_l, D_s):

$$\begin{aligned} &\varepsilon_0 = 1.756\cdot 10^{-1}\frac{A}{f_r^2}[\%] \;\; ; \;\; v_l^2 = \frac{\omega_0 h}{\Psi_l} \;\; ; \;\; E = \rho v_l^2 \;\; ; \;\; D_l = 0.25\cdot \frac{C}{A}\bigg|_{f=f_r}\cdot \frac{A}{C}\bigg|_{f=\sqrt{2}\cdot f_r}[\%] \\ &\gamma_0 = 6.59\cdot 10^{-2}\frac{A}{f_r^2}[\%] \;\; ; \;\; v_s^2 = \frac{\omega_0 h}{\Psi_s} \;\; ; \;\; G = \rho v_s^2 \;\; ; \;\; D_s = 0.25\cdot \frac{C}{A}\bigg|_{f=f_r}\cdot \frac{A}{C}\bigg|_{f=\sqrt{2}\cdot f_r}[\%] \end{aligned} , \qquad (2.1)$$

where ω_0 is the specimen natural pulsation, h is the specimen length, ρ is mass density of specimen, Ψ_l is the root of the longitudinal frequency equation and Ψ_s is the root of torsional frequency equation, both wit the same analytical form: $\Psi = \sqrt{R - \frac{1}{3}R^2 + \frac{4}{45}R^3}$ but with different R: in longitudinal case R is the ratio between weight of the specimen and weight of the top vibrator $R = W / W_{top}$ and in the torsional case R is the ratio between torsional inertia of the specimen and the torsional inertia of the top cap system : $R = J / J_{top}$.

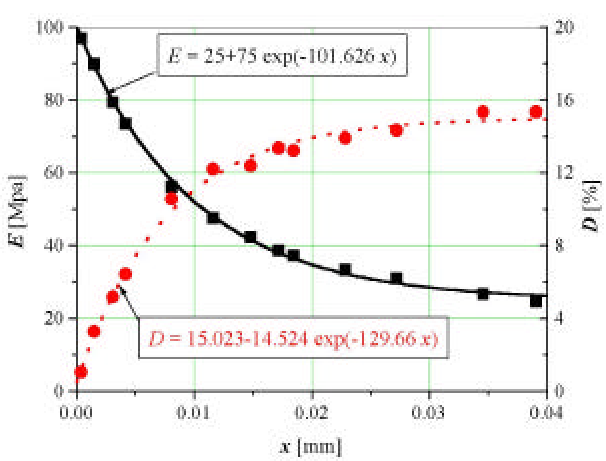

Fig. 2.1
Modulus and damping functions

In the above determinations, one has assumed constant amplitude of excitation and a constant cell pressure.

By changing these conditions, due to the nonlinear and dissipative behaviour of the material, another values of the above mechanical characteristics are obtained. As a result of several sequences of tests with varying loading level, a set of the modulus (E^i or G^i) values and damping (D_l^i or D_s^i) values corresponding to stain amplitude (ε_0 or γ_0) are obtained.

By statistical processing of these data one can obtain the modulus and damping functions in terms of strain: $E = E(\varepsilon_0)$ and $D_l = D_l(\varepsilon_0)$ or $G = G(\gamma_0)$ and $D_s = D_s(\gamma_0)$. Also, by using the relationships between strains and displacements of the sample, these material function can be express in terms of longitudinal displacement x or rotation θ: $E = E(x)$ and $D_l = D_l(x)$ or $G = G(\theta)$ and $D_s = D_s(\theta)$.

As an example, in fig. 2.1 are given the result of such determination using the test performed upon rubber sample in longitudinal harmonic loading conditions.

3. DYNAMIC BEHAVIOUR OF THE NKV MODEL

For the longitudinal harmonic vibration case, the differential equation governing the non-linear single degree-of-freedom of resonant column system can write as [5, 6]:

$$mx + c(x)\cdot \dot{x} + k(x)\cdot x = F_0 \cdot \sin \omega t\ , \tag{3.1}$$

where x is the system's displacement, m is the mass of vibration device, $c = c(x)$ is the non-linear damping function, $k = k(x)$ is the non-linear spring function, F_0 is external force amplitude and ω is the pulsation of this force.

In eq. (3.1) was used the same method that describes the nonlinearity by strain or displacement dependence of the material parameters:

$$k(x) = \frac{S}{h} E(x)\ \text{[N/m]}\ , \tag{3.2}$$

$$c(x) = 2m\omega_0 \cdot D(x)\ \text{[Ns/m]}.$$

where S is the cross sectional area of the specimen, h is the specimen's high, ω_0 is the natural pulsation of the specimen, $E(x)$ is the modulus function and $D(x)$ damping function of the specimen material.

By using the geometrical rubber sample and its mechanical characteristics from fig. 2.1 the strength and damping NKV functions results in the exponential form given in fig.3.1.

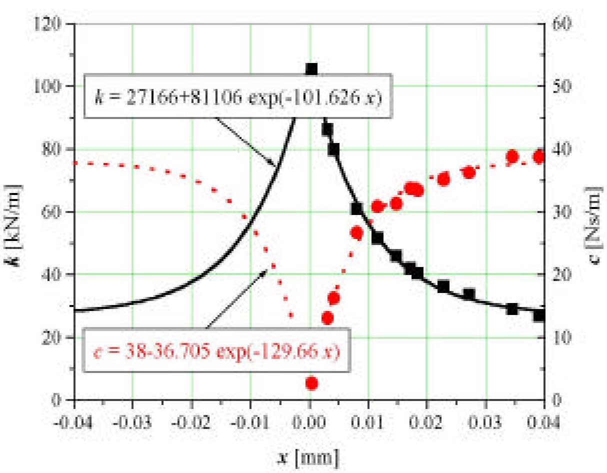

Fig. 3.1
Dynamic NKV characteristics

4. EVALUATION OF THE NON-LINEAR RESPONSE

By using the change of variable $\tau = \omega_0 t$ and by introducing a new "time" function [1]:

$$\varphi(\tau) = x(t) = x\left(\frac{\tau}{\omega_0}\right), \tag{4.1}$$

one obtains from eq. (1.2) a dimensionless form of the non-linear equation of motion:

$$\varphi'' + C(\varphi)\cdot \varphi' + K(\varphi)\cdot \varphi = \mu \cdot \sin \upsilon\tau\ , \tag{4.2}$$

where the superscript accent denotes the time derivative with respect to τ:

$$\varphi'(\tau) = \frac{\partial \varphi}{\partial \tau} = \frac{1}{\omega_0}\dot{x} \qquad ; \qquad \varphi''(\tau) = \frac{\partial^2 \varphi}{\partial \tau^2} = \frac{1}{\omega_0^2}\ddot{x}\ , \tag{4.3}$$

and:

$$C(\varphi)\equiv C(x)=\frac{c(x)}{m\omega_0}=2D(x)\quad ;\quad \mu=\frac{F_0}{m\omega_0^2}=\frac{F_0}{k(0)}=x_{st}$$
$$K(\varphi)\equiv K(x)=\frac{k(x)}{m\omega_0^2}=\frac{k(x)}{k(0)}=\frac{E(x)}{E(0)}=E_n(x)\quad ;\quad \upsilon=\frac{\omega}{\omega_0} \qquad (4.4)$$

For a given normalized amplitude μ and relative pulsation υ, the non-linear equation (4.2) can be numerically solved [9] and a solution of the form:

$$\varphi_{nl}(\tau;\mu,\upsilon)=\mu\cdot\Phi_{nl}(\upsilon;\mu)\cdot\sin(\upsilon\tau+\Psi) \qquad (4.5)$$

can be obtained. In eq. (4.5) $\Phi_{nl}(\upsilon;\mu)$ are the magnification function and Ψ the phase difference.

Since the magnification functions gives significant information about the dynamic system behaviour in fig. (4.1) several functions $\Phi_{nl}(\upsilon;\mu)$ obtained for tested rubber are shown.

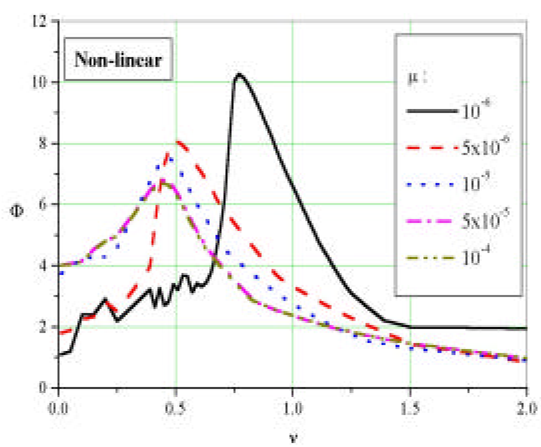

Fig. 4.1
Magnification functions for non-linear model

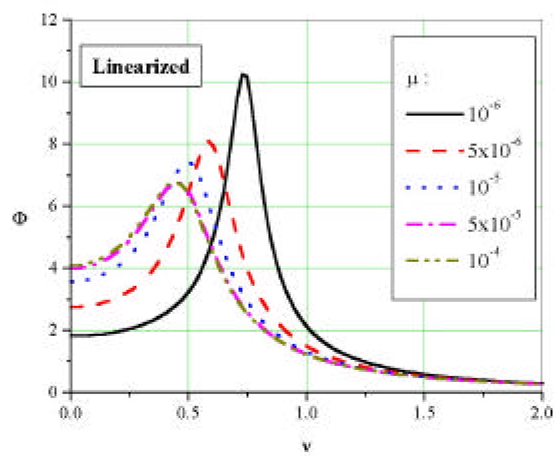

Fig. 5.1
Magnification functions for linearized model

5. EVALUATOIN OF THE EQUIVALENT LINEAR CONSTANTS

In many engineering problems the generation and solving of large non-linear systems is not justified. Also, in structural mechanics there are efficient computational methods, based upon the hypothesis of linear body, which are correct for the structures himself but are inadequate for the antivibratory materials that work together with these structures (especially in vibration control strategies where is no time for large non-linear system solving).

All this reasons justify the attempt to replace the NKV non-linear equation (4.2) by an equivalent linear equation:

$$\varphi''+\tilde{c}\cdot\varphi'+\tilde{k}\cdot\varphi=\mu\cdot\sin\upsilon\tau\,, \qquad (5.1)$$

provided that no large difference between the non-linear and equivalent linear solutions occurs.

By using a global linearization method in [3] the equivalent linear stiffness and damping coefficients c and k was obtained in the form:

$$\tilde{c}=\frac{1}{\varphi_m}\int_0^{\varphi_m} C(\varphi)\cdot \mathrm{d}\,\varphi \;\; ; \;\; \tilde{k}=\frac{2}{\varphi_m^2}\int_0^{\varphi_m} K(\varphi)\cdot\varphi\cdot \mathrm{d}\,\varphi\,, \tag{5.2}$$

where:

$$\varphi_m=\max\left|\varphi(\tau)\right|=\mu\Phi_{\max}=\frac{2\grave{\imath}}{\tilde{c}\sqrt{4\tilde{k}-\tilde{c}^2}}\,. \tag{5.3}$$

Because the integration limit φ_m is function in terms of $\tilde{c}$ and $\tilde{k}$, from eqs. (5.2) a non-linear system of two algebraic equations for the unknown's c and k are obtained.

Thus, for instance, if the material function $C(\varphi)$ and $K(\varphi)$ has the exponential form:

$$C(\varphi)=a_1-a_2\cdot\exp(-a_3\varphi) \;\;\; ; \;\; K(\varphi)=b_1+b_2\cdot\exp(-b_3\varphi) \tag{5.4}$$

as in the tested rubber sample, from eqs. (5.2) and (5.3) the following system result:

$$\begin{cases} a_1\varphi_m+\dfrac{a_2}{a_3}\left[\exp(-a_3\varphi_m)\right]-\dfrac{2\mu}{\sqrt{4\tilde{k}-\tilde{c}^2}}=0 \\ b_1\varphi_m-\dfrac{b_2}{b_3}\varphi_m\left[\exp(-b_3\varphi_m)\right]-\dfrac{4\tilde{k}\mu^2}{\tilde{c}^2\left(4\tilde{k}-\tilde{c}^2\right)}=0 \end{cases} \tag{5.5}$$

This system can be numerically solved [10] for different amplitudes μ and the equivalent linear values for $\tilde{c}$ and $\tilde{k}$ are obtained.

Table 5.1

μ	$\tilde{c}$	$\tilde{k}$
10^{-6}	0,131	0,550
$5\cdot10^{-6}$	0,208	0,364
10^{-5}	0,262	0,280
$5\cdot10^{-5}$	0,310	0,251
10^{-4}	0,314	0,245

Thus, using this method for tested rubber a set of equivalent linear constants $\tilde{c}$ and $\tilde{k}$ was obtained (table 5.1) and the corresponding magnification functions are given in fig. 5.1. The comparison between non-linear (fig. 4.1) and linearized (fig. 5.1) magnification functions have shown a reasonable agreement.

As can see from table 5.1 and fig. 5.1 the equivalent linear constants $\tilde{c}$ and $\tilde{k}$ are, in fact, functions in terms of normalized excitation amplitude μ: $\tilde{c}=\tilde{c}(\mu)$ and $\tilde{k}=\tilde{k}(\mu)$. Thus, apparently, the linearized NKV model is non-linear model too containing non-linear characteristic functions $\tilde{c}=\tilde{c}(\mu)$ and $\tilde{k}=\tilde{k}(\mu)$. But, the changing the variable x with the new variable μ is more convenient because x is unknown of the analytical solving method while the amplitude μ is not.

After that determining a set of equivalent linear constants (as those given in table 5.1) for practical applications it is possible to evaluate the dependence of these dynamic characteristics in terms of excitation amplitude in normalized form μ (fig. 5.2). Also, the non-linear damping and spring functions $c=c(x)$ and $k=k(x)$ of the initial NKV model can be expressed in terms of dimensional amplitude F_0 (fig.5.3).

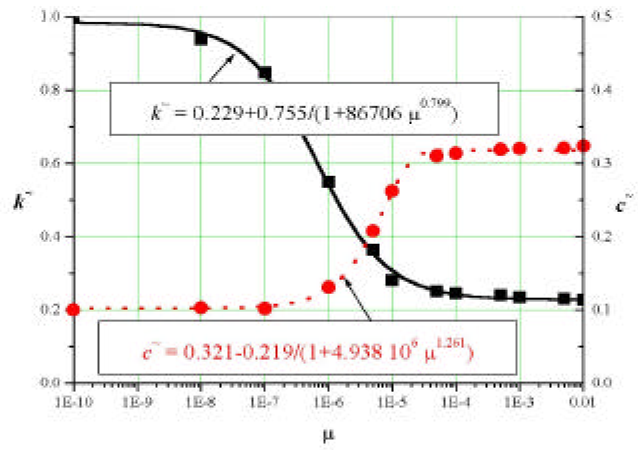

Fig. 5.2
Equivalent linear characteristics in terms of normalized excitation amplitude μ

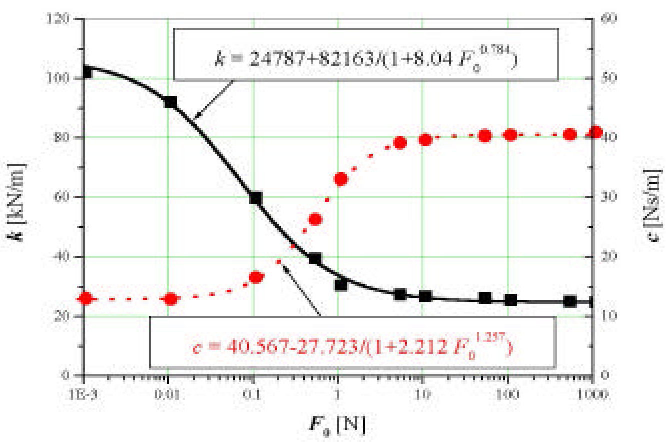

Fig. 5.3
Non-linear characteristics in terms of excitation amplitude F_0

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