# NUCLEAR THRESHOLD EFFECTS AND NEUTRON STRENGTH FUNCTION

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One proves that a Nuclear Threshold Effect is dependent, via Neutron Strength Function, on Spectroscopy of Ancestral Neutron Threshold State. The magnitude of the Nuclear Threshold Effect is proportional to the Neutron Strength Function. Evidence for relation of Nuclear Threshold Effects to Neutron Strength Functions is obtained from Isotopic Threshold Effect and Deuteron Stripping Threshold Anomaly

### **1. INTRODUCTION**

The problems of Nuclear Threshold Effects and of Neutron Strength Function were both formulated by Wigner, (Nobel Prize, 1963, see [1]), in the early times of Nuclear Physics, [2], [3]. Both subjects are still topical in Contemporary Physics of Nuclear Reactions as cardinal research topics, (see [4] and [5]). Apparently the two research subjects have been developed mainly as non-related ones. This Report does establish, theoretically and from experimental data, a relation between Nuclear Threshold Effects and Neutron Strength Functions.

# 2. NUCLEAR THRESHOLD EFFECTS

The basic law of Nuclear Reactions is conservation of the flux; if a new reaction channel opens, a redistribution of the flux appears in old open channels. The modification of an open-channel cross-section, due to opening of a new one, is called threshold effect.

The first (formulation and) solution to the problem of Threshold Effects was provided by Wigner and it is known as the Cusp Theory, [2]. The threshold effects observed in open channels depend on amount of flux absorbed by the new opening (threshold) channel. If the threshold channel exhibits no (coulombian and centrifugal) barriers, (*i.e.* it is a *s*-wave neutron one), then absorption of the flux is suddenly produced and it results in the Cusp Threshold Effect. The Wigner Threshold Cusp should be an universal effect, appearing at threshold of every new *s*-wave neutron channel. However, extensive experimental studies, along many decades, (see [4]), have shown that the nuclear threshold effects are rather rare and diverse. An alternative solution to problem of Nuclear Threshold Effects has to be developed; it has to explain the diversity and rare evidences of threshold effects and to include, as limit cases, the Cusp Theory and other theoretical threshold models.

The problem of Threshold Effects can be incorporated in a broader class of multichannel scattering problems studying the effect of an eliminated (invisible or unobserved) channel on the retained (observed) open channels. Formally, such multichannel scattering problems, could be considered as scattering problems in truncated space of retained channels.

The Dynamics of Quantum Scattering is described by S- or Scattering- Matrix. The S- Matrix elements describe distribution of the flux in different reaction channels. The conservation of the flux is expressed by the S- Matrix Unitarity. The flux transfer between retained r and eliminated e channels is described by the Reduced S- Matrix, [6]. The Reduced S- Matrix is the S- submatrix for retained channels, taking into account the effect of the eliminated one(s),

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$$\Delta S_{rr} = S_{re} (1 + S_{ee})^{-1} S_{er}$$

The "reduced" component  $\Delta S_{rr}$  of the *S*- Matrix displays transitions to and from eliminated channels. The Reduced Scattering Matrix was generalized, [7], in order to separate *S*- Matrix singularities, *i.e.* the resonant- and threshold- structures. The Reduced Scattering Matrix method was extended below threshold, by a joint approach with the Atomic Quantum Defect Method, [8], [9].

The Reduced S- Matrix describes, unitary, a broad class of multichannel problems, including the problem of Threshold Effects. The most important physical result is the relation between Threshold Effects in open channels,  $(\Delta S_{rr})$ , and Nuclear Reaction Dynamics,  $(S_{ee})$ , of the threshold channel. For example, the Cusp Theory is obtained in zero-energy limit of Potential Scattering, [6]. Other types of Threshold Anomalies are related to Resonant or Quasi-Resonant Scattering, [7], [10].

The flux transfer between reaction channels of a Direct Interaction Process is mainly controlled by kinematical and channel penetration factors. The *S*- Matrix dependence only on penetration factors is basic assumption of the Cusp Theory, [2], [11], namely,  $S_{an} = M_{an} k_n^{1/2}$ ,  $S_{nn} = 1 - N_{nn} k_n$ ,  $r = \{a, b\} =$  open (observed) channels, e = n = neutron threshold (invisible) channel, neutron penetration factor ~ neutron channel wave number,  $k_n \rightarrow 0$ ,  $(S_{nn} \rightarrow 1)$ ,  $\Delta S_{ab} = (1/2) S_{an} S_{nb}$ , [11]. One can prove, [6], that the energy dependence only via neutron penetration factors is not a sufficient condition for an observable nuclear cusp effect.

The flux transfer in reactions developing via resonances, is controlled by the resonance couplings to reaction channels. [A resonance is a quasistationary state of the compound system, decaying in different reaction channels.] The factor governing the flux leakage from a resonance ( $\lambda$ ) to reaction channel (n) is the partial resonance decay width,  $\Gamma_{\lambda n}^{1/2} = P_n^{1/2} \gamma_{\lambda n}$ ; it consists from the preformation factor of (*n*)-particle in ( $\lambda$ ) resonance, (particle reduced width  $\gamma_{\lambda n}$ ), and from the penetration factor ( $P_n$ ) of the channel barrier. The reduced width amplitude  $\gamma_{\lambda n}$  is proportional to single particle component (n) of the wave function of the resonant state (A); it is primary factor governing the flux leakage from compound state into reaction channels. The other factor governing the flux leakage into reaction channel is penetrability  $P_{n}$  through nchannel potential barrier, (favouring s- wave neutron channel). The resonance total width, i.e. sum of all channels partial widths,  $\Gamma_{\lambda} = \Gamma_{\lambda n} + \Gamma_{\lambda a} + \Gamma_{\lambda b} + \dots$ , gives the flux leakage in all reaction channels. The threshold effect is directly related to flux absorbed by threshold channel; this means that the resonance total width has to be dominated by neutron threshold partial width,  $\Gamma_{\lambda} \cong \Gamma_{\lambda n}$ , or resonance's neutron reduced width is very large approaching its maximal value,  $\gamma_{\lambda n} \cong \gamma_{W}$ , (reduced width Wigner unit). An obvious necessary condition is energy coincidence of the resonance with neutron threshold,  $|E_{\lambda} - E_{thr}| < \Gamma_{\lambda}$ . The two conditions,  $|E_{\pi} - E_{thr}| < \Gamma_{\lambda}$ .  $\Gamma_{\pi}$  and  $\gamma_{\pi n} \cong \gamma_{W}$ , define the ( $\pi$ ) "Neutron Threshold State". According to this representation the significant threshold effects are related to resonances, coincident with neutron threshold; these resonances have to decay preferentially in neutron threshold channel.

The magnitude of a threshold effect does depend not only on Reaction Mechanism but also on Spectroscopical Amplitude, (i.e. neutron reduced width), of the Neutron Threshold State. A threshold quasistationary state does act as an amplifier for flux transfer to threshold channel because state overlap of channel is very large, *i.e.* it has a large reduced width for decay in threshold channel. The neutron threshold states are vital in producing significant threshold effects. The threshold effects observed with light nuclei are related to resonant levels coincident with neutron channel thresholds and having large neutron reduced widths (spectroscopic amplitudes). This assertion could be verified by analysis of threshold effects observed with light nuclei, (1-p Shell), [4].

The threshold effects are determined by Reaction Dynamics which comprises, in addition to genuine Reaction Mechanisms, the Spectroscopy of the involved nuclear states.

# **3. NEUTRON STRENGTH FUNCTION**

The sharp resonances are observed in reactions on light nuclei at low energies; the resonance's spectroscopic parameters are reduced widths. For medium and heavy nuclei one observes no longer sharp resonances but rather smooth cross-sections; the resonant levels become very closely spaced and their widths are larger than their separations. It is therefore necessary to define a corresponding statistical spectroscopic quantity, by averaging over many levels. This is the Strength Function, which is defined as the total value of the reduced width per unit energy interval of ( $\lambda$ ) resonances,  $S_{\lambda n} = \langle \gamma_{\lambda n}^2 \rangle \rho_{\lambda}$ , where  $\rho_{\lambda}$  is the density of ( $\lambda$ ) levels. The Strength Function is ratio of averaged width to the mean spacing *D* between adjacent levels,  $\rho_{\lambda} = 1/D_{\lambda}$ . The Strength Function is an averaged quantity like the nuclear-level density. Regions where it is appropriate to discuss levels densities instead of single levels are also regions where it is useful to think in terms of Strength Function instead of individual reduced widths.

The (Bohr) Compound Nucleus Resonance is a quasistationary state of high complexity, involving multinucleon excitations. The formation of Compound Nucleus develops via succesive nuclear configurations culminating with resonant quasistationary state(s), which afterwards decay in different reaction channels. The first configuration of this sequential process corresponds to motion of incident nucleon in the self-consistent nuclear potential of the compound system. This motion is described by Single Particle Models as Shell Model (for negative energies) and Optical Model (for positive energies). The Single Particle (nucleon) Shell Model bound state's counterpart, at positive energies, is the Optical Model (Shape or) Single Particle Resonance. The next configurations following single particle motion, are approached via residual nuclear interactions; these are differences between the actual nuclear potential and the Shell Model/Optical Model Potential. By the residual interactions the Single Particle Resonant State is spread out over a group of actual levels. The group of actual levels, carrying out a substantial fraction of Single Particle State, constitues the Giant Resonance, (Micro-Giant Model of Lane-Thomas-Wigner, for Optical Model Resonance [3]). (If the residual interactions would a bigger spreading of the Single Particle State, one results the Uniform Model).

The Strength Function is a measure of the mean strength of reduced widths of actual compound nucleus resonances. This spectroscopic quantity is defined as the overlap of Single Particle State and the actual states, giving how much the Single Particle State is mixed with actual states of the nucleus. It is expected that the Strength Function will display maxima whenever a Single Particle State is present. The (broad) Giant Resonances correspond to each of the Single Particle States of the compound system when the residual interaction was neglected. The (neutron) single particle reduced width  $\gamma_{sp}^2$  is shared among the complicated levels of the compound nucleus in such a way that that  $\sum_{\lambda} \gamma_{hn}^2 = \gamma_{sp(n)}^2$ . The amplitude enhancement of the cross-section or of the Strength Function , resembling to a large width resonance, reveals existence of Single Particle States in nucleon scattering on nuclei. The Giant Resonances are (*e.g.* neutron) Single Particle Resonances which are split , by residual interactions, into complicated Compound Nucleus states. They are not more described by Single Particle reduced widths but rather by the statistical Neutron Strength Function. The spectroscopic parameter of the Neutron Giant Resonance is not more the neutron reduced width but rather its statistical corresponding, *i.e.* the Neutron Strength Function. Both the Reduced Width of an isolated resonance and the Strength Function of a Giant Resonance could influence the flux dynamics in threshold channel which, at its turn, is determining the magnitude of the threshold effects.

For this study, the Neutron Single Particle Resonance coincident with threshold are of interest. The Single Particle Resonance is a Single Channel Resonance. In next chapter we discuss how a single channel neutron resonance could induce, via Quasi-Resonant Scattering, a multichannel threshold effect and how this effect is related to Neutron Strength Function.

# 4. THRESHOLD EFFECTS, QUASI-RESONANT SCATTERING AND NEUTRON CHANNEL STRENGTH FUNCTION

The Compound Nucleus Resonances (involving multinucleon excitations) are multichannel resonances. The Single Particle Resonances, on contrary, should manifest, in first approximation, as Single Channel Resonances. A Direct Channel Coupling of a Single Particle resonance to other competing reaction channels could result into a Coupled Multichannel Resonance. This resonant scattering mechanism, as distinct from Compound Nucleus, is named Quasi-Resonant Scattering, [10], [8]. In last frame, a threshold channel Single Particle Resonance, could induce a significant threshold effect provided there is a strong and selective direct coupling to open channels.

The Quasi-Resonant Scattering process consists of direct inter-channel transitions, preceded or followed by a Single Channel Resonance. They are experimentally evinced as resonant structures in some reaction channels; other competing reaction channels show a (direct interaction) monotone energy dependence. An approach to Quasi-Resonant processes is here presented in terms of Reduced *S*- Matrix.

The Scattering Matrix is split into "fast" background ( $\beta$ ) and "delayed" resonant ( $\rho$ ) terms,  $S=S^{\beta}+S^{\rho}$ . The Unitarity of Scattering Matrix *S* and of "fast" background Scattering Matrix  $S^{\beta}$  results into a "modulation" of  $S^{\rho}$  resonant elements in terms of background Scattering Matrix, [7]. The resonance ( $\lambda$ ) could decay in a channel both directly or via other intermediate channels. If a single channel resonance from an unobserved channel *n* is involved, ( $\gamma_{\lambda n} \neq 0, \gamma_{\lambda a} = 0, a$  and *b*- observed channels), then the *S*- Matrix or the Transition *T*- Matrix, (*S*=1+2*iT*), becomes, [10],

$$T_{ab} = T_{ab}^{\beta} - 2i \frac{T_{an}^{\beta} \gamma_{\lambda n}^2 T_{nb}^{\beta}}{E_{\lambda} - E + \operatorname{Re} T_{nn}^{\beta} \gamma_{\lambda n}^2 - i(1 - \sum_{c} |T_{cn}^{\beta}|^2) \gamma_{\lambda n}^2}$$

A single channel resonance  $(E_{\lambda}, \gamma_{\lambda n})$  induces, via direct interaction couplings  $(T_{an}^{\beta}, T_{nb}^{\beta})$  resonant structures in other reactions (a,b). The Quasi-Resonance's reduced widths depend both on channel couplings strengths and on Spectroscopy of ancestral Single Particle State:  $\alpha_{\lambda a} = iT_{an}^{\beta}\gamma_{\lambda n}$ ,  $\alpha_{\lambda n} = (1 + iT_{nn}^{\beta})\gamma_{\lambda n}$ . A physical implication of the direct transitions in multichannel reaction system is "direct compression" of the Quasi-Resonance,  $\gamma_{\lambda n}^{2} \rightarrow \gamma_{\lambda n}^{2}$   $(1-\sum_{c}/T_{cn}^{\beta})^{2}$ ; the resonant structure width of the whole reaction system is narrower than single channel resonance width. A physical interpretation, [10], of this effect is related to Channel Coupling Pole; this originates in a distant pole in energy (or wave number) plane which is driven to physical region when the channel coupling becomes strong. This is a demonstration that the origine of the multichannel quasi-resonance is a single particle resonance state from an invisible channel.

Another peculiar compression of a quasi-resonant structure comes into play if the single particle state  $(\pi)$  is located near channel threshold. A *R*- Matrix parametrization of this formula displays a strong energy dependence of the resonance denominator via threshold channel logarithmic derivative  $L_n = S_n + iP_n$ .

$$\Delta T_{ab}^{\beta} = -2i \frac{T_{an}^{\beta} \gamma_{\pi n}^2 T_{nb}^{\beta}}{E_{\pi} - E + S_n \gamma_{\pi n}^2 - i(P_n \gamma_{\pi n}^2 + \Gamma')}$$

The threshold channel level shift,  $(S_n \gamma_{\pi n}^2)$ , and natural decay width,  $(P_n \gamma_{\pi n}^2)$ , are explicitly evinced because of their peculiar energy dependance near threshold. [The complementary width,  $\Gamma$ ', describes Quasi-Resonance's coupling to other open channels.] One has to mention a phenomenological formula proposed by Lane, [12], for Deuteron Stripping Threshold Anomaly,  $\alpha/[E_{\pi}-E+S_n\gamma_{\pi n}^2-i(P_n\gamma_{\pi n}^2+W)]$ , where *W* is Single Particle Resonance's spreading width and  $\alpha$  parameters are related only to isospin proton-neutron coupling strength. The non-linear energy dependence of the shift-function  $S_n$  and of penetration-factor  $P_n$ , results into

a "threshold compression factor",  $\beta(E)=1/[1+\gamma_{\pi\pi}^2 dS_{\pi}/dE]$ , [12]. The subunitary threshold factor,  $\beta(E) < 1$ , results into a shift to the threshold of the resonance,  $E_{\pi} \rightarrow \beta E_{\pi}$  and in compression of the total width,  $\Gamma \rightarrow \beta \Gamma$ . The magnitude of the quasiresonant process is proportional both to direct channel coupling strengths

 $(T_{an}^{\beta}T_{nb}^{\beta})$  and to the single channel neutron reduced width  $(\gamma_{\pi n}^{2})$ . The last property is subject of this chapter. As mentioned before, the Single Particle State is spread out , by residual interactions, over the actual (compound nucleus) levels. By averaging over actual levels one obtains the result the effect is proportional to Neutron Strength Function,  $S_{\pi n} \sim \langle \gamma_{\pi n}^2 \rangle$ . [The only fluctuant quantities are neutron reduced width and total resonance width; the other terms  $(T^{P}, S_{n}, P_{n})$  are monotone energy dependent and are not involded in averaging. The averaging method used here is Brown procedure, [13]; one has to avoid a small region at threshold-branch point.] The relation Threshold Effect magnitude - Neutron Strength Function was firstly obtained by another procedure, [14],

$$\alpha_{ab} = \Gamma_{an} < \gamma_{\pi n}^2 > \Gamma_{nb}$$

with  $\Gamma_{an}$  and  $\Gamma_{nb}$  as coupling strengths of the threshold channel *n* to open ones.

In order to extract from data the relation Threshold Effect - Neutron Strength Function one has to take into account the energy dependence on input (a) and exit (b) channels, *i.e.* those of  $T_{an}^{\ \beta}T_{nb}^{\ \beta}$  factors. Let consider the case of the exit proton channel (p=p) coupled by isospin interaction to neutron threshold channel; the proton and neutron channels are isospin analog-ones. The exit proton energy is fixed by its Coulomb relation to the threshold zero-energy of neutron analogue channel, Q(p,n) for analogue channels has nearly same value for all nuclei in a given mass area or with same zero energy neutron state]. Therefore the term  $T_{\mu\nu}^{\beta}$  could be considered as constant for the group of nuclei displaying same zero-energy neutron state. For a quasiresonant threshold effect in proton elastic scattering one obtains that the product  $T_{pn}^{\ \beta}T_{np}^{\ \beta}$  is nearly constant for target nuclei within mass area with a given zero-energy Neutron Single Particle State. The experimental threshold effect's magnitude  $\alpha_{nn}$  is, up to a constant factor, just Neutron Strength Function

 $\alpha_{pp} = const.S_{\pi n}$ . For transfer reactions populating same proton and neutron isospin analogue channels, [e.g. (*d*,*p*) and (d,n) ones], it could happen that the Q-values, [Q(d,p)], change significantly for different target nuclei of same mass area. Consequently one has to "correct" for the input channel energy dependence, the primary experimental data. One can overcome this situation by remarking that the transfer analog proton and neutron reactions have same kinematical structure; their energy dependence on input channel appears only in DWBA Radial Integrals via input (deuteron) channel distorted wave function. Consequently one can consider that the input channel energy dependence of threshold experimental data is same as of the background crosssection,  $\alpha_{dp} \sim T_{dp}^{\beta} \alpha'$ , with  $\alpha'$  nearly independent on input channel energy. The relation Threshold Effect -Neutron Strength Function is now shifted into relation  $\alpha' = const.S_{-}$ .

These procedures have been used in establishing relations Threshold Effect- Neutron Strength Function both for Isotopic Threshold Effect and Deuteron Stripping Threshold Anomaly.

# 5. EVIDENCE FOR RELATION OF AVERAGED NUCLEAR THRESHOLD EFFECTS AND **NEUTRON CHANNEL STRENGTH FUNCTION**

Threshold effects related to Quasi-Resonant Scattering require two conditions: (1) a zero-energy neutron single particle resonance, and (2) direct selective coupling of neutron threshold channel to the observed (proton) one. The first condition does select the mass area of target nuclei, while the second one does select the reaction populating neutron channel. Indeed, a Single Particle State of given energy is a global property of a whole mass region; for example the 2-p and 3-p Neutron Single Particle States are specific for  $A \gg 30$  and

*A»*90 mass nuclei, respectively. The Neutron Single Particle Resonance is then coupled selectively, by isospin interaction, only to analogue proton channel.

a)ISOTOPIC THRESHOLD EFFECT

The possible proton reactions , satisfying to above conditions, are (p,p) proton elastic scattering and (p,n), charge exchange reactions on mirror nuclei of *A*»30 mass area, [15]; (data for <sup>25</sup>Mg are taken from a work on statistical fluctuations, [16]). Experimental data of proton scattering on *A*»30 mass nuclei do indeed exhibit an averaged threshold effect; its magnitude is proportional to 2-*p* wave Neutron Strength Function, [17]. The "empirical" magnitude of the threshold effect,  $\Delta = (\sigma_{max} - \sigma_{min})/\sigma$ , is the maximal deviation in the threshold domain of the averaged proton cross-section data normalized to its corresponding integrated value. Another measure of threshold effect magnitude, extracted from analysis of data, could be the ("computational") resonant strength  $\alpha$ . The threshold anomaly's magnitude does follow the mass dependence of the 2-*p* wave Neutron Strength Function, Fig. 1.



Fig. 1. Strengths of the Isotopic Threshold Effect,  $\Delta$  ("empirical") and  $\alpha$  ("computational") and the 2-*p* wave Neutron Strength Function, versus atomic mass number *A*.

The "empirical" and "computational" results on Isotopic Threshold Effect support the idea of this work, according to the magnitude of the threshold effect is dependent on spectroscopic amplitude of the quasiresonant process, *i.e.* Neutron Strength Function. This result was also first experimental evidence that a Coupled Channel Resonance depends not only on channels-coupling strength but also on spectroscopical properties of ancestral (Neutron) Single Particle State.

#### b) DEUTERON STRIPPING THRESHOLD ANOMALY

A cross-section anomaly in (d,p) reactions on  $A \gg 90$  target nuclei was observed at threshold of (d,n) analogue channel, [18]. The two exit channels are selectively coupled by isospin interaction; the  $A \gg 90$  mass nuclei do exhibit the 3-p wave neutron single particle state at zero energy. The relation between the magnitude of deuteron stripping threshold anomaly and the 3-p wave neutron strength function was analyzed in work [19]. It is proved, within empirical approach and computational frameworks, that the magnitude of Deuteron Stripping Threshold Anomaly on  $A \gg 90$  mass nuclei is proportional to the 3-p wave Neutron Strength Function, in their mass dependence, Fig. 2.

# 6. CONCLUSIONS

The empirical and computational analysis of the Isotopic Threshold Effect and of the Deuteron Stripping Threshold Anomaly demonstrate their close relationship to Neutron Strength Functions. It was established that the Nuclear Threshold Effects depend, in addition to genuine Nuclear Reaction Mechanisms, on Spectroscopy of (Ancestral) Neutron Threshold State. The magnitude of the effect is proportional to the Neutron Strength Function, in their dependence on mass number. This result constitutes also a proof that the origins of these threshold effects are Neutron Single Particle States at zero energy.



Fig. 2. The mass dependence of the experimental 3-*p* wave Neutron Strength Function (•) and of the  $\alpha$ 's strength of Deuteron Stripping Threshold Anomaly: (a) "empirical" ( $\triangleright$ ); (b), (c) "computational" ( $\Diamond$ , o).

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