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RADIATION SPECTRA OF CHARGED PARTICLES MOVING IN A SPIRAL IN MAGNETIC FIELDS

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The expression for the averaged radiation power of the charged particles moving in a spiral in transparent isotropic medià and in vacuum are studied by using the Lorentz's self-interaction method. Special attention is given to the research of the structure of the synchrotron radiation spectral distribution of two electrons moving in a spiral in vacuum. The spectra of synchrotron, Cherenkov, and synchrotron-Cherenkov radiations for a separate electron are analyzed.

Key words: Cherenkov radiation, synchrotron radiation, synchrotron-Cherenkov radiation, Lorentz`s self-interaction.

1. INTRODUCTION

Investigations of the radiation spectra of charged particles moving in magnetic fields in transparent isotropic medium and in vacuum are important from the point of view of their applications in electronics, astrophysics, plasma physics, physics of storage rings [1-3].

A question requiring further investigations is the coherence of synchrotron radiation [2]. At moving an electron beam through a spiral undulator a laser radiation takes place [4]. Properties of free-electron lasers were considered in papers [5-7].

Using the exact integral relationships for the spectral distribution of radiation power of two electrons moving one after another along a spiral in vacuum, the structure of the synchrotron radiation spectrum was investigated by means of analytical and numerical methods. The Doppler effect influence on peculiarities of the radiation spectrum of a separate electron at its motion in a spiral in transparent media and in vacuum is investigated.

2. INSTANTANEOUS AND TIME-AVERAGED RADIATION POWER OF CHARGED PARTICLES

The instantaneous radiation power of charged particles $P^{rad}(t)$ in an isotropic transparent medium and in vacuum [8, 9] is expressed as

$$P^{\mathrm{rad}}(t)=\int_{\tau}\left(\vec{j}(\vec{r},t)\frac{\partial\vec{A}^{Dir}(\vec{r},t)}{\partial t}-\rho(\vec{r},t)\frac{\partial\varphi^{Dir}(\vec{r},t)}{\partial t}\right)\mathrm{d}\vec{r}. \tag{1}$$

Here $\vec{j}(\vec{r},t)$ is the current density and $\rho(\vec{r},t)$ is the charge density. The integration is over some volume τ. According to the hypothesis of Dirac [9-12], the scalar $\varphi^{Dir}(\vec{r},t)$ and vector $\vec{A}^{Dir}(\vec{r},t)$ potentials are defined as a half-difference of the retarded and advanced potentials:

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$$\varphi^{Dir} = \frac{1}{2}\left(\varphi^{ret} - \varphi^{adv}\right) \ , \ \vec{A}^{Dir} = \frac{1}{2}\left(\vec{A}^{ret} - \vec{A}^{adv}\right) \quad (2)$$

After substituting (2) into (1) we obtain the relationship for instantaneous radiation power of charged particles moving in isotropic transparent media as the function of spectral distribution

$$P^{rad}(t) = \int_0^{\infty} W(t,\omega)\ \mathrm{d}\omega, \quad (3)$$

$$\begin{gathered} W(t,\ \omega) = \frac{1}{\pi}\int_{-\infty}^{\infty}\mathrm{d}\vec{r}\int_{-\infty}^{\infty}\mathrm{d}\vec{r}'\times \\ \times\int_{-\infty}^{\infty}\omega\mu_r(\omega)\frac{\mu_0}{4\pi}\frac{\sin\left[\frac{n(\omega)\omega}{c}\left|\vec{r}-\vec{r}'\right|\right]}{\left|\vec{r}-\vec{r}'\right|}\cos\omega(t-t')\left\{\vec{j}(\vec{r},t)\vec{j}(\vec{r}',t') - \frac{c^2}{n^2(\omega)}\rho(\vec{r},t)\rho(\vec{r}',t')\right\}\mathrm{d}t' \end{gathered} \quad (4)$$

where $\mu_a(\omega) = \mu_r(\omega)\mu_0$ is the absolute magnetic permittivity, $n(\omega)$ is the refraction index, ω is the cyclic frequency, and c is the velocity of light in vacuum.

The time-averaged radiation power of charged particles is defined by the expression

$$\overline{P}^{rad} = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T} P^{rad}(t)\ \mathrm{d}t\ . \quad (5)$$

It can be obtained after substitution of the instantaneous radiation power expressed by relationships (3) and (4) into (5).

3. SYSTEMS OF NON-INTERACTING POINT CHARGED PARTICLES

Let us consider a system of point non-interacting particles with charges $q_1,\ q_2, ...,\ q_N$ and rest masses $m_{01},\ m_{02},\ ...,\ m_{0N}$ moving along an arbitrary defined trajectories. Then the source functions of N charged point particles are defined as

$$\vec{j}(\vec{r},t) = \sum_{l=1}^{N}\vec{V}_l(t)\rho_l(\vec{r},t) \ , \ \rho(\vec{r},t) = \sum_{l=1}^{N}\rho_l(\vec{r},t) \ , \ \rho_l(\vec{r},\ t) = q_l\ \delta\left(\vec{r} - \vec{r}_l(t)\right), \quad (6)$$

where $\vec{r}_l(t)$ and $\vec{V}_l(t)$ are the motion law and the velocity of the l^{th} particle, respectively.

Substituting relationships (6) into (3) and (4) we obtain the expression for the instantaneous radiation power of charged particles system in transparent media (relative magnetic $\mu_r(\omega)$ and relative dielectric $\varepsilon_r(\omega)$ permittivities are real):

$$\begin{gathered} P^{rad}(t) = \frac{1}{\pi}\int_0^{\infty}\omega\mu_r(\omega)\frac{\mu_0}{4\pi}\mathrm{d}\omega\int_{-\infty}^{\infty}\sum_{l,j=1}^{N}q_l\ q_j\ \frac{\sin\left\{\frac{n(\omega)}{c}\omega\left|\vec{r}_l(t) - \vec{r}_j(t')\right|\right\}}{\left|\vec{r}_l(t) - \vec{r}_j(t')\right|}\times \\ \times\cos\omega(t-t')\left\{\vec{V}_l(t)\vec{V}_j(t') - \frac{c^2}{n^2(\omega)}\right\}\mathrm{d}t' \end{gathered} \quad (7)$$

The time-averaged radiation power can be obtained from expression [13]

$$\bar{P}^{rad} = \frac{1}{\pi}\lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T} dt\int_{0}^{\infty}\omega\mu_r(\omega)\frac{\mu_0}{4\pi}d\omega\int_{-\infty}^{\infty}\sum_{l,j=1}^{N} q_l\, q_j\, \frac{\sin\left\{\frac{n(\omega)}{c}\omega\left|\vec{r}_l(t)-\vec{r}_j(t')\right|\right\}}{\left|\vec{r}_l(t)-\vec{r}_j(t')\right|}\times$$
$$\times\cos\omega(t-t')\left\{\vec{V}_l(t)\vec{V}_j(t')-\frac{c^2}{n^2(\omega)}\right\}dt' \quad (8)$$

Let us consider a system of point non-interacting electrons $(q_l = e,\ m_{0l} = m_0)$ moving one by one along an arbitrary defined trajectory. Then the motion law and the velocity of the l^{th} particle of this system are determined by the relationships

$$\vec{r}_l(t)=\vec{r}_p(t+\Delta t_l)\quad,\ \vec{V}_l(t)=\vec{V}(t+\Delta t_l). \quad (9)$$

In this case we obtain the averaged radiation power after substitution of expressions (9) into (8):

$$\bar{P}^{rad} = \frac{e^2}{\pi}\lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T} dt\int_{-\infty}^{\infty} dt'\int_{0}^{\infty}\mu_r(\omega)\frac{\mu_0}{4\pi}\ \omega S_N(\omega)\frac{\sin\left\{\frac{n(\omega)}{c}\omega\left|\vec{r}_p(t)-\vec{r}_p(t')\right|\right\}}{\left|\vec{r}_p(t)-\vec{r}_p(t')\right|}\times$$
$$\times\cos\omega(t-t')\left[\vec{V}(t)\ \ \vec{V}(t')-\frac{c^2}{n^2(\omega)}\right]d\omega \quad (10)$$

where the coherence factor $S_N(\omega)$ is defined as

$$S_N(w)=\sum_{l,j=1}^{N}\cos\{w(\Delta t_l-\Delta t_j)\}. \quad (11)$$

The coherence factor $S_N(\omega)$ determines a redistribution of the charged particles radiation power between frequencies.

4. STRUCTURE OF THE RADIATION SPECTRA OF TWO ELECTRONS MOVING ALONG A SPIRAL IN VACUUM

Peculiarities of the radiation spectra of two electrons moving one by one in a spiral in vacuum can be investigated combining analytical and numerical methods. The law of motion and the velocity of the l^{th} electron are given by the expressions

$$\vec{r}_l(t)=r_0\cos\{\omega_0(t+\Delta t_l)\}\vec{i}+r_0\sin\{\omega_0(t+\Delta t_l)\}\vec{j}+V_{\|}(t+\Delta t_l)\vec{k}\quad,\ \vec{V}_l(t)=\frac{d\vec{r}_l(t)}{dt}. \quad (12)$$

Here $r_0 = V_{\perp}\omega_0^{-1}$, $\omega_0 = ec^2B^{ext}\tilde{E}^{-1}$, $\tilde{E} = c\sqrt{p^2 + m_0^2c^2}$, the magnetic induction vector $\vec{B}^{ext}\|0Z$, $V_{\perp}$ and $V_{\|}$ are the components of the velocity, $\vec{p}$ and $\tilde{E}$ are the momentum and energy of the electron, e and m_0 are its charge and rest mass, respectively.

The time-averaged radiation power of two electrons in vacuum we obtain after substitution expressions (12) into (10). Then

$$\bar{P}^{rad} = \int_{0}^{\infty} W(\omega)\,d\omega, \quad (13)$$

$$W(\omega)=\frac{2e^2}{\pi}\int_0^\infty \omega\frac{\mu_0}{4\pi}S_2(\omega)\frac{\sin\left\{\frac{1}{c}\omega\eta(x)\right\}}{\eta(x)}\cos\omega x\left[V_\perp^2\cos(\omega_0 x)+V_\parallel^2-c^2\right]dx \tag{14}$$

where $\eta(x)=\sqrt{V_\parallel^2x^2+4\frac{V_\perp^2}{\omega_0^2}\sin^2\left(\frac{\omega_0}{2}x\right)}$.

The coherence factor $S_2(\omega)$ of two electrons is defined as

$$S_2(\omega)=2+2\cos(\omega\Delta t). \tag{15}$$

Here $\Delta t=\Delta t_2-\Delta t_1$ is the time shift of the electrons moving along a spiral.

At the frequencies $\omega=2i\pi/(\Delta t)$ (i=0, 1, 2,…) the coherence factor of two electrons (15) is equal to 4 and at the frequencies $\omega=(2i+1)\pi/(\Delta t)$ (i=0, 1, 2,…) the coherence factor is equal to zero. The analogous expression for the coherence factor was investigated by Bolotovskii [14].

From relationships (13) and (14) after some transformations the contributions of separate harmonics to the averaged radiation power can be written as

$$\begin{aligned}\bar{P}^{rad}=\frac{e^2}{c}\sum_{m=1}^{\infty}\int_0^\infty \omega^2\frac{\mu_0}{4\pi}d\omega\int_0^\pi \sin\theta\, d\theta\times 2\left[1+\cos\{\omega(\Delta t)\}\right]\times\delta\left\{\omega\left(1-\frac{1}{c}V_\parallel\cos\theta\right)-m\omega_0\right\}\times\\ \times\left\{V_\perp^2\left[\frac{m^2}{q^2}J_m^2(q)+J_m'^2(q)\right]+\left(V_\parallel^2-c^2\right)J_m^2(q)\right\}\end{aligned} \tag{16}$$

where $q=\frac{V_\perp}{c}\frac{\omega}{\omega_0}\sin\theta$, $J_m(q)$ and $J'_m(q)$ are the Bessel function with integer index and its derivative, respectively.

Each harmonic is a set of the frequencies, which are the solution of the equation

$$\omega\left(1-\frac{1}{c}V_\parallel\cos\theta\right)-m\omega_0=0. \tag{17}$$

The limits of the m^{th} harmonic are determined by the frequencies

$$\omega_m^{\min}=\frac{m\omega_0}{1+\frac{V_\parallel}{c}},\quad \omega_m^{\max}=\frac{m\omega_0}{1-\frac{V_\parallel}{c}}, \tag{18}$$

and the total radiation power emitted by a separate electron moving in a spiral in vacuum is determined according to [15] as

$$P_m^{tot}=\frac{2}{3}\frac{e^2}{c}\frac{\mu_0}{4\pi}\omega_0^2V_\perp^2\left(1-\frac{V^2}{c^2}\right)^{-2}, \tag{19}$$

where $\omega_0=\frac{eB^{ext}}{m_0}\sqrt{1-\frac{V^2}{c^2}}$.

Our numerical calculations of the radiation power spectral distribution were performed at $B^{ext}=10^{-4}$ T, $c=0.2997925\cdot10^9$ m/s.

For the velocities components $V_{\perp vac}=0.2\cdot10^8$ m/s and $V_{\parallel vac}=0.12\cdot10^9$ m/s the radiation power spectral distributions of two electrons in vacuum depending on their location along a spiral are shown in Figs 1–3 (curves 1–5).

It is interesting to compare the radiation power spectral distribution for two electrons with the radiation power spectral distribution of a separate electron (curve 0 in Fig.1). The radiation power of the separate electron in vacuum $P^{tot}_{vac0} = 0.8455 \cdot 10^{-24}$ W calculated according to relationship (19) is in good agreement to the power $P^{\rm int}_{vac0} = 0.8527 \cdot 10^{-24}$ W determined after integration of relationships (13) and (14). For the time difference $\Delta t_1 = 0.01\pi / \omega_{01}$ the coherence factor $S_2(\omega) = 4$ and two electrons radiate as a charged particle with the charge $2e$ and the rest mass $2m_0$, i.e. by a factor of four more than a separate electron.

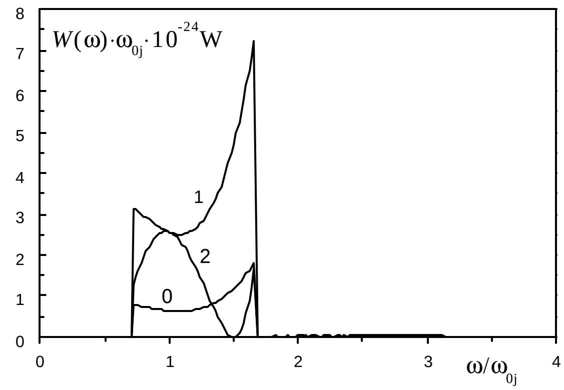

Fig. 1. Spectral distribution of radiation power for two electrons moving one by one in a spiral. ($V_{\perp vac} = 0.2 \cdot 10^8$ m/s, $V_{||vac} = 0.12 \cdot 10^9$ m/s, curves 0–5). Curve 0 – the radiation spectrum of a separate electron, $P^{tot}_{vac0} = 0.8455 \cdot 10^{-24}$ W, $P^{\rm int}_{vac0} = 0.8527 \cdot 10^{-24}$ W, Curve 1: $\Delta t_1 = 0.01\pi / \omega_{01}$, $P^{\rm int}_{vac1} = 0.3409 \cdot 10^{-23}$ W. Curve 2: $\Delta t_2 = 2\pi / \omega_{02}$, $P^{\rm int}_{vac2} = 0.1409 \cdot 10^{-23}$ W,. $\omega_{00} = \omega_{01} = \omega_{02} = \omega_{03} = \omega_{04} = \omega_{05} = 0.1607 \cdot 10^8$ rad/s, $r_{00} = r_{01} = r_{02} = r_{03} = r_{04} = r_{05} = 1.244$ m.

In the case $\Delta t_2 = 2\pi / \omega_{02}$ the function of the radiation power spectral distribution has the maxima approximately at the frequencies ω_{03}, whereas the radiation at $1.5\,\omega_{03}$ is absent.

For the time difference $\Delta t_3 = 4\pi / \omega_{03}$ (curve 3 in Fig. 2) we have found the maxima of the spectral distribution function located approximately at the frequencies ω_{03}, $1.5\omega_{03}$, $2\omega_{03}$, $2.5\omega_{03}$ and $3\omega_{03}$ but at the frequencies $0.75\omega_{03}$, $1.25\omega_{03}$, $1.75\omega_{03}$, $2.25\omega_{03}$, and $2.75\omega_{04}$ the radiation is absent.

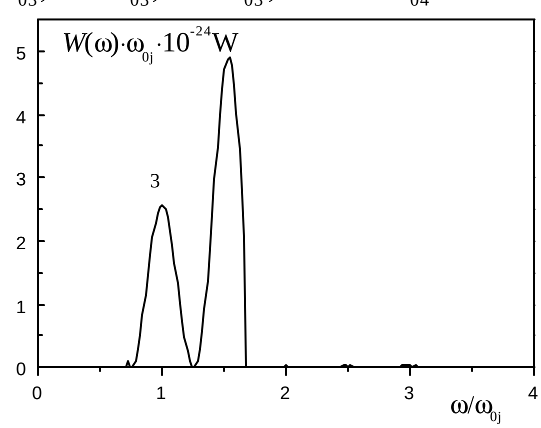

Fig. 2. Spectral distribution of radiation power for two electrons moving one by one in a spiral. Curve 3: $\Delta t_3 = 4\pi / \omega_{03}$, $P^{\rm int}_{\rm vac\ 3} = 0.1844 \cdot 10^{-23}$ W

For the time differences $\Delta t_4 = \pi / \omega_{04}$ (curve 4 in Fig. 3) and $\Delta t_5 = 3\pi / \omega_{05}$ (curve 5 in Fig. 3) the radiation at the basic frequencies $\omega_{04} = \omega_{05}$ is absent.

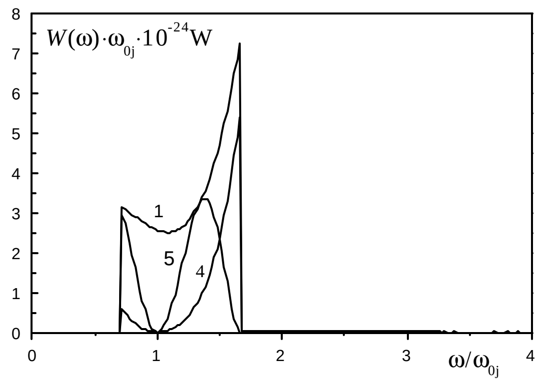

Fig. 3. Spectral distribution of radiation power for two electrons moving one by one in a spiral.
Curve 4: $\Delta t_4 = \pi / \omega_{04}$, $P^{\text{int}}_{vac4} = 0.1053 \cdot 10^{-23}$ W. Curve 5: $\Delta t_5 = 3\pi / \omega_{05}$, $P^{\text{int}}_{vac5} = 0.1530 \cdot 10^{-23}$ W.

At the basic frequency ω_{0j} the function of the radiation power spectral distribution of two electrons is equal to zero if the time difference between them in a spiral is equal to $(2i+1)\pi / \omega_{0j}$ (i=0, 1, 2,…).

5. SPECTRAL DISTRIBUTION OF SYNCHROTRON-CHERENKOV RADIATION POWER IN LOW- FREQUENCY RANGE

Let us consider a Doppler effect influence on synchrotron-Cherenkov radiation in transparent media. The synchrotron-Cherenkov radiation in a medium is the only process [16]. The expressions for the synchrotron-Cherenkov radiation power in such a medium can be obtained starting from (10). Then for the separate electron moving in a spiral we have

$$\bar{P}^{rad} = \int_0^\infty W(\omega) \mathrm{d}\omega , \tag{20}$$

$$W(\omega) = \frac{2e^2}{\pi} \int_0^\infty \mathrm{d}x \;\; \mu_r(\omega) \frac{\mu_0}{4\pi} \omega \frac{\sin\left\{ \dfrac{n(\omega)\omega}{c} \eta(x) \right\}}{\eta(x)} \cos(\omega x) \left[V_\perp^2 \cos(\omega_0 x) + V_\parallel^2 - \frac{c^2}{n^2(\omega)} \right], \tag{21}$$

where $\eta(x) = \sqrt{V_\parallel^2 x^2 + 4 \dfrac{V_\perp^2}{\omega_0^2} \sin^2\left(\dfrac{\omega_0}{2} x \right)}$.

In the case of transparent media in low-frequency spectral range, i.e. at $\varepsilon_r = const$ and $\mu_r = 1$, the power of the Cherenkov radiation at rectilinear motion in a medium (n is the constant) is determined as:

$$P_{ch}^{tot} = \frac{e^2}{2} V \frac{\mu_0}{4\pi} \omega_{\max}^2 \left(1 - \frac{c^2}{V^2 n^2} \right). \tag{22}$$

For the refraction index $n = 2$ at the velocities $V_{\perp m} = 0.15 \cdot 10^8$ m/s, $V_{\parallel m} = 0.1493 \cdot 10^9$ m/s, and $V_{\perp m} = 0.12 \cdot 10^8$ m/s, $V_{\parallel m} = 0.1496 \cdot 10^9$ m/s (curves 6 and 7 in Fig. 4) the conditions for existence of the synchrotron-Cherenkov radiation are fulfilled.

The power of the Cherenkov radiation at rectilinear motion $P_{ch8}^{tot} = 0.6979 \cdot 10^{-18}$ W (relation (22)) is in good agreement to the power of the synchrotron-Cherenkov radiation $P_m^{\text{int}} = 0.699 \cdot 10^{-18}$ W calculated in conforming the relationships (20) and (21) at the motion of the charged particle having a small ($V_{\perp m} = 0.1 \cdot 10^6$ m/s) transverse velocity component (the absolute values of the velocities are the same).

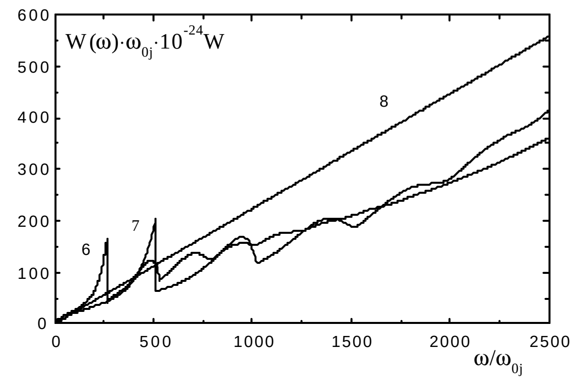

Fig. 4. Spectral distribution of synchrotron-Cherenkov radiation power with relative frequency.
(For the curves 6-8: $n = 2$, $B^{ext} = 10^{-4}$T, $\omega_{06} = \omega_{07} = \omega_{08} = 0.1523 \cdot 10^8$ rad/s). Curve 6: $V_{\perp m} = 0.15 \cdot 10^8$ m/s, $V_{\parallel m} = 0.1493 \cdot 10^9$ m/s, $P_{m6}^{\text{int}} = 0.4688 \cdot 10^{-18}$ W, $r_{06} = 0.985$ m. Curve 7: $V_{\perp m} = 0.12 \cdot 10^8$ m/s, $V_{\parallel m} = 0.1496 \cdot 10^9$ m/s, $r_{07} = 0.788$ m, $P_{m7}^{\text{int}} = 0.469 \cdot 10^{-18}$ W. Curve 8: $V_{\perp m} = 0.1 \cdot 10^6$ m/s, $V_{\parallel m} = 0.1500839 \cdot 10^9$ m/s, $P_{m8}^{\text{int}} = 0.699 \cdot 10^{-18}$ W, $P_{ch8}^{tot} = 0.6979 \cdot 10^{-18}$ W, $r_{08} = 0.007$ m.

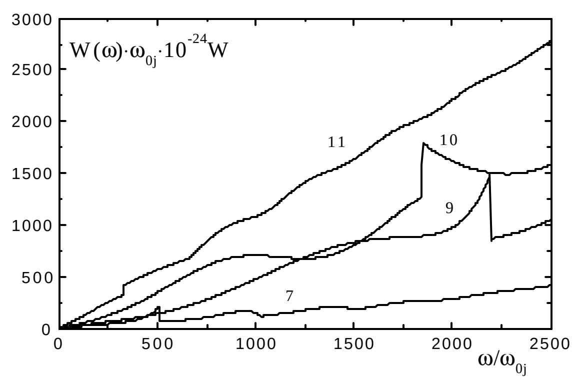

Fig. 5. Spectral distribution of synchrotron-Cherenkov radiation power with relative frequency. For curves 9–11 $V_{\perp m} = 0.12 \cdot 10^8$ m/s, $V_{\parallel m} = 0.1496 \cdot 10^9$ m/s, $\omega_{09} = \omega_{010} = \omega_{011} = 0.1523 \cdot 10^8$ rad/s, $r_{09} = r_{010} = r_{011} = 0.788$ m, Curve 9: $n = 2.003$, $P_{m9}^{\text{int}} = 0.1469 \cdot 10^{-17}$ W. Curve 10: $n = 2.005$, $P_{m10}^{\text{int}} = 0.2099 \cdot 10^{-17}$ W, Curve 11: $n = 2.01$, $P_{m11}^{\text{int}} = 0.3450 \cdot 10^{-17}$ W.

The performed high-accuracy calculations of relationships (20) and (21) for the spectral distribution of the synchrotron-Cherenkov radiation power of electrons showed that the spectral distributions at $V_{\parallel} < c/n$ (curves 6 and 7 in Fig.4 and curves 7 and 9 in Fig. 5) essentially differed from that at $V_{\parallel} > c/n$ (curves 10 and 11 in Fig. 5). The analytical studies and numerical calculations showed that the Doppler effect influence on the peculiarities of the radiation power spectral distribution of the electrons was essential near the Cherenkov threshold.

The allowance of frequency dispersion does not change essentially the radiation power spectral distribution in low-frequency range in transparent media. In the case of the Cherenkov radiation in nontransparent media the allowance of frequency dispersion leads to some interesting peculiarities in high-frequency spectral range [17, 18].

6. CONCLUSIONS

The coherence factor leads to essential changes in the radiation power spectral distribution of a system of charged particles depending on charged particles location in orbit.

The Doppler effect influence on the form of spectral distribution power is essential near the Cherenkov threshold.

REFERENCES

1. KONSTANTINOVICH, A.V., FORTUNA, V.V. *On the theory of noninteracting charged particles system moving in constant magnetic field in vacuum*, Izv. Vuzov. Fizika, **26**, No 12, P. 102–104, 1983 (in Russian).
2. TERNOV, I.M. *Synchrotron radiation*, Usp. Fiz. Nauk, **165**, No 4, P. 429–456,1995 (in Russian).
3. KONSTANTINOVICH, A.V., MELNYCHUK, S.V., RARENKO, I.M., KONSTANTINOVICH, I.A., ZHARKOI, V.P. *Radiation spectrum of the system of charged particles moving in nonabsorbable isotropic medium*, J. Physical Studies, **4**, No 1., P. 48–56, 2000 (in Ukrainian).
4. DEACON, D.A., ELIAS, L.R., MADEY, J.M.J., RAIMAN, G.J., SCHWETTMAN, H.A., SMITH T.I., *First operation of a free-electron laser*, Phys. Rev. Lett., **38**, No 16, p. 892–894, 1977.
5. FEDOROV, M.V, *Interaction of electrons with electromagnetic field in free-electron lasers*, Usp. Fiz. Nauk, **135**, ¹2, p. 213 – 236, 1981 (in Russian).
6. BULYAK, Ye.V, Kurylko, V.I., *Undulator radiation coherence and efficiency of free-electron lasers*, Dopovidi NAN of Ukraine. Mathematics. Nature Sciences. Technical Sciences, No 7, p. 82– 84 , 1998 (in Ukrainian).
7. DATTOLI, G., MEZA, L., MIGLIORATI, M., PALUMBO, L., *Storage ging free-electron laser and microwave type instabilities*, Nuovo Cimento, **115B**, No 6, p. 639–644, 2000.
8. SCHWINGER, J. *On the classical radiation of accelerated electrons*, Phys. Rev., **75,** No 12, P. 1912–1925, 1949.
9. KONSTANTINOVICH, A.V., MELNYCHUK, S.V., KONSTANTINOVICH I.A. *Classical radiation theory of charged particles. I. Retarded and advanced potentials and electromagnetic fields intensites and the Lorentz's self-interaction method*, Bulletin of Chernivtsi National University. Physics and Electronics, No 102, P. 5-13, 2001 (in Ukrainian).
10. DIRAC, P. A. M. *Classical theory of radiating electrons*, Proc Roy. Soc., **167A**, No 1, P. 148–169, 1938.
11. SOKOLOV, A. A. *On the classical theory of particles elementary (point electron*), Bulletin of Moscow State University, No 2, P. 33–48, 1947, (in Russian).
12. KONSTANTINOVICH A.V., NITSOVICH V.M. *The loss energy of electron moving in a spiral in transparent dielectric*, Izv. Vuzov. Fizika, No 2., P. 59-62, 1973 (in Russian).
13. KONSTANTINOVICH I.A., MELNYCHUK S.V., KONSTANTINOVICH A.V. *Classical radiation theory of charged particles. II. Medium influence on the radiation spectrum of charged particles moving in electromagnetic fields*, Bulletin of Chernivtsi National University, Physics and Electronics, No 132, P. 24-39, 2002 (in Ukrainian).
14. BOLOTOVSKII, B.M. *The theory of the Vavilov-Cherenkov effect*, Usp. Fiz. Nauk, **62**, No 3, P. 201-246, 1957 (in Russian).
15. SOKOLOV, A.A., ZHUKOVSKII, V.Ch., KOLESNIKOVA, M.M., NIKITINA, N.S., SHISHANIN O.E. *On the synchrotron radiation of the electron moving in a spiral*, Izv. Vuzov. Fizika, No 2, P.108–116, 1969 (in Russian).
16. SCHWINGER J., TSAI WU-YANG, ERBER T. *Classical and quantum theory of synergic synchrotron-Cherenkov radiation*, Ann. of Phys., **96**, No 2, P. 303–332, 1976.
17. AFANASIEV, G.N., KARTAVENKO, V.G., *Radiation of a point charge uniformly moving in a dielectric medium*, J. Phys. D: Appl. Phys., **31,** No 20, 21, P. 2760 – 2776, 1998.
18. AFANASIEV, G.N., SHILOV, V.M., STEFANOVSKY Yu.P. *New results in the Vavilov-Cherenkov radiation theory*, Nuovo Cimento, **117B.** No 7, P. 815-837, 2002.

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