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COMPUTATION OF SEPARATING LAMINAR BOUNDARY-LAYER FLOWS

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The purpose of this paper is to evaluate the accuracy with which the location of laminar separation can be predicted on two-dimensional and axisymmetric bodies using new boundary-layer equations. The evaluation was made by studying several flows for which comparisons with previously published results are possible. Predictions were also made for the separation points of some general classes of external flows for which complete solutions do not yes exist, including one that exhibits incipient separation. It was concluded from the study that the method predicts separation points with the reliability and accuracy needed for aerodynamic design purposes.

Key words: Laminar boundary-layer, viscous-inviscid interaction, separation.

1. INTRODUCTION

In many problems, it is necessary to know whether the boundary layer (either laminar or turbulent) will separate from the surface of a specific body. If it does, it is also necessary to know accurately where the flow separation will occur. This is quite important in many design problems. In the design of airfoils or hydrofoils, it is necessary to avoid flow separation in order to keep drag levels low. In designing for high lift, predicting separation points is a crucial part of the design problem.

The determination of the separation point in boundary-layer flows has been subject of many investigations over the past few decades. The usual procedure is to apply numerical methods to the governing partial differential equation, compute the full-field solution, and thereby obtain the streamwise station at which the wall shear stress becomes zero. This solution procedure is not without its difficulties; it is well know that the wall shear stress approaches zero in a singular fashion at the separation point, a fact that invariably gives rise to problems of numerical convergence there [1], [2], [3]. In the following sections, a new boundary-layer model, the basic features of which were previously introduced in Ref. [4], is applied for the determination of the separation point. The mathematical model is presented, a key assumption on the inviscid solution near the surface is corroborated, and solutions for several external flows (including one that exhibits incipient separation) are yielded.

2. BOUNDARY-LAYER MODEL

Consider the steady incompressible two-dimensional laminar boundary-layer flow past solid bodies for which the governing equation is [4]

$$f''' + ff'' - 2\xi\left[f'\, f'_{\xi} - f''\, f_{\xi}\right] - \beta f'^{2} = F''' + FF'' - 2\xi\left[F'\, F'_{\xi} - F''\, F_{\xi}\right] - \beta F'^{2} \tag{1}$$

where:

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$$\xi = \frac{1}{L}\int_0^x \frac{U_w}{U_\infty}\left(\frac{r}{R}\right)^{2k} \mathrm{d}\,x', \quad \eta = \left(\frac{r}{R}\right)^k \frac{U_w y}{\sqrt{2\nu U_\infty L\xi}}, \quad f = \int_0^\eta \frac{u}{U_w}\,\mathrm{d}\,\eta',$$

$$F = F_w(\xi) + \int_0^\eta \frac{U}{U_w}\,\mathrm{d}\,\eta', \quad F_w = \int_0^\delta (f' - F')\,\mathrm{d}\,\eta, \quad \beta = 2\xi(\ln U_w)_\xi.$$

Here, x and y are physical coordinates along the surface of the body and normal to it, respectively, with $x = 0$ at the leading edge and $y = 0$ at the body. The prime denotes the differentiation with respect to η and subscript ξ the differentiation with respect to ξ. L and R are reference lengths of the body, $k = 0$ for coplanar flows and $k = 1$ for axisymmetric flows. The subscripts w, δ and ∞ refer to conditions at the wall, at the local edge of the boundary layer and far upstream, respectively.

The associated boundary conditions are:

$$\eta = 0 : f(\xi,0) = f'(\xi,0) = 0 \tag{2}$$

$$\eta = \eta_\delta : f = F,\ f' = F' \tag{3}$$

Here the inviscid solution $F'(\xi,\eta)$ is known from the external flowfield and then the new boundary-layer equations (Eqs. (1-3)) provide the functions $f''(\xi,0), f'(\xi,\eta)$ and $F_w(\xi)$, that represent the wall shear stress, the boundary-layer velocity, and the wall mass flux, respectively. Generally, the inviscid solution is given rather as the velocity distribution on the body surface $U_w(x)$ than as the velocity field $U(x,y)$. However, by examination of several inviscid solutions evidence is given that the inviscid velocity field in close vicinity of body is slightly variable. Hence, it can assume for that a simple linear distribution:

$$F' = \frac{u}{U_w} = 1 + \left.\frac{\partial U}{\partial y}\right|_w \frac{y}{U_w} \tag{4}$$

Integrating the x-momentum equation from $y = 0$ to δ and substituting into Eq. (4) for the slope $\left.\frac{\partial U}{\partial y}\right|_w$, we obtain the intrinsic relationship:

$$F'(\xi,\eta) = 1 + \omega(\xi)\eta \tag{5}$$

where:

$$\omega(\xi) = f''(0) + \left(F_w^1 - F_w^2\right) + 2\xi\left(F_w^1 - F_w^2\right)_\xi - \beta F_w^2, \quad F_w^1 = \int_0^{\eta_\delta}(F' - f')\mathrm{d}\eta, \quad F_w^2 = \int_0^{\eta_\delta}\left(F'^2 - f'^2\right)\mathrm{d}\eta.$$

Now the solution of the equation (1), which contains information on the location of the separation point, is determinate if the value of the parameter β is known.

3. NUMERICAL RESULTS

We consider now the determination of the separation points for some general classes of external flows, previously studied.

Symmetric Flow Past Elliptic Cylinders and Ellipsoids. Consider the ellipse defined in the $x'y'$-plane with x'- axis intersects at –1 and +1 and y'- axis intersects at $-\varepsilon$ and $+\varepsilon$ and for which the equation is:

$$x'^2 + y'^2/\varepsilon^2 = 1$$

The elliptic cylinder is created by an infinite extension of the ellipse in the $+z$ and $-z$ directions and the ellipsoid is created by a rotation of the ellipse about the x- axis. If a uniform stream is approaching from $-x'$ direction the potential flow is given by:

$$\frac{U_w}{U_\infty}=\frac{1+\varepsilon P(\varepsilon)}{\sqrt{1+\frac{\varepsilon^2 x'^2}{1-x'^2}}} \quad ; \quad \beta=\frac{-2\varepsilon^2 x' Q(x')}{(1-x')\left[1-\left(1-\varepsilon^2\right)x'^2\right]}$$

where, for cylinders

$$P(\varepsilon)=1 \quad ; \quad Q(x')=1$$

and, for ellipsoids

$$P(\varepsilon)=\frac{\varepsilon\left(\operatorname{sech}^{-1}\varepsilon-\sqrt{1-\varepsilon^2}\right)}{\sqrt{1-\varepsilon^2}-\varepsilon^2\operatorname{sech}^{-1}\varepsilon} \quad ; \quad Q(x)=\frac{2-x'}{3(1-x')}$$

The parameter β at the leading edge for cylinders is β_o=1 and for ellipsoids is β_o=1/2, regardless of the value assigned to ε. The present method yields at separation for any value of ε the solutions β_s=-0.664 for cylinders and β_s=-0.439 for ellipsoids. Through the known external parameter $\beta(x',\varepsilon)$, we can compute for any value of ε the separation coordinate x'_s or, from the geometric relationship,

$$\Phi=\pi-\cos^{-1}\frac{x'}{\sqrt{\varepsilon^2+\left(1-\varepsilon^2\right)x'^2}}$$

the separation angle Φ_s.

The results are shown in Fig. 1 together with the solutions for the circular cylinder [5] and for the sphere [6].

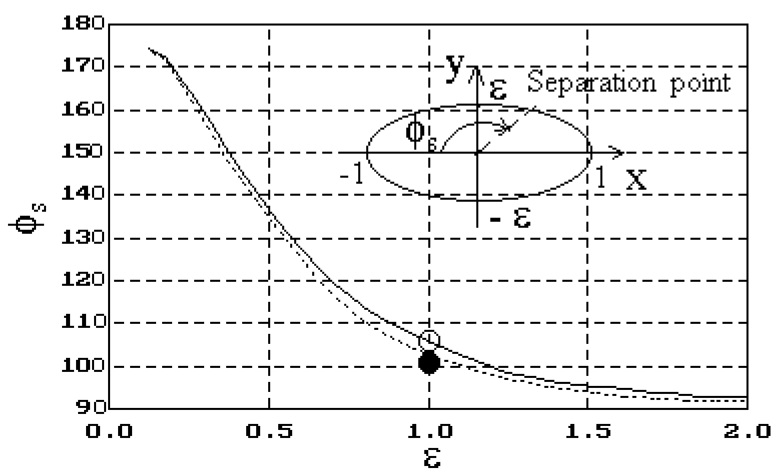

Fig. 1 – Separation points of the potential flow past elliptical cylinders (--- present prediction, • [5]) and ellipsoids (--- present prediction, o [6]).

Retarded Flows of the Howarth-Tani Type. For this class of retarded flows, the external parameters are given by:

$$\frac{U_w}{U_\infty}=1-x^n \quad ; \quad \beta=\frac{-2nx^n\left[1-x^n/(n+1)\right]}{\left(1-x^n\right)^2}$$

where we shall consider values of $n > 0$. It is easy to see that β_o=0 and the present method yields a different value of β_s for each value of n. Through the known external flow parameter $\beta(x,n)$ the location of the physical separation point can be determined. The results are shown in Fig. 2, together with the solutions of Howarth [7] for n=1 and Tani [8] for n=2,4 and 8.

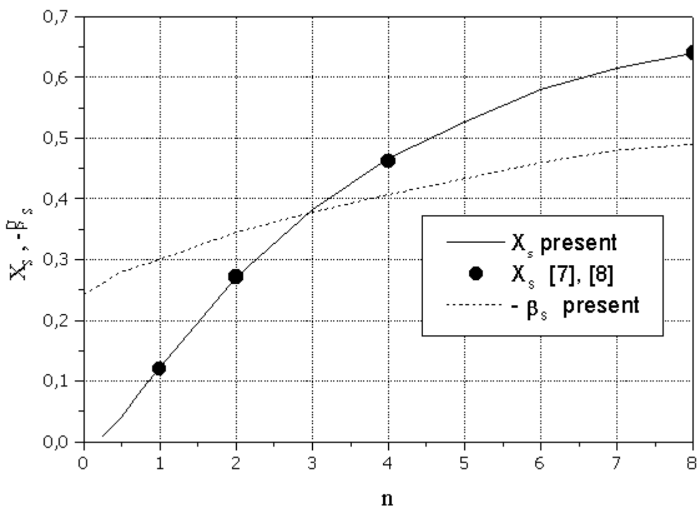

Fig. 2 – Separation points of Howarth-Tani flows (---- present prediction, • [7], [8]).

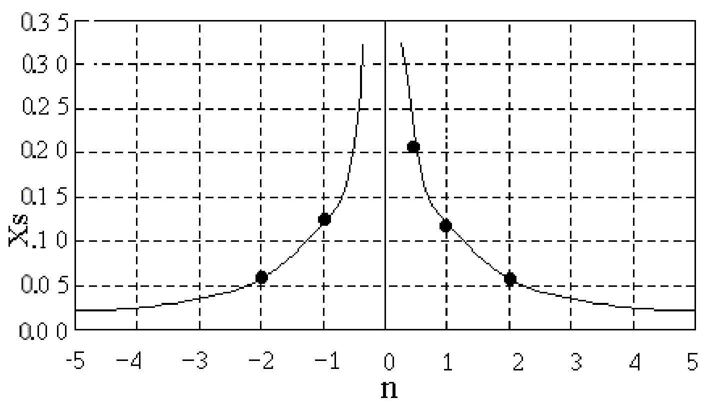

Fig. 3 – Separation points of the Görtler flows (--- present prediction, • [9]).

Retarded Flows of the Görtler Type. For this class of retarded flows the external parameters are given by

$$\frac{U_w}{U_\infty} = \left[1 - S(n)x\right]^n \quad ; \quad \beta = \frac{2n}{n+1}\left[1 - \frac{1}{\left[1 - S(n)x\right]^{n+1}}\right], \ n \neq -1 \quad ; \quad \beta = -2\ln(1+x) \quad ; \quad n = -1,$$

where $S(n)$ stands for sign of n. Regardless of the value of n, β_o=0 and the solution for β_s is the same as that for Howarth's linearly retarded flow (n=1), i. e. β_s=-0.3033. Through the known external flow parameter $\beta(x,n)$, the location of the physical separation point can be determined for any value of n. The results are shown in Fig. 3 together with the solutions of Görtler for n=1/2, 1, 2, -1 and –2 [9]. The present method yields $x_s \to 1$ as $n \to 0$ from above and $x_s \to \infty$ as $n \to \beta_s/(2-\beta_s)$ from below. In the range $\beta_s/(2-\beta_s) < n < 0$, the flow remains attached for all finite positive x.

Accelero-retarded Flows of the Falkner Type. For this class of flows, the external parameters are given by

$$\frac{U_w}{U_{\max}} = (\mathrm{e}x/n)^n \,\mathrm{e}^{-x}; \quad \left(\frac{U_w}{U_{\max}} = 1 \text{ for } x = n\right)$$
$$\beta = -2\left(\mathrm{e}^x - 1\right), \quad n = 0 \text{ (sharp leading edge)},$$
$$\beta = 2\frac{\left(nx^{-1} - 1\right)}{(\mathrm{e}x/n)^n \,\mathrm{e}^{-x}} \int_0^x \left(\frac{\mathrm{e}x'}{n}\right)^n \mathrm{e}^{-x'} \mathrm{d}x', \quad n \neq 0, 1 \text{ (wedged leading edge)},$$
$$\beta = 2(1-x)\left[\mathrm{e}^x - (1+x)\right]/x^2, \quad n = 1 \text{ (blunt leading edge)}.$$

The solution for β_s by the present method is different for each value of n. The results are shown in Fig. 4 together with the numerical results of Ref. [10] for $n = 0, 1/5, 1/3, 2/5, 3/5, 4/5$ and 1. Figure 4 evidences that the laminar boundary layer tolerates, without separating, smaller and smaller adverse pressure gradients the longer the accelerated path.

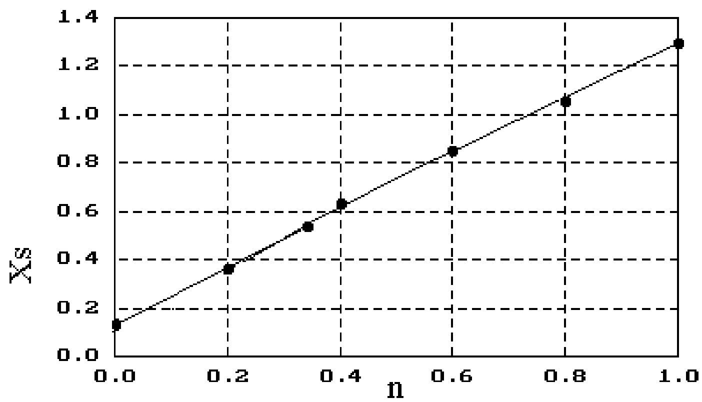

Fig. 4 – Separation points of the Falkner flows (--- present prediction, • [10])

Retarded Flows of the Curle Type (Incipient Separation). For this class of flows, the external parameters are given by

$$\frac{U_w}{U_\infty} = x - x^3 + \alpha x^5 \quad ; \quad \beta = \frac{\left(1 - x^2/2 + \alpha x^4/3\right)\left(1 - 3x^2 + 5\alpha x^4\right)}{\left(1 - x^2 + \alpha x^4\right)^2}.$$

Regardless of the value of the parameter $\alpha, \beta_0 = 1$ and the present yields the solution β_s=0.664. Through the known external flow parameter $\beta(x, \alpha)$, the location of the physical separation point can be determined for any value of α. The results are shown in Fig. 5 together with the solutions of Curle [11] for α=-0.12156, 0.0 and 0.07885. The results from the present method reveal that Curle's family of retarded flows contains a flow that exhibits incipient separation for the particular value $\alpha_i = 0.379$ at the streamwise station $x_{s,i} = 0.922$. For values $\alpha > \alpha_I$ separation does not occur at all, whereas for values $\alpha < \alpha_i$ separation occurs with decreasing sensitivity of the solution for x_s to the value of α.

4. CONCLUSIONS

Based on the calculations shown in this paper, the following conclusions can be made on the accuracy of calculating the laminar boundary-layer separation on the two-dimensional and axisymmetric bodies:

1. A direct method involving the particular inviscid velocity field in the neighbourhood of the body enables the accurate determination of the separation point from velocity distributions on the body surface alone.

2. The present method describes in a way the transition of the flow structure, in the immediate vicinity of the separation point, from the classical boundary-layer to that of a triple deck.
3. With back flow near the wall past separation, the method is stable in downstream direction so long as the thickness of the reversed-flow region is comparable to the boundary-layer thickness. For larger regions of reversed flow, the equations become unstable. In a consistent way, the flow in these larger separated-flow regions physically becomes unstable, and an unsteady boundary-layer model has to be used.

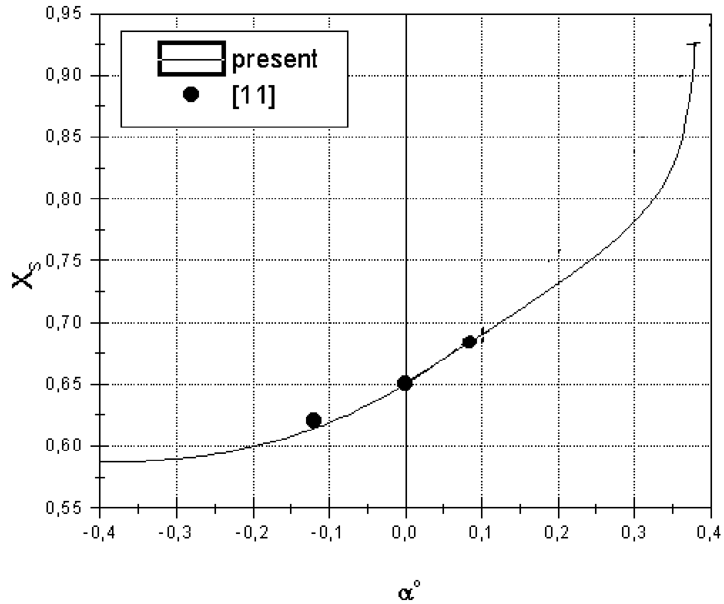

Fig. 5 – Separation points of the Curle flows (--- present method, • [11])

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