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NON-LINEAR EFFECTS IN SEISMIC BASE ISOLATION

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The base-isolated structural systems contain an isolatory layer composed by rubber like materials with obvious non-linear mechanical properties. The objective of this paper is to analyses the effects of these properties on base-isolation structure response. Using a non-linear one-degree-of-freedom model for superstructure-base system by numerical non-linear simulation one presents some features of the non-linear structural response.

1. INTRODUCTION

The objective of seismic isolation systems is to decouple the building structure from the damaging components of the earthquake impute motion. By interposing a layer with low horizontal stiffness but with high damping characteristics between the structure and the foundation, the superstructure is partially decoupled from the horizontal components of the earthquake ground motion.

In the last decade several base isolation systems have developed for seismic protection of the structure with the special destinations as hospitals, emergency communication centers, firestations, traffic management centers, historical buildings, a.s.o [12], [17], [21]. Performance of the base isolated buildings in different parts of the world during earthquakes in the recent past established that the base isolated technology is a viable alternative to the conventional earthquake resistant design of a large category of buildings.

Some of the commonly used isolation system is the laminated steel-rubber bearings (LRB) installed between structure and the foundation [12]. The LRB system consists of alternating layer of rubber and steel vulcanized between them. These elements supports entire vertical statically loads of the building and play a dissipative roll for horizontal seismic excitations. The rubber material supported permanent vertical compression and horizontal shearing loading during earthquake.

Another notable base-isolation system is based on pendulum column, which are introduced between structure and bloc foundations (PRB system) [22]. The system consists of a series of short pendulum column, with spherically steel calottes, placed between the superstructure and the foundation and laterally embedded in a mass of rubber. The gravitational loads are transported only throughout columns and rubber material is subjected to compression only during earthquake, therefore the fatigue risk is diminished. This system combines the advantages of kinematics systems with those of the laminated-rubber bearings.

The main performance of these isolation systems is governed by the dynamic behaviour of the isolatory materials as rubber, neoprene or another material, which play the same role. The dynamic properties of these materials such as horizontal stiffness and damping capacity determine the filtering role of the isolation layer and, finally, the structural dynamic response.

All of materials used in isolation layer systems exhibit, more or less, a non-linear behaviour. This fact is an indubitable experimental reality [1], [6], [16], [24]. But, many times this non-linearity is just like that ignored, other times the structural effects are presume inessential.

Depending of loading tip, one can encounter either softening or hardening non-linearity. Thus, for example, in shear or torsional loading the rubber exhibit a softening non-linearity and in compression tests the same material show hardening behaviour. Thus, it is possible that the structural response will depend not only on non-linearity himself but non-linearity tip also.

Reccomended by Panaite MAZILU, member of the Romanian Academy

The objective of this paper is to evaluate the effects of the non-linear behaviour of isolatory materials on dynamic building response. Using a simplified method a non-linear one-degree-of-freedom model for base-isolation structure are obtained and with the aid of this model a numerical comparative linear versus non-linear study are performed. As can see in the next, the non-linear behaviour of the isolatory material lead to the qualitative and quantitative different structural response by comparison with linear hypothesis. More than, the kind of non-linearity, the softening or hardening non-linearity has a considerable influence.

2. SIMPLIFIED METHOD FOR STRUCTURAL RESPONSE EVALUATION

Let a structure isolated from this base by a certain isolation system. Due to large differences between mechanical characteristics of the structural materials and isolatory materials two degree of freedom (2dof) simplified model can be used to predict the dynamic response of a such base isolated structure [20], [23].

The superstructure is assimilated to single degree of freedom (sdof) system (characterized by its generalized mass m_s, its generalized damping c_s and its generalized stiffness k_s) mounted on the base assimilated to another sdof system, characterized by its generalized mass m_b, its generalized damping c_b and its generalized stiffness k_b (fig. 2.1).

If x_g, x_b and x_s are ground, base and superstructure absolute displacements, the base and superstructure displacements relative to the ground is (fig. 2.2):

$$u_b = x_b - x_g \quad ; \quad u_s = x_s - x_g \tag{2.1}$$

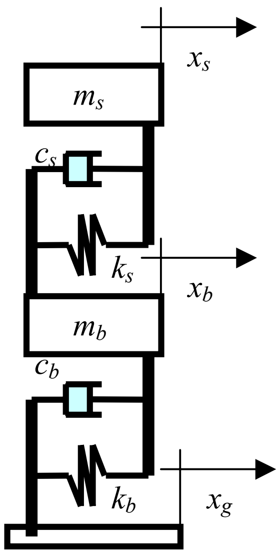


Fig. 2.1

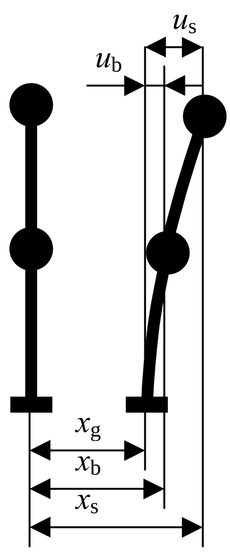


Fig. 2.2

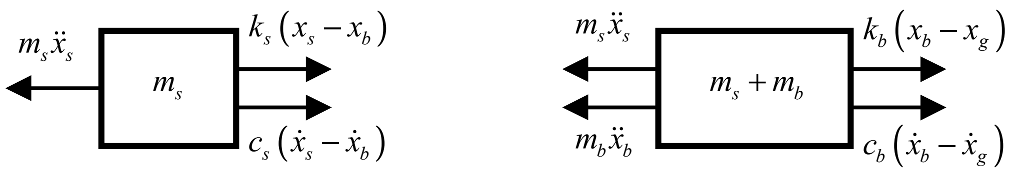


Fig. 2.3

In order to determine the motion equations one can disconnect the two masses as in fig. 2.3 and thus result:

$$\begin{cases} m_b\ddot{x}_b + m_s\ddot{x}_s + c_b\left(\dot{x}_b - \dot{x}_g\right) + k_b\left(x_b - x_g\right) = 0 \\ m_s\ddot{x}_s + c_s\left(\dot{x}_s - \dot{x}_b\right) + k_s\left(x_s - x_b\right) = 0 \end{cases} \tag{2.2}$$

Because:

$$x_b = u_b + x_g \quad ; \quad x_s = u_s + u_b + x_g \tag{2.3}$$

the system (2.2) become:

$$\begin{cases} m_b\ddot{u}_b + m_s\left(\ddot{u}_s + \ddot{u}_b\right) + c_b\dot{u}_b + k_b u_b = -\left(m_b + m_s\right)\ddot{x}_g \\ m_s\left(\ddot{u}_s + \ddot{u}_b\right) + c_s\dot{u}_s + k_s u_s = -m_s\ddot{x}_g \end{cases} \tag{2.4}$$

or, in matricial form:

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} = -\mathbf{M}\delta\ddot{x}_g \tag{2.5}$$

where the mass, damping and stiffness matrices are, respectively:

$$\mathbf{M} = \begin{bmatrix} m_t & m_s \\ m_s & m_s \end{bmatrix} \quad ; \quad \mathbf{C} = \begin{bmatrix} c_b & 0 \\ 0 & c_s \end{bmatrix} \quad ; \quad \mathbf{K} = \begin{bmatrix} k_b & 0 \\ 0 & k_s \end{bmatrix} \tag{2.6}$$

m_t being the total mass: $m_t = m_b + m_s$ and δ a position vector: $\delta = \{1 \quad 0\}^T$.

Assuming the structure together with its base as perfectly rigid body mounted on isolators, a single degree of freedom model result (fig. 2.4), with the circular frequency:

$$\omega_b^2 = \frac{k_b}{m_t} \tag{2.7}$$

Also, assuming the structure with fixed base another sdof system result (fig. 2.5), with the circular frequency:

$$\omega_s^2 = \frac{k_s}{m_s} \tag{2.8}$$

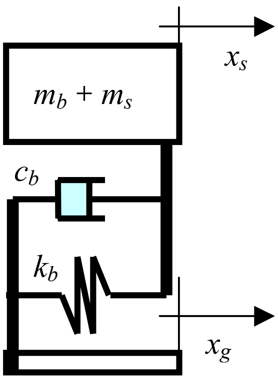


Fig. 2.4

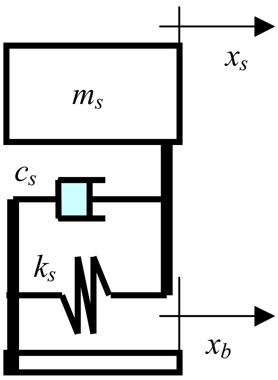


Fig. 2.5

By introducing the frequential ratio:

$$\varepsilon = \frac{\omega_b^2}{\omega_s^2} \tag{2.9}$$

and the mass ratio:

$$\mu = \frac{m_b}{m_s} \tag{2.10}$$

the dynamic matrix **D** become:

$$\mathbf{D} = \omega_s^2 \begin{bmatrix} \varepsilon\frac{1+\mu}{\mu} & -\frac{1}{\mu} \\ -\varepsilon\frac{1+\mu}{\mu} & \frac{1+\mu}{\mu} \end{bmatrix} \tag{2.11}$$

The fundamental frequencies of 2dof system result now as the eigenvalues of dynamic matrix in the form:

$$p_{1,2}^2 = \frac{\omega_s^2}{2}\left[\frac{(\mu+1)(\varepsilon+1)}{\mu} \mp \frac{(\mu+1)(\varepsilon-1)}{\mu}\sqrt{1+\frac{4\varepsilon}{(\mu+1)(\varepsilon-1)^2}}\right] \tag{2.12}$$

By developing the term under the radical in binomial series the fundamental frequencies of the 2dof system become:

$$p_1^2 \cong \omega_s^2 \frac{\varepsilon}{\varepsilon+1} = \omega_b^2 \frac{1}{\varepsilon+1} \quad ; \quad p_2^2 \cong \omega_s^2 \frac{(\mu+1)(\varepsilon+1)}{\mu} \tag{2.13}$$

and the normalized form of the fundamental modes result:

$$\mathbf{X}_1 = \{1 \quad \varepsilon+1\}^T \quad ; \quad \mathbf{X}_2 = \left\{1 \quad -\frac{\mu}{\varepsilon+1}\right\}^T \tag{2.14}$$

Because $\omega_s \gg \omega_b$ the frequential ratio ε has small values. In these conditions, the first fundamental frequency becomes: $p_1^2 \cong \omega_b^2$. This means that the first fundamental frequency of the base isolated buildings is close to the frequency of the sdof system constituted by rigid superstructures mounted on flexible base isolators and the isolated buildings tends to behave globally as a sdof system. Also, the expression for the second fundamental frequency denotes that the addition of the base isolators increases the structural frequency, in particular for small values of the mass ratio.

The modal participation factors are [14]:

$$L_i = \frac{\mathbf{X}_i^T \mathbf{M} \boldsymbol{\delta}}{\mathbf{X}_i^T \mathbf{M} \mathbf{X}_i} \tag{2.15}$$

thus, by replacement of the appropriate fundamental modes values (2.15) result:

$$L_1 = \frac{\mu+(\varepsilon+1)}{\mu+(\varepsilon+1)^2} \quad ; \quad L_2 = \frac{\varepsilon(\varepsilon+1)}{\mu+(\varepsilon+1)^2} \tag{2.16}$$

and by series development become:

$$L_1 = 1 - \frac{\varepsilon}{\mu+1} \quad ; \quad L_2 = \frac{\varepsilon}{\mu+1} \tag{2.17}$$

These relations show that the participation factor for the first mode approaches one, which denote the sdof behaviour. The participation factor for the second mode is very small, therefore even if the second mode frequency p_2^2 falls in the range of high spectral acceleration, the smallness of the participation factor ensures that the second mode is not highly excited by the ground motion.

In order to exemplify the above-simplified method and the behaviour of the base isolated structure in the next will be presented a comparative study of a structure with and without base isolation.

Mechanical characteristics of the superstructure are choose as in [23]:

$$m_s = 29485 \text{ kg} \quad ; \quad c_s = 23710 \text{ Ns/m} \quad ; \quad k_s = 11912000 \text{ N/m}$$

Assuming the structure with fixed base a sdof system result (as in fig. 2.5), with the pulsation ω_s, the natural frequency f_s and the natural period T_s:

$$\omega_s = \sqrt{\frac{k_s}{m_s}} = \sqrt{\frac{11912000}{29485}} = 20 \text{ rad/s} \quad ; \quad f_s = \frac{\omega_s}{2\pi} = \frac{20}{2\pi} \cong 3.2 \text{ Hz} \quad ; \quad T_s = \frac{1}{f_s} = \frac{1}{3.2} = 0.313 \text{ s}$$

We notice that these dynamic characteristics of superstructure are close to predominant dynamics characteristics for a usual site (consolidated aluvionary deposits). Therefore, this structure is a proper candidate for isolated base technology.

Mechanical characteristics of the base with the isolated layer (supposed linear in [23]) are:

$$m_b = 6800 \text{ kg} \quad ; \quad c_b = 3740 \text{ Ns/m} \quad ; \quad k_b = 232000 \text{ N/m}$$

Assuming the structure together with its base as perfectly rigid body mounted on isolators, a sdof model result (as in fig. 2.5), with the following dynamic characteristics:

$$\omega_b = \sqrt{\frac{k_b}{m_b + m_s}} = \sqrt{\frac{232000}{36285}} = 2.53 \text{ rad/s} \quad ; \quad f_b = \frac{\omega_b}{2\pi} = \frac{2.53}{2\pi} \cong 0.4 \text{ Hz} \quad ; \quad T_s = \frac{1}{f_s} = \frac{1}{0.4} = 2.5 \text{ s}$$

The massic rapport is in this case:

$$\mu = \frac{m_b}{m_s} = \frac{6800}{29485} = 0.23$$

and pulsation rapport:

$$\varepsilon = \frac{\omega_b^2}{\omega_s^2} = \frac{6.394}{404} = 0.016$$

are small enough to make acceptable the approximations used in eq. (2.16) and from these relations, the natural pulsation of the 2dof-system result:

$$p_1 = \sqrt{\frac{1}{\varepsilon + 1}} \cdot \omega_b = \sqrt{\frac{1}{0.016 + 1}} \cdot \omega_b = 0.992\omega_b = 0.992 \cdot 2.53 = 2.51 \text{ rad/s}$$

$$p_2 = \sqrt{\frac{(\mu + 1)(\varepsilon + 1)}{\mu}} \cdot \omega_s = \sqrt{\frac{(0.23 + 1)(0.016 + 1)}{0.23}} \cdot \omega_s = 2.331\omega_s = 2.331 \times 20 = 46.62 \text{ rad/s}$$

Thus, the natural frequency and natural period of the 2dof system are:

$$f_1 = \frac{p_1}{2\pi} = \frac{2.51}{2\pi} \cong 0.4 \text{ Hz} \quad ; \quad T_1 = \frac{1}{f_1} = \frac{1}{0.4} = 2.5 \text{ s}$$

$$f_2 = \frac{p_2}{2\pi} = \frac{46.62}{2\pi} \cong 7.424 \text{ Hz} \quad ; \quad T_2 = \frac{1}{f_2} = \frac{1}{7.424} = 0.135 \text{ s}$$

and from eq. (2.20) the participation factors are:

$$L_1 = 1 - \frac{\varepsilon}{\mu + 1} = 1 - \frac{0.016}{0.23 + 1} = 1 - 0.013 = 0.987$$

$$L_2 = \frac{\varepsilon}{\mu + 1} = \frac{0.016}{0.23 + 1} = 0.013$$

One can observed that due to the isolator layer the dynamic characteristics undergo a jump:

$$f_s = 3.2 \text{ Hz} \quad \Rightarrow \quad \begin{cases} f_1 = 0.4 \text{ Hz} \\ f_2 = 7.424 \text{ Hz} \end{cases} \quad ; \quad T_s = 0.314 \text{ s} \quad \Rightarrow \quad \begin{cases} T_1 = 2.5 \text{ s} \\ T_2 = 0.135 \text{ s} \end{cases}$$

which for first vibration mode take out the structure from dangerous zone (fig.2.7). For second vibration mode the structure remain in the dangerous zone but due to small participation factor ($L_2 = 0.013$) the structural effects are negligible.

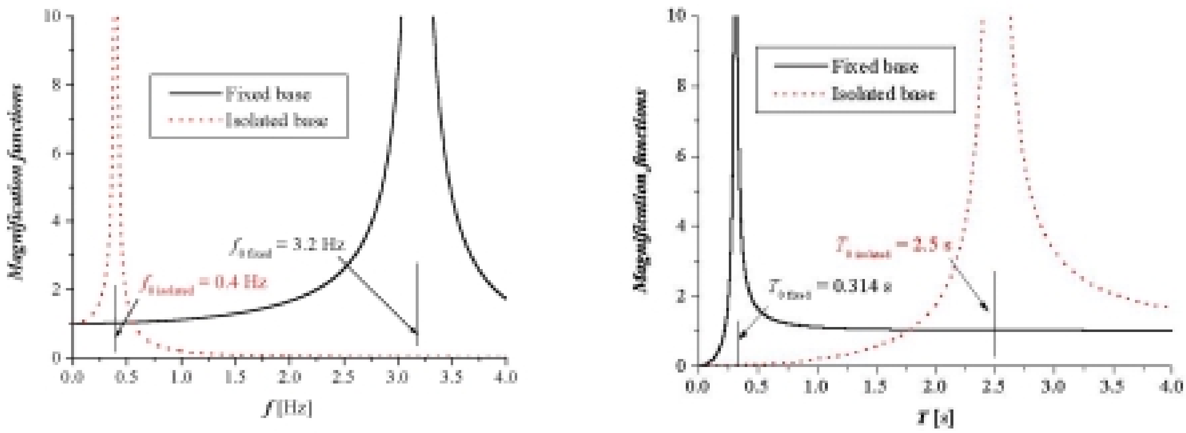


Fig. 2.7 Magnification functions in term of frequency and period

The above example prove that the quasi-rigid behaviour of the building from base isolated structures allows to use a simplified single degree of freedom modell to predict the dynamic behaviour. In order to simplify this demonstration in this example we preserve the linear mechanical characteristics for superstructure and isolated layer used in [23]. For the structural materials this hypothesis is acceptable but for isolatory material is at least moot. Experimental studies performed upon either material sample or whole isolator prove evident strain dependence behaviour [1], [6], [7], [9], [16], [24]. For this reason the simplified sdof system must be nonlinear.

Finally, we notice that:

- The isolated layer gives the structure a fundamental frequency that is much lower than its fixed-base frequency and also much lower than the usual predominant frequencies of the ground motion.
- The first dynamic mode of the isolated structure involves deformation only in the isolation system, the structure above being to all intents and purposes rigid.
- The higher modes have high frequencies but do not participate in the motion due to its small participation factor.
- The quasi-rigid behaviour of the building from base isolated structures allows using a simplified single degree of freedom modell. But, due to nonlinear characteristics of the isolated layer materials this sdof system must be nonlinear.

3. NONLINEAR SINGLE-DEGREE-OF-FREEDOM MODEL

As can see in the previous chapter, for simplified evaluation of the structure-base isolated system a non-linear sdof model are needful. Also, all devices used for experimental determination of the dynamic characteristics of the isolated layer materials are in fact sdof systems subjected to steady-harmonic excitations at top side (e. g. resonant column apparatus [7], [15], [25]) or by base displacement (e. g. electrodynamics shaker)[24]. For this reasons, in this chapter will be present a nonlinear single-degree-of-freedom obtained as an extension in non-linear domain of the linear Kelvin-Voigt model.

In linear dynamics a usual description of a solid single-degree-of-freedom behaviour is given by the Kelvin-Voigt model consisting of a spring (with a stiffness *k*) and a dashpot (with a viscosity *c*) connected in parallel. The governing equation of this system for harmonic vibrations is [7], [10], [13]:

$$M\ddot{x} + c \cdot \dot{x} + k \cdot x = A_0 \cdot \sin\ \omega t\,, \tag{3.1}$$

where x is the system's displacement (linear displacement x or rotation θ), M is the mass characteristics (mass m or massic moment of inertia J), A_0 is external force amplitude (F_0 or M_0) and ω is the pulsation of this force.

Dynamic characteristics c and k have for linear elastic materials known expression. Thus, for a rod subjected to longitudinal excitations we have:

$$k = \frac{S}{h}E \quad , \quad c = 2m\omega_0\zeta_{long}\,, \tag{3.2}$$

where S is the cross sectional area, h is the high, E is the Young's modulus, m is the mass, ω_0 is the natural pulsation and ζ_{long} is the damping ratio. Also, for the same circular rod but subjected to torsional excitation we have:

$$k = \frac{I_p}{h}G \quad , \quad c = 2J\omega_0\zeta_{tors}\,, \tag{3.3}$$

where I_p is the rotational moment of inertia, G is the shear modulus, J is the massic moment of inertia and ζ_{tors} is the damping ratio for this type of loading.

Using the method that describes the non-linearity by strain or displacement dependence of the material moduli: $E = E(\varepsilon)$ or $E = E(x)$ and $G = G(\gamma)$ or $G = G(\theta)$ [2], [3], [4], [7] we shall assume that the spring stiffness k and the damper viscosity c are functions in terms of displacement:

$$\begin{aligned} k(x) &= \frac{S}{h}E(x) \quad , \quad c(x) = 2m\omega_0\zeta_{long}(x), \\ k(\theta) &= \frac{I_p}{h}G(\theta) \quad ; \quad c(\theta) = 2J_0\omega_0\zeta_{tors}(\theta) \end{aligned} \tag{3.4}$$

We mention that as in this non-linear case the spring stiffness is a function, we shall define the undamped natural pulsation in terms of initial value of stiffness function $k(0) : \omega_0 = \sqrt{k(0)/M}$.

Therefore the most expected form of the governing equation for non-linear behaviour of a single-degree-of-freedom system is:

$$M\ddot{x} + c(x) \cdot \dot{x} + k(x) \cdot x = A_0 \cdot \sin\ \omega t\,, \tag{3.5}$$

with the analogic non-linear Kelvin-Voigt (NKV) model from fig. 3.1 [11], [19].

For complete determination of the eq. (3.5) the material functions $c(x)$ and $k(x)$ from eqs. (3.4) must be evaluation from experimental data.

In order to exemplify this nonlinear extension in the next will use the experimental data obtained from torsional resonant column test performed upon rubber sample.

The modulus and damping functions in terms of rotation θ was obtained with a reasonable accuracy in exponential form (fig.3.2).

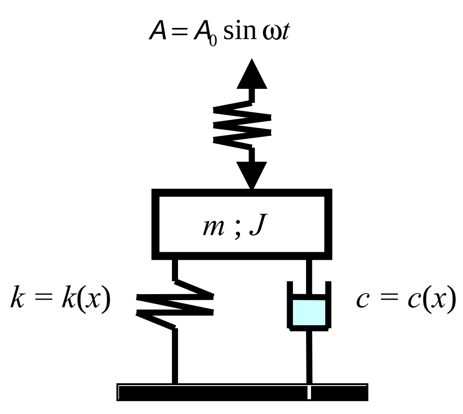
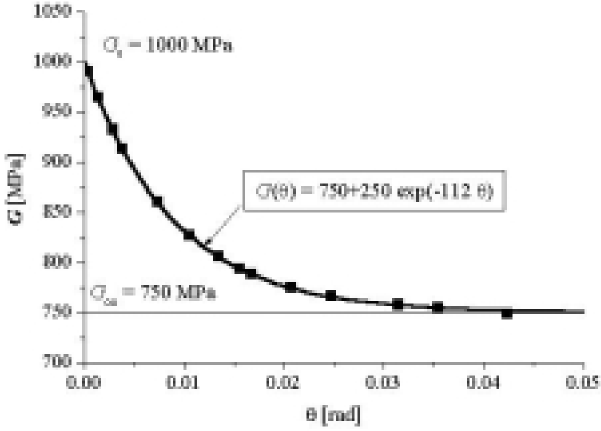


Fig. 3.1 NKV model



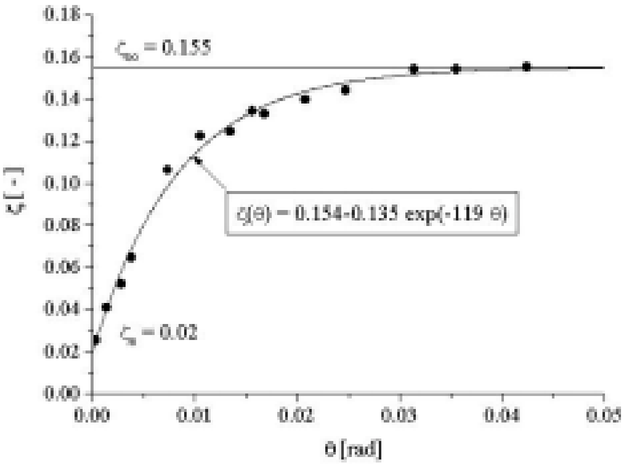


Fig. 3.2 Modulus $G = G(\theta)$ and damping $\zeta = \zeta(\theta)$ functions for tested rubber

In these figures one can see that material functions have similar forms with the relaxation and creep functions of the analogic standard viscoelastic solid [11], [19]. This can denote that a solid standard model is more adequate then NKV model. But, changing the parameters of the linear Kelvin-Voigt model with the non-linear material functions with initial values leads in fact to a nonlinear standard model. Because in the next will not use analogic models, the denomination *non-linear Kelvin-Voigt model* (NKV model), as an extension of the linear Kelvin-Voigt model, was preserve.

Take into account the sample dimensions (height h =7.4 cm and diameter ϕ = 3.2 cm) and mass characteristics ($m = 147$g, $J_0 = 2.8 \cdot 10^{-3}$ Nms^2) from eqs. (3.9) the dynamics nonlinear function of the NKV model result as in fig.3.3.

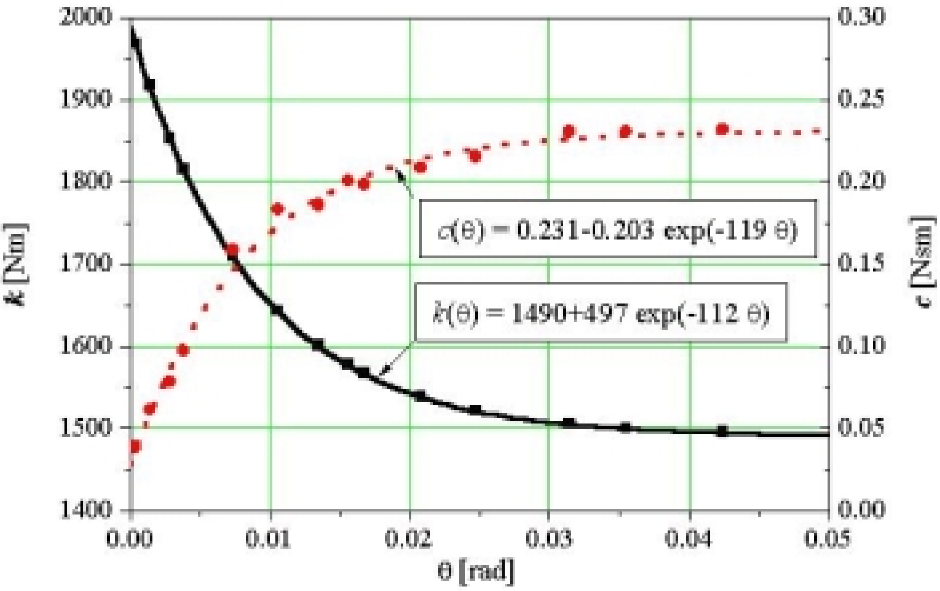


Fig. 3.3 NKV functions

4. EVALUATION OF THE NON-LINEAR SDOF RESPONS

After the qualitative and quantitative determination of the dynamic material functions $c(x)$ and $k(x)$ the differential equation of the non-linear sdof system (3.5):

$$M\ddot{x} + c(x)\cdot\dot{x} + k(x)\cdot x = A_0 \sin\omega t \tag{4.1}$$

can be numerical solved for system displacement x. Because the dimensionless equations assure a better numerical accuracy solving it is preferable to transform the eq. (4.1) into a dimensionless equation.

Thus, by using the change of variable $\tau = \omega_0 t$ and by introducing a new "time" function [5]:

$$\varphi(\tau) = x(t) = x\left(\frac{\tau}{\omega_0}\right) \tag{4.2}$$

one obtain for eq. (4.1) another form:

$$\varphi'' + C(\varphi)\cdot\varphi' + K(\varphi)\cdot\varphi = \mu\cdot\sin\upsilon\tau \tag{4.3}$$

where the superscript accent denotes the time derivative with respect to τ:

$$\varphi'(\tau) = \frac{\partial\varphi}{\partial\tau} = \frac{1}{\omega_0}\dot{x} \quad ; \quad \varphi''(\tau) = \frac{\partial^2\varphi}{\partial\tau^2} = \frac{1}{\omega_0^2}\ddot{x} \tag{4.4}$$

and:

$$\begin{aligned} &C(\varphi) \equiv C(x) = \frac{c(x)}{M\omega_0} = 2\zeta(x) \quad ; \quad K(\varphi) \equiv K(x) = \frac{k(x)}{M\omega_0^2} = \frac{k(x)}{k(0)} = k_n(x) \\ &\mu = \frac{A}{M\omega_0^2} = \frac{A}{k(0)} = x_{st} \quad ; \quad \upsilon = \frac{\omega}{\omega_0} \end{aligned} \tag{4.5}$$

For torsional excitation $M = M_0 \sin\omega t$ when the displacement is the rotation θ the (4.3) form is a dimensionless one. But, if the excitation is of longitudinal tip $F = F_0 \sin\omega t$ the transformed equation (4.3) have length dimension. In this case one can obtain a dimensionless form by introducing a new τ function:

$$\psi(\tau) = \varphi(\tau)/\mu \tag{4.6}$$

and the dimensionless longitudinal equation result in the form:

$$\psi'' + 2\zeta(\mu\psi)\cdot\psi' + k_n(\mu\psi)\cdot\psi = \sin\upsilon\tau \tag{4.7}$$

where:

$$\varphi(\tau) = \mu\psi(\tau) \quad ; \quad \varphi'(\tau) = \mu\psi'(\tau) \quad ; \quad \varphi''(\tau) = \mu\psi''(\tau) \tag{4.8}$$

For a given normalized amplitude μ and relative pulsation υ, the non-linear dimensionless equation (4.3) or (4.7) can be numerically solved and a solution of the form:

$$\varphi(\tau;\mu,\upsilon) = \mu\psi(\upsilon;\tilde{\mu}) = \mu\cdot\Phi(\upsilon;\mu)\cdot\sin(\upsilon\tau + \Psi), \tag{4.9}$$

can be obtained in n discreet points [5]. In eq. (4.9) $\Phi(\upsilon;\mu)$ are *the maximum magnification functions* (one function $\Phi(\upsilon)$ for each normalized amplitude μ) and Ψ is *the phase difference*. Finally, from inverse transformations the quested solution $x = x(t;A_0,\omega)$ is obtained.

When the non-linear system are subjected to a harmonic acceleration applied at the system base (as in seismic case):

$$\ddot{u}(t)=\ddot{u}_0 \sin \omega t \tag{4.10}$$

the movement equation has the form:

$$m\ddot{x}+c(x)\cdot\dot{x}+k(x)\cdot x=-m\ddot{u}_0 \sin \omega t \tag{4.11}$$

or by using eqs. (3.4):

$$\ddot{x}+2\omega_0\zeta(x)\cdot\dot{x}+\omega_0^2 k_n(x)\cdot x=-\ddot{u}_0 \sin \omega t \tag{4.12}$$

By the same variable change (4.2) a new transformed equation like eq. (4.3) but with length dimension, are obtained:

$$\varphi''+2\zeta(\varphi)\cdot\varphi'+k_n(\varphi)\cdot\varphi=-\mu\cdot\sin\upsilon\tau \tag{4.13}$$

where the normalized amplitude μ is in this case:

$$\mu=\frac{\ddot{u}_0}{\omega_0^2} \tag{4.14}$$

Because the excitation has harmonic form the base displacement may be in the form:

$$u(t)=-u_0 \sin\omega t=-\frac{\ddot{u}_0}{\omega^2}\sin\omega t \tag{4.15}$$

thus:

$$\mu=\frac{\ddot{u}_0}{\omega_0^2}=u_0\frac{\omega^2}{\omega_0^2}=u_0\upsilon^2 \tag{4.16}$$

The dimensionless form of the eq. (4.13) can be obtained by introducing in eq. (4.13) the τ function:

$$\psi(\tau)=\varphi(\tau)/u_0 \tag{4.17}$$

that lead to:

$$\psi''+2\zeta(u_0\psi)\cdot\psi'+k_n(u_0\psi)\cdot\psi=\tilde{\mu}\sin\upsilon\tau \tag{4.18}$$

where $\tilde{\mu}=\mu/u_0=\upsilon^2$.

Now, the dimensionless equation (4.18) can be numerical solved and solution $\psi=\psi(\tau;\tilde{\mu})$ can be used for obtained the solution of the eq. (4.13):

$$\varphi(\tau;\mu,\upsilon)=u_0\psi(\upsilon;\tilde{\mu})=u_0\cdot\Phi(\upsilon;\tilde{\mu})\cdot\sin(\upsilon\tau+\Psi), \tag{4.19}$$

and then $x=x(t;\ddot{u}_0,\omega)$

5. NON-LINEARITY INFLUENCE

The modelling of the base isolated structures as nonlinear single-degree-of-freedom and numerical solving possibility of this system allows as to qualitative evaluate the influence of the isolatory material non-linearity on structural response.

All of materials used in isolation layer systems of the base isolated structures exhibit, more or less, a non-linear behaviour. This fact is an indubitable experimental reality. Many times this non-linearity is just like that ignored, other times are neglected. Depending of loading tip, one can encounter either softening or hardening non-linearity. Thus, for example, in shear or torsional loading the rubber exhibit a softening non-linearity and in compression tests the same material show hardening behaviour. Thus, it is possible that the structural response depend not only on non-linearity himself but non-linearity tip also.

In order to evaluate the non-linear behaviour and linear - non-linear differences, a numerical simulation study was performed. For this, the same base-isolated structure was using, first neglecting the isolatory layer non-linearity and then discard this simplification. Also, the structural response was determinate using isolators with softening non-linearity as well as with hardening non-linearity. For detect the softening - hardening differences the same structure mounted upon two different isolator layers was used. In the first case was provided the LRB (Laminated Rubber Bearings) isolators where the rubber is subjected to shear and exhibit softening non-linearity and in the second case, the PRB (Pendulum Rubber Bearing) isolators where the rubber is subjected to compression and exhibit hardening non-linearity. In both cases a comparison with linear approximations will be made.

The structure used in these simulations was the same structures used in chapter 2 for exemplify the difference between fixed and isolated base [23]. Recall that in the chapter's 2 example for isolated layer was assumed a linear behaviour with the dynamic characteristics:

$$m_b = 6800 \text{ kg} \quad ; \quad c_b = 3740 \text{ Ns/m} \quad ; \quad k_b = 232000 \text{ N/m}$$

The non-linear stiffness $k = k(x)$ was builder by extension starting to own test [6] and experimental data given in [17] for softening behaviour and [24] for hardening behaviour:

$$k_{softening} = 186000 + 46000 \exp(-450x) \quad \text{[N/m]}$$
$$k_{hardening} = 232000 + 2760 \exp(450x) \quad \text{[N/m]}$$

Also, for detect the structural differences between softening and hardening behaviour in both cases the same damping function $\zeta = \zeta(x)$ was used:

$$\zeta(x) = 0.05 - 0.03 \exp(-25x)$$

In order to compare the non-linear results with the linear calculus from chapter 2 the initial values of these non-linear material functions was scaled for coincide with constant values of the linear behaviour hypothesis. We notice that material functions $k = k(x)$ and $\zeta = \zeta(x)$ was choose with middle non-linearity as observed in tests performed upon whole isolator [16] though in laboratory test performed upon rubber sample a more pronounced strain dependence was obtained [6].

The dimensionless material functions (see eq.4.5) are given in fig.5.1.

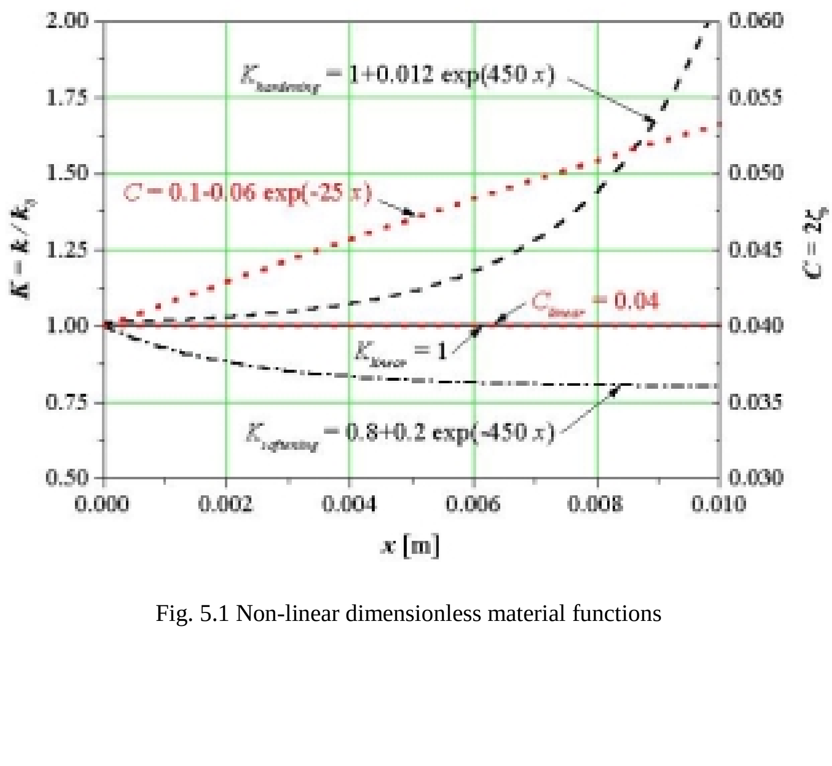


Fig. 5.1 Non-linear dimensionless material functions

The abutment excitation used in the simulation process was of harmonic type: $\ddot{x}_g = \ddot{x}_g^0 \sin \omega t$ with the amplitude values $\ddot{x}_g^0$ corresponding to peak ground acceleration observed during some eartqkuakes [23].

The structural non-linear response was obtained using a computer program based on Newmark algorithm [18] and solving method from chapter 3. The simulation result is summarized in fig. 5.2. From this figure, result some observations:

- Both non-linearity types take out the structure from dangerous zone. But, the jump toward high periods is different from linear one. Whereas the linear calculus lead to the unique resonance value the real non-linear solving lead to the multiple resonant values (in terms of excitation amplitudes) situated before and after linear resonant value.
- The non-linear magnification functions are different shapes in comparison with the linear one.
- At resonance, the amplitude of the non-linear magnification functions are inferior vis-a-vis the maximum amplitude of linear magnification function, thus non-linear calculus makes apparently the damping capacity neglected by linear calculus.
- For period values until resonance the linear calculus may underestimate the dynamic magnification.

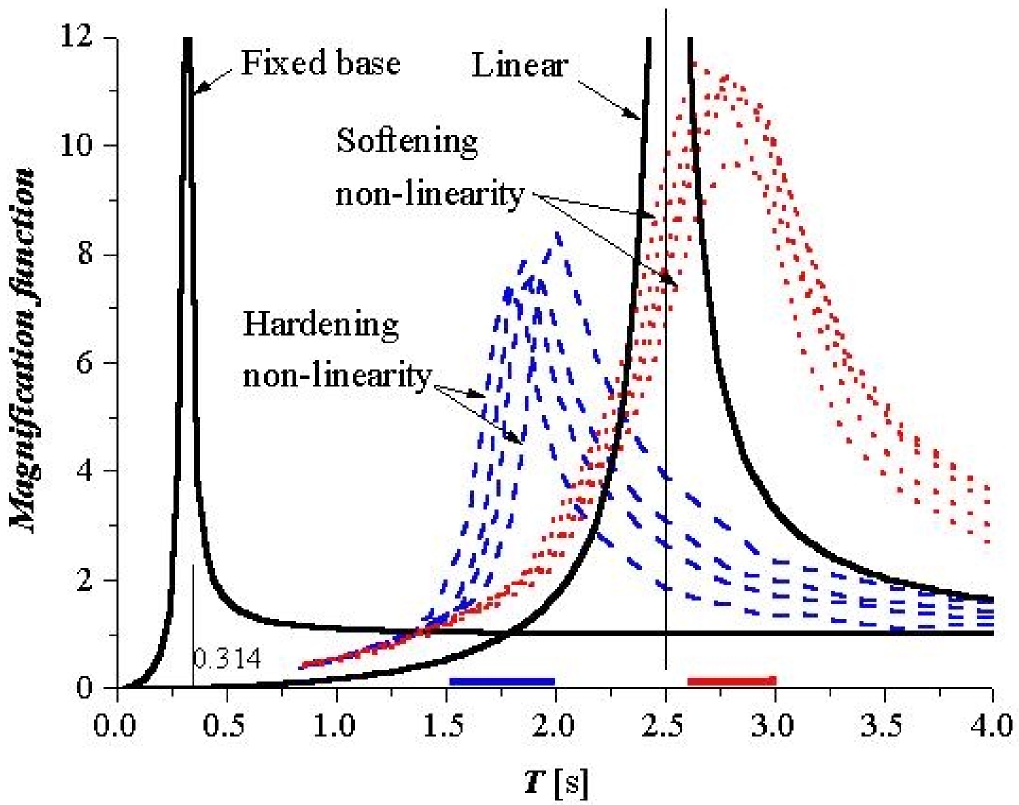


Fig. 5.2 Non-linear magnification functions

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