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BEHAVIOR IN IDEAL DIAGNOSTIC TESTS OF ROCK LIKE MATERIALS DESCRIBED BY DIFFERENT CONSTITUTIVE MODELS

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The aim of this paper is to realise a comparision among the theoretical results for some diagnostic tests on rock like materials, performed using two constitutive models: linear viscoelastic and elasto/viscoplastic. There are mentioned some qualitative properties of the behavior of the solutions.

Key words: creep, transient creep, relaxation,constitutive models

1. INTRODUCTION

For practical reasons, there is required a more and more complete and complex description of the rheological behavior of rocks. For some simplier constitutive models, like the linear viscoelastic model, there exist analytical solutions for many fundamental problems associated to the practical applications like the calculus of mining structures [1]. For more complex constitutive models, like Cristescu´s elasto/viscoplastic model [2], an analytical solution for the same fundamental problems is impossible to be obtained and only the numerical approach of these is suitable.

The theoretical solutions obtained for the linear viscoelastic model afford qualitative interpretations based on the constitutive restrictions, therefore the numerical values of the material constants do not have a signifiant influence on the qualitative aspects delivered by the analytical solutions. On the contrary, even the determination of the material functions for the Cristescu´s elasto/viscoplastic model is very difficult and therefore the results are more or less sure (the numerical results are very dependent on the values involved in the material functions, the material functions are not subject to constitutive restrictions).

The aim of this paper is to realise a comparision among the theoretical and the numerical results for some diagnostic tests performed using the two constitutive models mentioned before.

2. THE CONSTITUTIVE EQUATIONS

The linear viscoelastic model for isotropic and homogeneous materials has the form

$$\begin{cases} \dot{\boldsymbol{\varepsilon}}^{/} = -k\left(\boldsymbol{\varepsilon}^{/} - \dfrac{1}{2G_o}\boldsymbol{\sigma}^{R/}\right) + \dfrac{1}{2G}\dot{\boldsymbol{\sigma}}^{R/}, \\ \dot{\varepsilon} = -k_v\left(\varepsilon - \dfrac{1}{3K_o}\sigma^{R}\right) + \dfrac{1}{3K}\dot{\sigma}^{R}, \end{cases} \tag{2.1}$$

where

$$\boldsymbol{\sigma}^{R} = \boldsymbol{\sigma} - \boldsymbol{\sigma}^{K}, \sigma^{R} = \frac{1}{3}\mathrm{tr}\,\boldsymbol{\sigma}^{R}, \boldsymbol{\sigma}^{R/} = \boldsymbol{\sigma}^{R} - \sigma^{R}\mathbf{1}, \varepsilon = \frac{1}{3}\mathrm{tr}\boldsymbol{\varepsilon}, \boldsymbol{\varepsilon}^{/} = \boldsymbol{\varepsilon} - \varepsilon\mathbf{1}, \tag{2.2}$$

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$\boldsymbol{\sigma}$ – the current stress tensor, $\boldsymbol{\sigma}^{\mathrm{K}}$ – the stress tensor in the reference configuration, $\boldsymbol{\sigma}^{\mathrm{R}}$ – the relative stress tensor, $\boldsymbol{\varepsilon}$ – the infinitesimal strain tensor with respect to the reference configuration, " $\dot{}$ " denotes time derivatives. For this model the reference configuration correspond to a preloading for the diagnostic tests or for the primary stress state for the calculus of the mining structures, in any case, generally, not free of stress. The material constants are subject to the following constitutive restrictions ([2], [3])

$$0 < 2G < 3K,\ 0 < k \le k_v,\ k\left(\frac{1}{2G} - \frac{1}{2G_o}\right) \le k_v\left(\frac{1}{3K} - \frac{1}{3K_o}\right) < 0. \tag{2.3}$$

The inequality $k > 0$ is in agreement to the hypothesis of using the infinitesimal strain tensor and thefore only the compressibility of the volume is admited; other situations for the coefficients of viscosity k and k_v may be tolerated if the material constants are depending on the reference configuration, but only as long the hypothesis of using the infinitesimal strain tensor comes up to. The other inequalities are binding upon all situations.

The Cristescu´s elasto/viscoplastic model, for transient creep, for isotropic and homogeneous materials has the form

$$\dot{\boldsymbol{\varepsilon}} = \dot{\boldsymbol{\varepsilon}}^{\mathrm{E}} + \dot{\boldsymbol{\varepsilon}}^{\mathrm{I}},\quad \begin{cases} \dot{\boldsymbol{\varepsilon}}^{\mathrm{E}} = \dfrac{1}{2G}\boldsymbol{\sigma} + \left(\dfrac{1}{3K} - \dfrac{1}{2G}\right)\sigma\mathbf{1}, \\ \dot{\boldsymbol{\varepsilon}}^{\mathrm{I}} = k_T\left\langle 1 - \dfrac{W(t)}{H(\boldsymbol{\sigma})}\right\rangle \dfrac{\partial F}{\partial \boldsymbol{\sigma}}, \end{cases} \tag{2.4}$$

where it was used the notation $\langle A\rangle = \frac{1}{2}\left(A + |A|\right)$ for the positive part of the function A,

$$\sigma = \frac{1}{3}\mathrm{tr}\boldsymbol{\sigma},\ \boldsymbol{\sigma}' = \boldsymbol{\sigma} - \sigma\mathbf{1},\ \overline{\sigma} = \sqrt{\frac{3}{2}\boldsymbol{\sigma}'\cdot\boldsymbol{\sigma}'}, \tag{2.5}$$

$H(\boldsymbol{\sigma}) = H(\sigma, \overline{\sigma})$ is a yield function with

$$H(\boldsymbol{\sigma}(t)) = W(t) \tag{2.6}$$

the equation of the stabilization boundary (when $\dot{\boldsymbol{\varepsilon}}^{\mathrm{I}} = \mathbf{O}$ and $\dot{\boldsymbol{\sigma}} = \mathbf{O}$) with

$$W(t) = W(0) + \int_0^t \boldsymbol{\sigma}(s)\cdot\dot{\boldsymbol{\varepsilon}}^{\mathrm{I}}(s)\,\mathrm{d}s \tag{2.7}$$

the irreversible stress power used as a work-hardening parameter, $F(\boldsymbol{\sigma}) = F(\sigma, \overline{\sigma})$– a viscoplastic potential establishing the orientation of $\dot{\boldsymbol{\varepsilon}}^{\mathrm{I}}$, k_T a kind of viscosity coefficient (the true viscosity coefficient $k_T\,\dfrac{\partial F}{\partial \boldsymbol{\sigma}}$).

The parameter W is not a real state variable because his evolution equation is a consequence of ()

$$\dot{W} = k_T\left\langle 1 - \frac{W}{H}\right\rangle \frac{\partial F}{\partial \boldsymbol{\sigma}}\cdot\boldsymbol{\sigma} \tag{2.8}$$

In the representation of this model, there is not clear which reference configuration is used. In some of the applications the argument of H or F is a relative stress, in others the current stress. Further it will be used the current stress. Habitually,

$$H(\sigma, \overline{\sigma}) = \begin{cases} H_1(\sigma, \overline{\sigma}) + H_0(\sigma) & \text{for } \sigma \le \sigma_o, \\ H_1(\sigma, \overline{\sigma}) + H_0(\sigma_o) & \text{for } \sigma \le \sigma_o \end{cases} \tag{2.9}$$

In order to illustrate by diagrams the results from the theoretical approach, it can be used the constants from [1] for Borod, Racos or Baraolt coal as follows in [6], [7].

For the ideal tests that will be considered, the specimens are supposed to be cylindrical and no special real conditions of fastening are considered (like these considered in [4], [5]). In these conditions, the constitutive laws will be formulated in principal components both for the infinitesimal strain tensor $\varepsilon_1, \varepsilon_2 = \varepsilon_3$ and for the current or relative stress tensor $\sigma_1, \sigma_2 = \sigma_3$ respectively $\sigma_1^R, \sigma_2^R = \sigma_3^R$.

In these conditions the constitutive equations for the linear viscoelastic model take the form

$$\begin{cases} \dot{\varepsilon}_1 - \dot{\varepsilon}_2 = -k\left[\varepsilon_1 - \varepsilon_2 - \frac{1}{2G_o}\left(\sigma_1^R - \sigma_2^R\right)\right] + \frac{1}{2G}\left(\dot{\sigma}_1^R - \dot{\sigma}_2^R\right), \\ \dot{\varepsilon}_1 + 2\dot{\varepsilon}_2 = -k_v\left[\varepsilon_1 + 2\varepsilon_2 - \frac{1}{3K_o}\left(\sigma_1^R + 2\sigma_2^R\right)\right] + \frac{1}{3K}\left(\dot{\sigma}_1^R + 2\dot{\sigma}_2^R\right), \end{cases} \tag{2.10}$$

and for the Cristescu´s elasto/viscoplastic model the form

$$\begin{cases} \dot{\varepsilon}_1 = \frac{1}{2G}\dot{\sigma}_1 + \frac{1}{3}\left(\frac{1}{3K} - \frac{1}{2G}\right)\left(\dot{\sigma}_1 + 2\dot{\sigma}_2\right) + k_T\left\langle 1 - \frac{W}{H}\right\rangle\left(\frac{1}{3}\frac{\partial F}{\partial \sigma} + \frac{\sigma_1 - \sigma_2}{\overline{\sigma}}\frac{\partial F}{\partial \overline{\sigma}}\right), \\ \dot{\varepsilon}_2 = \frac{1}{2G}\dot{\sigma}_2 + \frac{1}{3}\left(\frac{1}{3K} - \frac{1}{2G}\right)\left(\dot{\sigma}_1 + 2\dot{\sigma}_2\right) + k_T\left\langle 1 - \frac{W}{H}\right\rangle\left(\frac{1}{3}\frac{\partial F}{\partial \sigma} - \frac{1}{2.}\frac{\sigma_1 - \sigma_2}{\overline{\sigma}}\frac{\partial F}{\partial \overline{\sigma}}\right). \end{cases} \tag{2.11}$$

Because the analyzed tests in this paper the stress state will be an uniaxial one, it will be used the following notation

$$h(\sigma_1) = H(\sigma, \overline{\sigma})\Big|_{\sigma=\frac{1}{3}\sigma_1, \overline{\sigma}=|\sigma_1|},$$

$$f_1(\sigma_1) = \left(\frac{1}{3}\frac{\partial F}{\partial \sigma} + \frac{\sigma_1}{\overline{\sigma}}\frac{\partial F}{\partial \overline{\sigma}}\right)\Bigg|_{\sigma=\frac{1}{3}\sigma_1, \overline{\sigma}=|\sigma_1|}, \quad f_2(\sigma_1) = \left(\frac{1}{3}\frac{\partial F}{\partial \sigma} - \frac{1}{2}\frac{\sigma_1}{\overline{\sigma}}\frac{\partial F}{\partial \overline{\sigma}}\right)\Bigg|_{\sigma=\frac{1}{3}\sigma_1, \overline{\sigma}=|\sigma_1|}. \tag{2.12}$$

3. UNCONFINED UNIAXIAL COMPRESSION TESTS UNDER CONSTANT LOAD

In this case, the reference configuration is characterized by

$$\sigma_1^K \geq 0, \ \sigma_2^K = \sigma_3^K = 0 \tag{3.1}$$

and the relative stress tensor has the components

$$\sigma_1^R = \sigma_1^o = \text{const.} > 0, \ \sigma_2^R = \sigma_3^R = 0. \tag{3.2}$$

The initial conditions are given by the instantaneous response

$$\varepsilon_1^o = \frac{1}{3}\left(\frac{1}{3K} + \frac{1}{G}\right)\sigma_1^o, \ \varepsilon_2^o = \frac{1}{3}\left(\frac{1}{3K} - \frac{1}{2G}\right)\sigma_1^o. \tag{3.3}$$

In this situation, the differential equations which describe the process of deformation through unconfined uniaxial compression constant load for the linear viscoelastic model with the initial conditions () admit the solution

$$\begin{cases} \varepsilon_1 = \frac{1}{3}\left(\frac{1}{3K_o} + \frac{1}{G_o}\right)\sigma_1^o + \frac{1}{3}\left(\frac{1}{G} - \frac{1}{G_o}\right)\sigma_1^o \exp(-kt) + \frac{1}{9}\left(\frac{1}{K} - \frac{1}{K_o}\right)\sigma_1^o \exp(-k_v t), \\ \varepsilon_1 = \frac{1}{3}\left(\frac{1}{3K_o} - \frac{1}{2G_o}\right)\sigma_1^o - \frac{1}{6}\left(\frac{1}{G} - \frac{1}{G_o}\right)\sigma_1^o \exp(-kt) + \frac{1}{9}\left(\frac{1}{K} - \frac{1}{K_o}\right)\sigma_1^o \exp(-k_v t). \end{cases} \tag{3.4}$$

The constitutive restrictions (2.3) for $0 < k \leq k_v$ point out the following properties [2], [3]

– horizontal assymptotes

$$\begin{cases}\varepsilon_1 = \dfrac{1}{3}\left(\dfrac{1}{3K_o} + \dfrac{1}{G_o}\right)\sigma_1^o, \\ \varepsilon_2 = \dfrac{1}{3}\left(\dfrac{1}{3K_o} - \dfrac{1}{2G_o}\right)\sigma_1^o;\end{cases} \tag{3.5}$$

– the load is bigger, the strains in absolute values are bigger;
– for a creep in steps, it appears a hardening ([6],[7]);
– for an inverse creep, the strain is complete recoverable ([6],[7]).

In the same situation, the differential equations for Cristescu´s elasto/viscoplastic model with the initial conditions (3.3) admit the solution

$$\begin{cases}\varepsilon_1 = \dfrac{1}{3}\left(\dfrac{1}{3K} + \dfrac{1}{G}\right)\sigma_1^o + \left(1 - \dfrac{W(0)}{h(\sigma_1^K + \sigma_1^o)}\right)\dfrac{h(\sigma_1^K + \sigma_1^o)}{\sigma_1^K + \sigma_1^o}\left[1 - \exp\left(-k_T\,\dfrac{(\sigma_1^K + \sigma_1^o)f_1(\sigma_1^K + \sigma_1^o)}{h(\sigma_1^K + \sigma_1^o)}\,t\right)\right], \\ \varepsilon_2 = \dfrac{1}{3}\left(\dfrac{1}{3K} - \dfrac{1}{2G}\right)\sigma_1^o - \\ \qquad - \left(1 - \dfrac{W(0)}{h(\sigma_1^K + \sigma_1^o)}\right)\dfrac{h(\sigma_1^K + \sigma_1^o)}{\sigma_1^K + \sigma_1^o}\,\dfrac{f_2(\sigma_1^K + \sigma_1^o)}{f_1(\sigma_1^K + \sigma_1^o)}\left[1 - \exp\left(-k_T\,\dfrac{(\sigma_1^K + \sigma_1^o)f_1(\sigma_1^K + \sigma_1^o)}{h(\sigma_1^K + \sigma_1^o)}\,t\right)\right],\end{cases} \tag{3.6}$$

because

$$W(t) = W(0) + \int_0^t \sigma_1(s)\dot{\varepsilon}_1^I(s)\,\mathrm{d}s = W(0) + (\sigma_1^K + \sigma_1^o)\left[\varepsilon_1^I(t) - \varepsilon_1^I(0)\right] = W(0) + (\sigma_1^K + \sigma_1^o)\left[\varepsilon_1(t) - \varepsilon_1(0)\right] \tag{3.7}$$

The creep is stabilized for

$$k_T\,\frac{(\sigma_1^K + \sigma_1^o)f_1(\sigma_1^K + \sigma_1^o)}{h(\sigma_1^K + \sigma_1^o)} > 0 \tag{3.8}$$

The stabilization boundary (horizontal assymptotes for the graphs $\varepsilon_1 = \varepsilon_1(t)$, $\varepsilon_2 = \varepsilon_2(t)$) is defined through

$$\begin{cases}\varepsilon_1 = \dfrac{1}{3}\left(\dfrac{1}{3K} + \dfrac{1}{G}\right)\sigma_1^o + \left(1 - \dfrac{W(0)}{h(\sigma_1^K + \sigma_1^o)}\right)\dfrac{h(\sigma_1^K + \sigma_1^o)}{\sigma_1^K + \sigma_1^o}, \\ \varepsilon_2 = \dfrac{1}{3}\left(\dfrac{1}{3K} - \dfrac{1}{2G}\right)\sigma_1^o + \left(1 - \dfrac{W(0)}{h(\sigma_1^K + \sigma_1^o)}\right)\dfrac{h(\sigma_1^K + \sigma_1^o)}{\sigma_1^K + \sigma_1^o}\,\dfrac{f_2(\sigma_1^K + \sigma_1^o)}{f_1(\sigma_1^K + \sigma_1^o)}.\end{cases} \tag{3.9}$$

Natural, in the case of stabilized creep, ε_1 and $|\varepsilon_2|$ must increase with σ_1^o, therefore the conditions

$$\begin{gathered}\frac{1}{3}\left(\frac{1}{3K} + \frac{1}{G}\right) + \frac{\partial}{\partial\sigma_1^o}\left[\left(1 - \frac{W(0)}{h(\sigma_1^K + \sigma_1^o)}\right)\frac{h(\sigma_1^K + \sigma_1^o)}{\sigma_1^K + \sigma_1^o}\right] > 0, \\ \frac{1}{3}\left(\frac{1}{3K} - \frac{1}{2G}\right) + \frac{\partial}{\partial\sigma_1^o}\left[\left(1 - \frac{W(0)}{h(\sigma_1^K + \sigma_1^o)}\right)\frac{h(\sigma_1^K + \sigma_1^o)}{\sigma_1^K + \sigma_1^o}\,\frac{f_2(\sigma_1^K + \sigma_1^o)}{f_1(\sigma_1^K + \sigma_1^o)}\right] < 0\end{gathered} \tag{3.10}$$

It is less easy to give a theoretical proof of the hardening of the rock subject to a creep in steps. An idea of this phenomena will appear in the example in [6], [7].

4. UNCONFINED UNIAXIAL COMPRESSION TESTS WITH CONSTANT LOADING RATE

While the reference configuration is the same as in the previous paragraph, the relative stress tensor has the components

$$\sigma_1^{\mathrm{R}} = \dot{\sigma}_1^{o} t,\ \dot{\sigma}_1^{o} = \text{const.},\ \sigma_2^{\mathrm{R}} = \sigma_3^{\mathrm{R}} = 0. \tag{4.1}$$

The test begins from the reference configuration, so that the initial conditions are

$$\varepsilon_1^{o} = \varepsilon_2^{o} = \varepsilon_3^{o} = 0\,. \tag{4.2}$$

In this situation, the differential equations which describe the deformation process for the linear viscoelastic model (2.10) with the initial conditions (4.2) admit the solution

$$\begin{cases} 3\varepsilon_1 = \left(\dfrac{1}{3K_o} + \dfrac{1}{G_o}\right)\dot{\sigma}_1^{o} t + \dfrac{1}{k}\left(\dfrac{1}{G} - \dfrac{1}{G_o}\right)\dot{\sigma}_1^{o}\left[1 - \exp(-kt)\right] + \dfrac{1}{k_v}\left(\dfrac{1}{3K} - \dfrac{1}{3K_o}\right)\dot{\sigma}_1^{o}\left[1 - \exp(-k_v t)\right], \\ 3\varepsilon_2 = \left(\dfrac{1}{3K_o} - \dfrac{1}{2G_o}\right)\dot{\sigma}_1^{o} t - \dfrac{1}{k}\left(\dfrac{1}{2G} - \dfrac{1}{2G_o}\right)\dot{\sigma}_1^{o}\left[1 - \exp(-kt)\right] + \dfrac{1}{k_v}\left(\dfrac{1}{3K} - \dfrac{1}{3K_o}\right)\dot{\sigma}_1^{o}\left[1 - \exp(-k_v t)\right] \end{cases} \tag{4.3}$$

In terms of relative stresses, the equations for the characteristic curves are

$$\begin{cases} 3\varepsilon_1 = \left(\dfrac{1}{3K_o} + \dfrac{1}{G_o}\right)\sigma_1^{\mathrm{R}} + \\ \qquad + \dfrac{1}{k}\left(\dfrac{1}{G} - \dfrac{1}{G_o}\right)\dot{\sigma}_1^{o}\left[1 - \exp\left(-k\dfrac{\sigma_1^{\mathrm{R}}}{\dot{\sigma}_1^{o}}\right)\right] + \dfrac{1}{k_v}\left(\dfrac{1}{3K} - \dfrac{1}{3K_o}\right)\dot{\sigma}_1^{o}\left[1 - \exp(-k_v\dfrac{\sigma_1^{\mathrm{R}}}{\dot{\sigma}_1^{o}}\right], \\ 3\varepsilon_1 = \left(\dfrac{1}{3K_o} + \dfrac{1}{G_o}\right)\sigma_1^{\mathrm{R}} + \\ \qquad + \dfrac{1}{k}\left(\dfrac{1}{G} - \dfrac{1}{G_o}\right)\dot{\sigma}_1^{o}\left[1 - \exp\left(-k\dfrac{\sigma_1^{\mathrm{R}}}{\dot{\sigma}_1^{o}}\right)\right] + \dfrac{1}{k_v}\left(\dfrac{1}{3K} - \dfrac{1}{3K_o}\right)\dot{\sigma}_1^{o}\left[1 - \exp(-k_v\dfrac{\sigma_1^{\mathrm{R}}}{\dot{\sigma}_1^{o}}\right]. \end{cases} \tag{4.4}$$

The constitutive restrictions (2.3) point out the following properties ([2],[3])

– the slope of the tangents to the characteristic curves in origin corresponds to the instantaneous response

$$\frac{d\varepsilon_1}{d\sigma_1^{\mathrm{R}}}(0) = \frac{1}{3}\left(\frac{1}{3K} + \frac{1}{G}\right),\ \frac{d\varepsilon_2}{d\sigma_1^{\mathrm{R}}}(0) = \frac{1}{3}\left(\frac{1}{3K} - \frac{1}{2G}\right); \tag{4.5}$$

– the characteristic curves admit oblic assymptotes

$$\begin{cases} 3\varepsilon_1 = \left(\dfrac{1}{3K_o} + \dfrac{1}{G_o}\right)\sigma_1^{\mathrm{R}} + \left[\dfrac{1}{k_v}\left(\dfrac{1}{3K} - \dfrac{1}{3K_o}\right) + \dfrac{1}{k}\left(\dfrac{1}{G} - \dfrac{1}{G_o}\right)\right]\dot{\sigma}_1^{o}, \\ 3\varepsilon_2 = \left(\dfrac{1}{3K_o} - \dfrac{1}{2G_o}\right)\sigma_1^{\mathrm{R}} + \left[\dfrac{1}{k_v}\left(\dfrac{1}{3K} - \dfrac{1}{3K_o}\right) - \dfrac{1}{k}\left(\dfrac{1}{2G} - \dfrac{1}{2G_o}\right)\right]\dot{\sigma}_1^{o}; \end{cases} \tag{4.6}$$

the slope of these assymptotes is these of the creep frontieres; the ordinates in origin of the assymptotes increase with $\dot{\sigma}_1^{o}$

– for $\dot{\sigma}_1^{o} \to \infty$ the characteristic curves tend to the lines of instantaneous response and for $\dot{\sigma}_1^{o} \to 0$ to the creep frontiers.

In the same situation, the differential equations for Cristescu´s elasto/viscoplastic model

$$\begin{cases} \dot{\varepsilon}_1 = \frac{1}{3}\left(\frac{1}{3K} + \frac{1}{G}\right)\dot{\sigma}_1^o + k_T\left(1 - \frac{W(t)}{h(\dot{\sigma}_1^o t)}\right) f_1(\dot{\sigma}_1^o t), \\ \dot{\varepsilon}_2 = \frac{1}{3}\left(\frac{1}{3K} - \frac{1}{2G}\right)\dot{\sigma}_1^o + k_T\left(1 - \frac{W(t)}{h(\dot{\sigma}_1^o t)}\right) f_2(\dot{\sigma}_1^o t), \end{cases} \tag{4.7}$$

$$\dot{W} = k_T\left(1 - \frac{W(t)}{h(\dot{\sigma}_1^o t)}\right) m(\dot{\sigma}_1^o t),\ m(\sigma_1) = \left(\frac{\partial F}{\partial \sigma}\sigma + \frac{\partial F}{\partial \bar{\sigma}}\bar{\sigma}\right)\Big|_{\sigma = \frac{1}{3}\sigma_1,\, \bar{\sigma} = |\sigma_1|}. \tag{4.8}$$

It results

$$\begin{cases} \varepsilon_1 = \frac{1}{3}\left(\frac{1}{3K} + \frac{1}{G}\right)\dot{\sigma}_1^o\left[1 - \exp\left(\frac{1}{\dot{\sigma}_1^o}\int_0^{\sigma_1}\frac{\sigma_1 f_1(\sigma_1)}{m(\sigma_1)}\,\mathrm{d}\sigma_1\right)\right], \\ \varepsilon_2 = \frac{1}{3}\left(\frac{1}{3K} - \frac{1}{2G}\right)\dot{\sigma}_1^o\left[1 - \exp\left(\frac{1}{\dot{\sigma}_1^o}\int_0^{\sigma_1}\frac{\sigma_1 f_2(\sigma_1)}{m(\sigma_1)}\,\mathrm{d}\sigma_1\right)\right]. \end{cases} \tag{4.9}$$

5. UNCONFINED UNIAXIAL COMPRESSION TESTS UNDER CONSTANT AXIAL STRAIN

The reference configuration is the same as before. It is supposed that through an uniaxial compression is obtained an axial strain which is conserved constant

$$\varepsilon_1 = \varepsilon_1^o = \text{const.} \tag{5.1}$$

The test being unconfined

$$\sigma_2^{\mathrm{R}} = \sigma_3^{\mathrm{R}} = 0. \tag{5.2}$$

It will be determined the evolution in time of the axial stress σ_1 and the radial strain $\varepsilon_2 = \varepsilon_3$.

The initial conditions are obtained through instantaneous response

$$\sigma_1^{\mathrm{R}o} = \frac{9GK}{G + 3K}\varepsilon_1^o,\ \varepsilon_2^o = \frac{2G - 3K}{2G + 6K}\varepsilon_1^o. \tag{5.3}$$

For the linear viscoelastic model, the differential equations for this deformation process are

$$\begin{cases} \frac{G + 3K}{3KG}\dot{\sigma}_1^{\mathrm{R}} = -\left(\frac{k}{G_o} + \frac{k_v}{3K_o}\right)\sigma_1^{\mathrm{R}} + 2(k_v - k)\varepsilon_2 + (2k + k_v)\varepsilon_1^o, \\ \frac{G + 3K}{3KG}\dot{\varepsilon}_2 = -\left(\frac{k}{6KG_o} - \frac{k_v}{6GK_o}\right)\sigma_1^{\mathrm{R}} - \left(\frac{k}{3K} + \frac{k_v}{G}\right)\varepsilon_2 + \left(\frac{k}{3K} - \frac{k_v}{2G}\right)\varepsilon_1^o. \end{cases} \tag{5.4}$$

The general solution of this system of differential equations is

$$\begin{cases} \sigma_1^{\mathrm{R}} = A_1\exp(\lambda_1 t) + A_2\exp(\lambda_2 t) + \frac{9G_oK_o}{G_o + 3K_o}\varepsilon_1^o, \\ \varepsilon_2 = B_1\exp(\lambda_1 t) + B_2\exp(\lambda_2 t) + \frac{2G_o - 3K_o}{2G_o + 6K_o}\varepsilon_1^o, \end{cases} \tag{5.5}$$

where λ_1, λ_2 are the real negative roots (for proof, see [2]) of the characteristic polynomial

$$P(\lambda)=\left(\frac{1}{3K}+\frac{1}{G}\right)\lambda^2+\left[k\left(\frac{1}{3K}+\frac{1}{G_o}\right)+k_v\left(\frac{1}{3K_o}+\frac{1}{G}\right)\right]\lambda+kk_v\left(\frac{1}{3K_o}+\frac{1}{G_o}\right), \tag{5.6}$$

the constants A_1, A_2, B_1, B_2 the roots of the algebric system formed from the initial conditions and the relations existing among these constants

$$\begin{cases} A_1+A_2=\left(\dfrac{9GK}{G+3K}-\dfrac{9G_oK_o}{G_o+3K_o}\right)\varepsilon_1^o, \\ B_1+B_2=\left(\dfrac{2G-3K}{2G+6K}-\dfrac{2G_o-3K_o}{2G_o+6K_o}\right)\varepsilon_1^o, \end{cases} \tag{5.7}$$

$$\begin{cases} A_1=\dfrac{1}{2(k_v-k)}\left[\dfrac{G+3K}{3GK}\lambda_1+\left(\dfrac{k}{G_o}+\dfrac{k_v}{3K_o}\right)\right]B_1, \\ A_2=\dfrac{1}{2(k_v-k)}\left[\dfrac{G+3K}{3GK}\lambda_2+\left(\dfrac{k}{G_o}+\dfrac{k_v}{3K_o}\right)\right]B_2. \end{cases} \tag{5.8}$$

For the linear viscoelastic model, there exist the limits

$$\begin{aligned} &\lim_{t\to\infty}\sigma_1=\sigma_1^{\mathrm{K}}+\frac{9G_oK_o}{G_o+3K_o}\varepsilon_1^o<\sigma_1^{\mathrm{K}}+\sigma_1^{\mathrm{R}o}, \\ &\lim_{t\to\infty}\varepsilon_2=\frac{2G_o-3K_o}{2G_o+6K_o}\varepsilon_1^o>\varepsilon_2^o. \end{aligned} \tag{5.9}$$

For the Cristescu´s elasto/viscoplastic model, the deformation process under constant axial strain is described by

$$\begin{cases} \dot\sigma_1=-\dfrac{9GK}{G+3K}k_T\left(1-\dfrac{w(\sigma_1)}{h(\sigma_1)}\right)f_1(\sigma_1), \\ \dot\varepsilon_2=k_T\left(1-\dfrac{w(\sigma_1)}{h(\sigma_1)}\right)\left[f_2(\sigma_1)+\dfrac{2G-3K}{2G+6K}f_1(\sigma_1)\right], \end{cases} \tag{5.10}$$

where

$$\begin{aligned} w(\sigma_1)&=W(0)+\int_0^t\sigma_1(s)\dot\varepsilon_1^{\mathrm{I}}(s)\mathrm{d}s= \\ &=W(0)+\int_0^t\sigma_1(s)\left[\dot\varepsilon_1(s)-\frac{1}{3}\left(\frac{1}{3K}+\frac{1}{G}\right)\dot\sigma_1(s)\right]\mathrm{d}s=W(0)-\frac{1}{3}\left(\frac{1}{3K}+\frac{1}{G}\right)\left[\sigma_1^2-\left(\sigma_1^o\right)^2\right] \end{aligned} \tag{5.11}$$

The final value of $\sigma_1^\infty=\lim\limits_{t\to\infty}\sigma_1(t)$ is root of the algebric equation

$$h(\sigma_1^\infty)=w(\sigma_1^\infty), \tag{5.12}$$

concretely, this value is the real positive root, close by σ_1^o and smaller than this. In the numerical integration of the system (5.10), it must be taken into account the position of ε_1^o with respect to the critical value

$$\varepsilon_1^{o\,\mathrm{cr}}=\frac{G+3K}{3GK}\sigma_o. \tag{5.13}$$

We mention that in the integration of differential equation system for both constitutive models it must take care of the H expresion with respect of σ values.

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