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TESTING OF ADCS BY FREQUENCY-DOMAIN ANALYSIS IN MULTI-TONE MODE

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In this paper the testing of analog-to-digital converters (ADCs) by frequency-domain analysis in multi-tone mode is investigated. The theoretical expressions of some of the most important ADC dynamic parameters are derived. Also, the theoretical expression of the difference between the ADC effective number of bits (ENOB) obtained by testing in multi-tone mode and the ENOB obtained by testing in the single-tone mode is derived. From the theoretical expression of this difference some important conclusions concerning the accuracy in the evaluation of ENOB by testing in multi-tone mode are drawn. Some experimental results are presented in order to validate these conclusions. The experiments were carried out using the test system MULTI-TONE ADC TEST, specially developed for this purpose.

Key words: Multi-tone mode, testing of ADCs by frequency-domain analysis, estimation of the effective number of bits of an ADC.

1. INTRODUCTION

Due to its advantages the frequency-domain analysis methods are mostly employed to determine the dynamic parameters of an analog-to-digital converter (ADC). These methods frequently use single-tone mode [1]-[3]. In this mode the signal used to test the ADC is a high purity sinewave (i.e. the single-tone signal). Then, a certain algorithm, based on the discrete Fourier transform (DFT) of the corresponding output codes, is used to estimate the dynamic parameters of the ADC.

In this paper the ADC characterization by frequency-domain analysis in multi-tone mode is described. In multi-tone mode the test signal is composed of the sum of m ($m \geq 2$) high purity sinewaves with the same amplitude (i.e. the m-tone signal). First, the theoretical expressions of some of the most important dynamic parameters of an ADC are derived. These parameters are: signal-to-noise and distortion ration (SINAD), effective number of bits (ENOB), total harmonic distortion (THD) and intermodulation distortion (IMD).

The global dynamic performances of an ADC are evaluated by means of ENOB. Thus, it is very important to compare the ENOB obtained by testing in multi-tone mode with the one obtained by testing in single-tone mode. From this reason, then, the theoretical expression of the difference between the values of ENOB, ΔENOB, obtained by testing in these modes is derived.

The derived expressions represent a generalization of the expressions presented in [4] for the dual-tone mode ($m = 2$).

Based upon the ΔENOB expression some important conclusions concerning the accuracy in the evaluation of ENOB by testing in multi-tone mode are drawn. In order to validate these conclusions some experiments were carried out. The experimental results were obtained by means of the test system named MULTI-TONE ADC TEST, specially implemented for this purpose.

2. ADC DYNAMIC PARAMETERS OBTAINED IN MULTI-TONE MODE

A real ADC can be modeled by an ideal ADC with the same resolution, followed by a nonlinear system [4], [5]. The nonlinear system is characterized by a polynomial transfer function given by

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$$y(t) = ax(t) + bx^2(t) + cx^3(t) + \dots \tag{1}$$

in which x is the input term;

y is the output term;

a, b, c,… are the linear, squared, third,… gain terms.

The order of the polynomial transfer function is 3 or 5 [4], [6]. In order to simplify the calculation a three-order transfer function was considered.

The m-tone signal is composed by a sum of m ($m \geq 2$) high purity sinewaves with the amplitude A and frequency f_i ($i = 1,2,\dots,m$)

$$x(t) = \sum_{i=1}^{m} A \sin(2\pi f_i t) \tag{2}$$

For testing of all ADC codes A is equal with

$$A = \frac{FSR}{2m} \tag{3}$$

where FSR is the ADC full-scale range.

By substituting (2) in (1) the following harmonic and intermodulation components are obtained

- harmonic components

$$\begin{aligned} H(2f_i) &= \frac{bA^2}{2}, \qquad i = 1,2,\dots m \\ H(3f_i) &= \frac{cA^3}{4}, \qquad i = 1,2,\dots m \end{aligned} \tag{4}$$

where $H(kf_i)$, $i = 1, 2, \dots, m$, $k = 2, 3$ is the amplitude of the harmonic component at frequency kf_i;

- intermodulation components

$$\begin{aligned} IM(f_i \pm f_j) &= bA^2, \qquad i, j = 1,2,\dots m,\ i < j \\ IM(f_i \pm 2f_j) &= \frac{3cA^3}{4}, \qquad i, j = 1,2,\dots m,\ i \neq j \\ IM(f_i \pm f_j \pm f_k) &= \frac{3cA^3}{2}, i, j, k = 1,2,\dots m,\ i < j < k \end{aligned} \tag{5}$$

where: $IM(f_i \pm f_j)$, $i, j = 1, 2, \dots, m$, $i < j$ is the amplitude of the intermodulation component at frequency $f_i \pm f_j$;

$IM(f_i \pm 2f_j)$, $i, j = 1, 2, \dots, m$, $i \neq j$ is the amplitude of the intermodulation component at frequency $f_i \pm 2f_j$;

$IM(f_i \pm f_j \pm f_k)$, $i, j, k = 1, 2, \dots, m$, $i < j < k$ is the amplitude of the intermodulation component at frequency $f_i \pm f_j \pm f_k$;

The number of the components $IM(f_i \pm f_j)$ is $m(m$-$1)$, of the components $IM(f_i \pm 2f_j)$ is $2m(m$-$1)$ and of the components $IM(f_i \pm f_j \pm f_k)$ is $2m(m$-$1)(m$-$2)/3$.

From (4) THD in multi-tone mode becomes

$$THD_{mT}^2 = \frac{\sum_k \left(H_k^2\right)_{mT}}{S_{mT}^2} = \frac{\sum_{i=1}^{m} H^2(2f_i) + \sum_{i=1}^{m} H^2(3f_i)}{S_{mT}^2} = \frac{m\frac{b^2A^4}{4} + m\frac{c^2A^6}{16}}{m\frac{A^2}{2}} = \frac{b^2A^2}{2} + \frac{c^2A^4}{8} \tag{6}$$

in which S_{mT} is the effective value of the m-tone signal, $S^2_{mT} = mA^2/2$.

Based on (5) IMD in multi-tone mode is given by

$$IMD_{mT}^2 = \frac{\sum\limits_{i,j,k}\left(IM_{ijk}^2\right)_{mT}}{S_{mT}^2} = \frac{\sum\limits_{\substack{i,j=1\\ i<j}}^{m} IM^2\left(f_i \pm f_j\right) + \sum\limits_{\substack{i,j=1\\ i\neq j}}^{m} IM^2\left(f_i \pm 2f_j\right) + \sum\limits_{\substack{i,j,k=1\\ i<j<k}}^{m} IM^2\left(f_i \pm f_j \pm f_k\right)}{S_{mT}^2} =$$
$$\frac{m(m-1)bA^2 + \frac{9}{8}m(m-1)c^2A^6 + \frac{3}{2}m(m-1)(m-2)c^2A^6}{m\frac{A^2}{2}} = \tag{7}$$
$$2(m-1)b^2A^2 + \frac{3}{4}(m-1)(4m-5)c^2A^4.$$

From (6) and (7) we obtain

$$IMD_{mT}^2 = 4(m-1)THD_{mT}^2 + \frac{(m-1)(12m-17)}{4}c^2A^4. \tag{8}$$

Because the second term of the expression (8) is positive it follows that

$$IMD_{mT}^2 \geq 4(m-1)THD_{mT}^2, \tag{9}$$

which shows that IMD^2_{mT} is much greater than THD^2_{mT}.
The signal-to-noise ratio (SNR) of an ideal *n*-bit ADC tested by frequency-domain in multi-tone mode is

$$SNR_{mT} = 10\log\left(\frac{S_{mT}^2}{\sigma_q^2}\right) = 6.02n + 1.76 - 10\log(m) \quad (dB) \tag{10}$$

where σ^2_q is the variance of the quantization noise; σ^2_q is given by $q^2/12$ (where q is the quantization step, $q = FSR/2^n$).
In the case of a real ADC besides the quantization noise other noise sources can be appear such as harmonic, intermodulation and spurious components, jitter of sampling clock, additive noise. Therefore, the ADC is characterized by another dynamic parameter named SINAD. In multi-tone mode SINAD is given by

$$SINAD_{mT} = 10\log\left(\frac{S_{mT}^2}{\sum\limits_{i,j,k}\left(IM_{ijk}^2\right)_{mT} + \sum\limits_{k}\left(H_k^2\right)_{mT} + \sigma_q^2 + \sigma_{ex}^2}\right) \quad (dB) \tag{11}$$

in which σ^2_{ex} is the variance of the excess noise (jitter, additive noise,…).

ENOB is calculated from (10) by replacing SNR_{mT} by $SINAD_{mT}$. Thus, it is obtained

$$ENOB_{mT} = \frac{SINAD_{mT}(dB) - 1.76 + 10\log(m)}{6.02}. \tag{12}$$

ENOB is the most important dynamic parameter of an ADC because it evaluates the global dynamic performances of the ADC. From this reason it is very important to know how the testing in multi-tone affects the evaluation of the ENOB by comparison with the one obtained by testing in single-tone mode. The difference ΔENOB between the $ENOB_{mT}$ and the ENOB obtained by testing in single-tone mode $ENOB_{1T}$ is given by

$$\Delta ENOB = ENOB_{mT} - ENOB_{1T} = \frac{SINAD_{mT}(dB) - SINAD_{1T}(dB) + 10\log(m)}{6.02}. \tag{13}$$

$SINAD_{1T}$ is given by the following expression

$$SINAD_{1T} = 10\log\left(\frac{S_{1T}^2}{\sum_k \left(H_k^2\right)_{1T} + \sigma_q^2 + \sigma_{ex}^2}\right) \quad (dB) \tag{14}$$

where: S_{1T} is the effective value of the single-tone signal;

$(H_k)_{1T}$ is the k harmonic in single-tone mode.

To test all the ADC output codes the amplitude of the single-tone signal is equal with FSR/2. So, S_{1T} is equal with

$$S_{1T}^2 = \frac{m^2 A^2}{2} = mS_{mT}^2. \tag{15}$$

From (11), (14) and (15) ΔENOB becomes

$$\Delta ENOB = \frac{10}{6.02}\log\left(\frac{\sum_k \left(H_k^2\right)_{1T} + \sigma_q^2 + \sigma_{ex}^2}{\sum_{i,j,k}\left(IM_{ijk}^2\right)_{mT} + \sum_k \left(H_k^2\right)_{mT} + \sigma_q^2 + \sigma_{ex}^2}\right). \tag{16}$$

For an ideal ADC the harmonic and intermodulation components and, also, the noise in excess do not exist. Thus, we obtain

$$\Delta ENOB = 0 \Rightarrow ENOB_{mT} = ENOB_{1T}. \tag{17}$$

For a real ADC the harmonic and intermodulation components are much greater than the rest of the noise. In this case ΔENOB can be approximated by the expression

$$\Delta ENOB \cong \frac{10}{6.02}\log\left(\frac{\sum_k \left(H_k^2\right)_{1T}}{\sum_{i,j,k}\left(IM_{ijk}^2\right)_{mT} + \sum_k \left(H_k^2\right)_{mT}}\right). \tag{18}$$

Moreover, from (9) it follows that in multi-tone mode the intermodulation distortion is much greater than the harmonic distortion. Thus, (18) becomes

$$\Delta ENOB \cong \frac{10}{6.02}\log\left(\frac{\sum_k \left(H_k^2\right)_{1T}}{\sum_{i,j,k}\left(IM_{ijk}^2\right)_{mT}}\right). \tag{19}$$

Also, (19) can be written as

$$\Delta ENOB \cong \frac{10}{6.02}\log\left(\frac{mTHD_{1T}^2}{IMD_{mT}^2}\right) \tag{20}$$

in which THD_{1T} is the total harmonic distortion in single-tone mode.

The single-tone signal is $x(t) = mA\sin(2\pi ft)$, where f is the frequency of the single-tone signal. By substituting this signal in (1) the following THD_{1T} is obtained

$$THD_{1T}^2 = \frac{m^2}{2}b^2A^2 + \frac{m^4}{8}c^2A^4. \tag{21}$$

By replacing (8) and (21) in (20) and after some algebra we have

$$\Delta ENOB \cong \frac{10}{6.02}\log\left(\frac{m^3}{4(m-1)}\right) + \frac{10}{6.02}\log\left(\frac{8b^2 + 2m^2c^2A^2}{8b^2 + 3(4m-5)c^2A^2}\right). \tag{22}$$

If the second harmonic distortion is the most important, situation frequently encountered in practice, (22) becomes

$$\Delta ENOB \cong \frac{10}{6.02}\log\left(\frac{m^3}{4(m-1)}\right). \tag{23}$$

In dual-tone mode (m = 2) from (23) it follows that the ΔENOB ≅ 0.5 bits, result which was obtained also in [4].

From (23) it follows that the accuracy in the evaluation of ENOB increases as the number of tones increases. Another important conclusion deduced from (23) is that it is possible to use sinewave generators with distortion performances inferior to those of the converter to estimate with high precision the ENOB. In this situation, known the distortion performances of the generators, based on (23), we can establish for a given ADC resolution the number of the generators to be used. For example, according to (23), by means of four sinewave generators with distortion performances that permit to test ADCs with resolution up to 9 bits it is possible to estimate with high accuracy the ENOB of the ADCs with resolution up to 10 bits. So, there is a 1-bit gain in resolution.

In next section some experimental results were carried out in order to validate the conclusions that were drawn above.

3. EXPERIMENTAL RESULTS

The experimental results were obtained by means of the test system named MULTI-TONE ADC TEST. This test system has the following key features:

- The acquisition system is based on TMS320C5x DSK board [7].
- Maximum record length of 4096 samples.
- Maximum sampling frequency of 20 MHz.
- Control of the signal generators via the IEEE-488 bus.
- The software is easy to use; it interacts with the user through mouse driven graphic interfaces.
- Designed for testing the ADCs when the sampling frequency is noncoherent with the sinewave input frequency [1] because this is the most encountered situation in practical applications of ADCs. However, MULTI-TONE ADC TEST permits, also, the testing of ADCs when these frequencies are coherent.
- Four modes were employed for testing:
  - single-tone mode;
  - dual-tone mode;
  - three-tone mode;
  - four-tone mode.
- For each mode two graphical pages are available, which provide a large amount of information concerning the sinewaves parameters and the ADC dynamic performances.
- Saving in ASCII format data files of the parameters that characterize the sinewaves and of the ADC dynamic parameters.
- Possibility to process also data files obtained by simulation or by means of other acquisition systems.

The sinewave parameters (amplitude, frequency and phase) were determined by means of the interpolated fast Fourier transform (IFFT) [8]. The δ values corresponding to the sinewave frequencies were, also, estimated by IFFT [8].

The ADC dynamic parameters estimated by MULTI-TONE ADC TEST were SINAD and ENOB. These parameters were estimated by the algorithm proposed in [3].

The block diagram of the MULTI -TONE ADC TEST is presented in Figure 1.

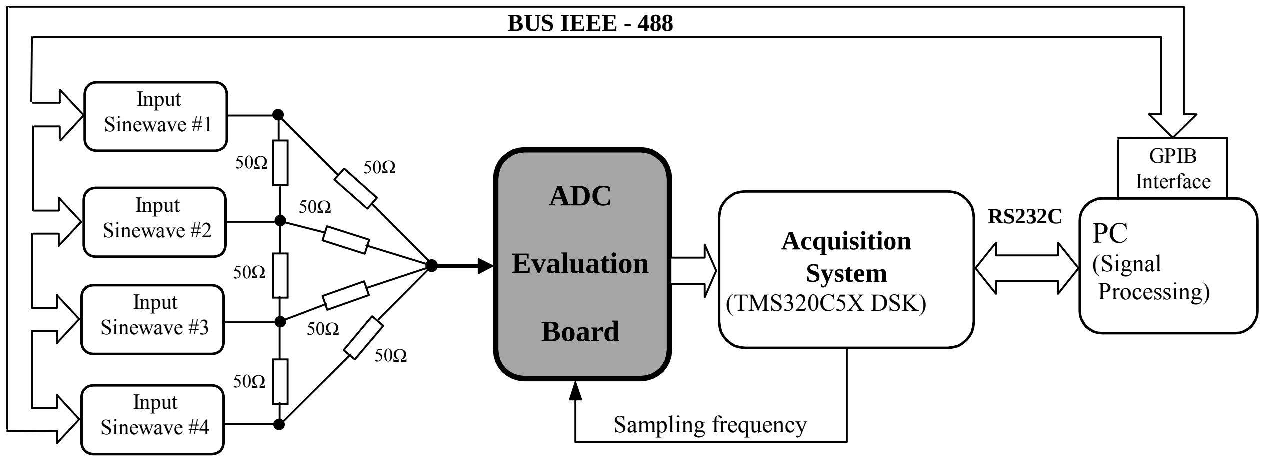


Fig. 1 - Block diagram of the MULTI –TONE ADC TEST.

The TMS320C5x DSK is a low-cost, simple, stand-alone application board equipped with a 16-bit fixed-point digital signal processor (DSP) TMS320C50 DSP. DSK contains an analog interface circuit (AIC) - TLC32040, which provides the necessary conversion between the analogue and digital domain. For this purpose TLC32040 incorporates a band-pass antialiasing input filter, a 14-bit ADC, a serial port by which the AIC communicates with the TMS320C50 DSP, a 14-bit digital-to-analog converter (DAC) and a low-pass output reconstruction filter. The DSK is connected to a PC via a RS232 interface.

The data acquisition programs were written in C and in assembly language of the TMS320C5x DSK [9]. The data processing and the interactive graphical pages were realized by means of the MATLAB 4.2.

Two ADCs were tested: TLC0820 [10] and the ADC of the TLC32040. TLC0820 is a high-speed 8-bit unipolar half-flash converter, realized in LinCMOS technology, with a minimum access and conversion time of 1.18 μs in the most rapid write-read mode. The sinewave generators employed for testing the ADCs were the programmable function generator Hameg - HM8130 [11]. The total harmonic distortion of the HM8130 in the frequency domain used is smaller than 0.5% (i.e. smaller than –46 dBc). A sinewave with this total harmonic distortion permits to test ADCs with a maximum resolution of 7 bits.

TLC0820 was tested at three frequency sets:

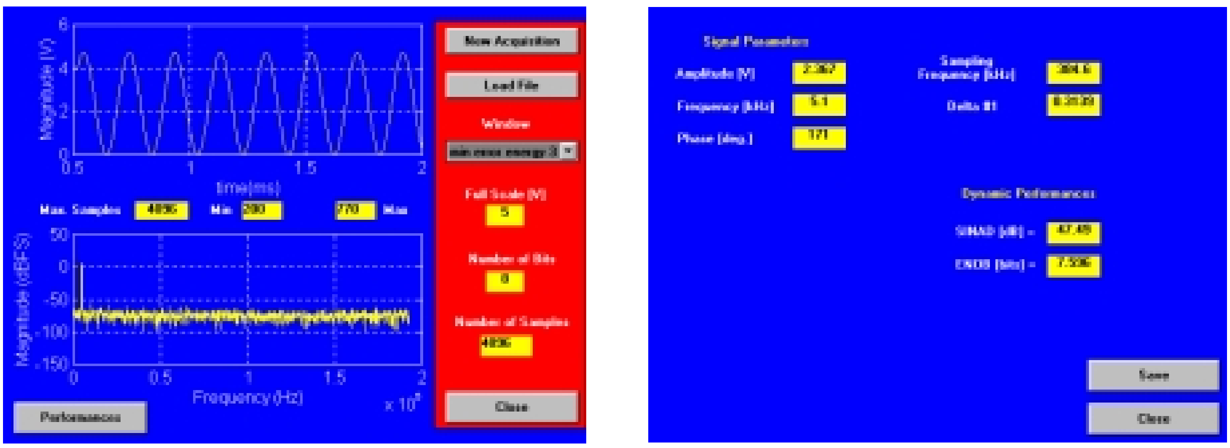
a) f_1 = 5.1 kHz, f_2 = 7.3 kHz, f_3 = 9.9 kHz, f_4 = 12.7 kHz;

b) f_1 = 16.1 kHz, f_2 = 18.8 kHz, f_3 = 21.9 kHz, f_4 = 25.2 kHz;

a) f_1 = 35.1 kHz, f_2 = 38.6 kHz, f_3 = 42.3 kHz, f_4 = 45.7 kHz.

Because the coherent sampling relationships are not met [1] to eliminate the spectral leakage errors the 3-term minimum energy error was used [12].

Figures 2-5 show the results obtained after the testing with these frequencies sets.



(a) (b)

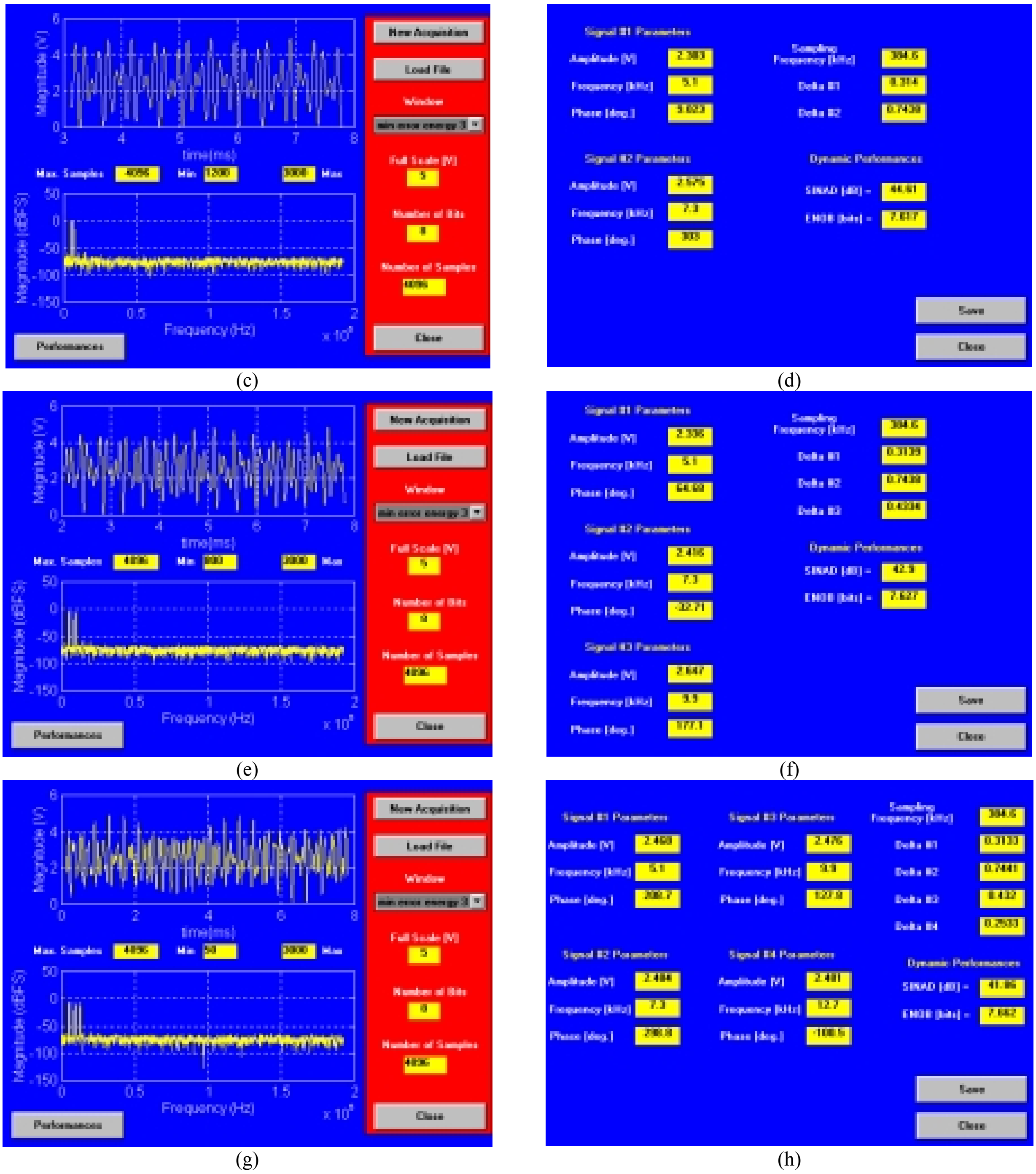


Fig. 2 - The results obtained after the testing of TLC0820 with the first frequency set in:
(a)-(b) single-tone mode; (c)-(d) dual-tone mode; (e)-(f) three-tone mode; (g)-(h) four-tone mode.

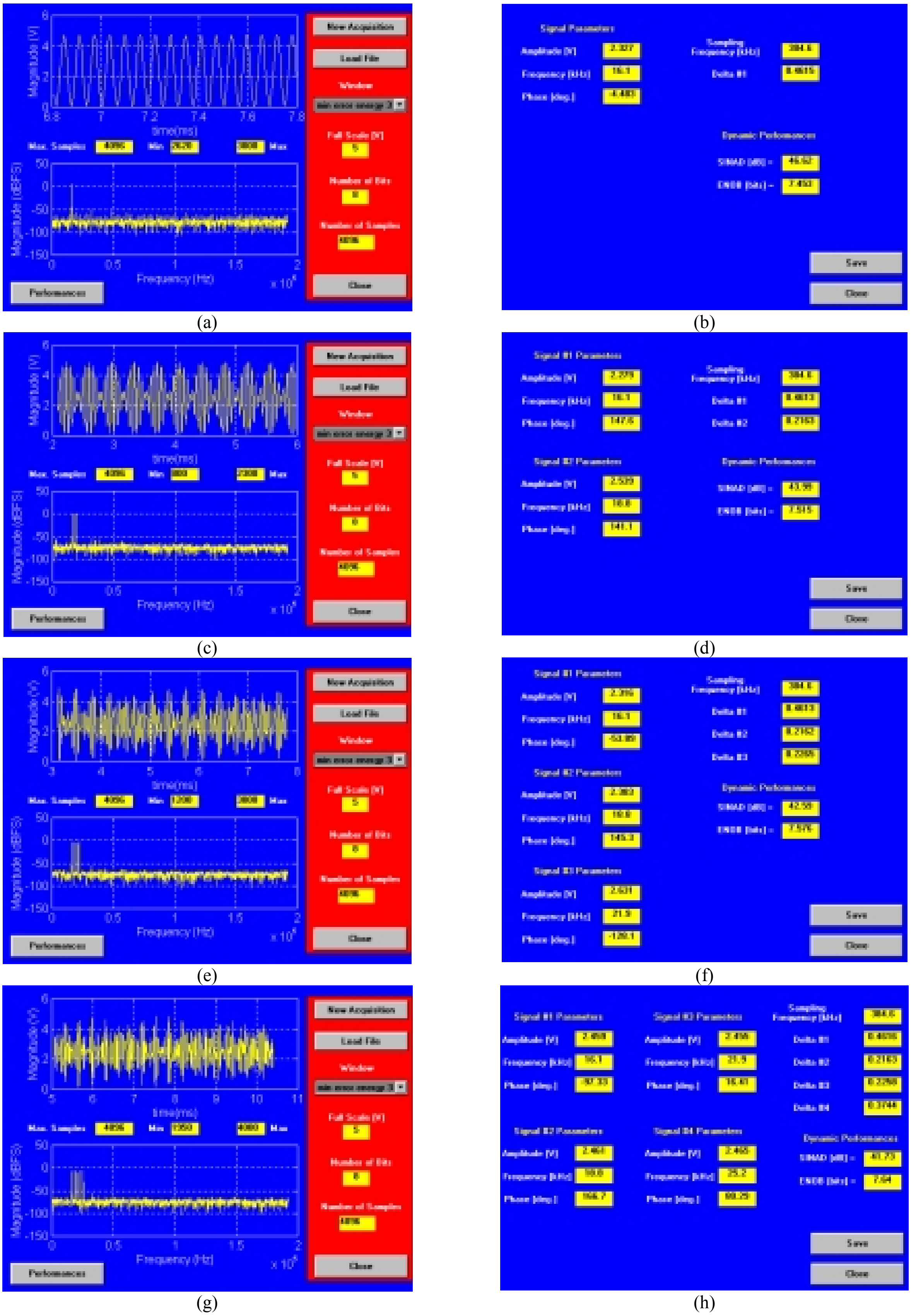


Fig. 3 - The results obtained after the testing of TLC0820 with the second frequency set in:
(a)-(b) single-tone mode; (c)-(d) dual-tone mode; (e)-(f) three-tone mode; (g)-(h) four-tone mode.

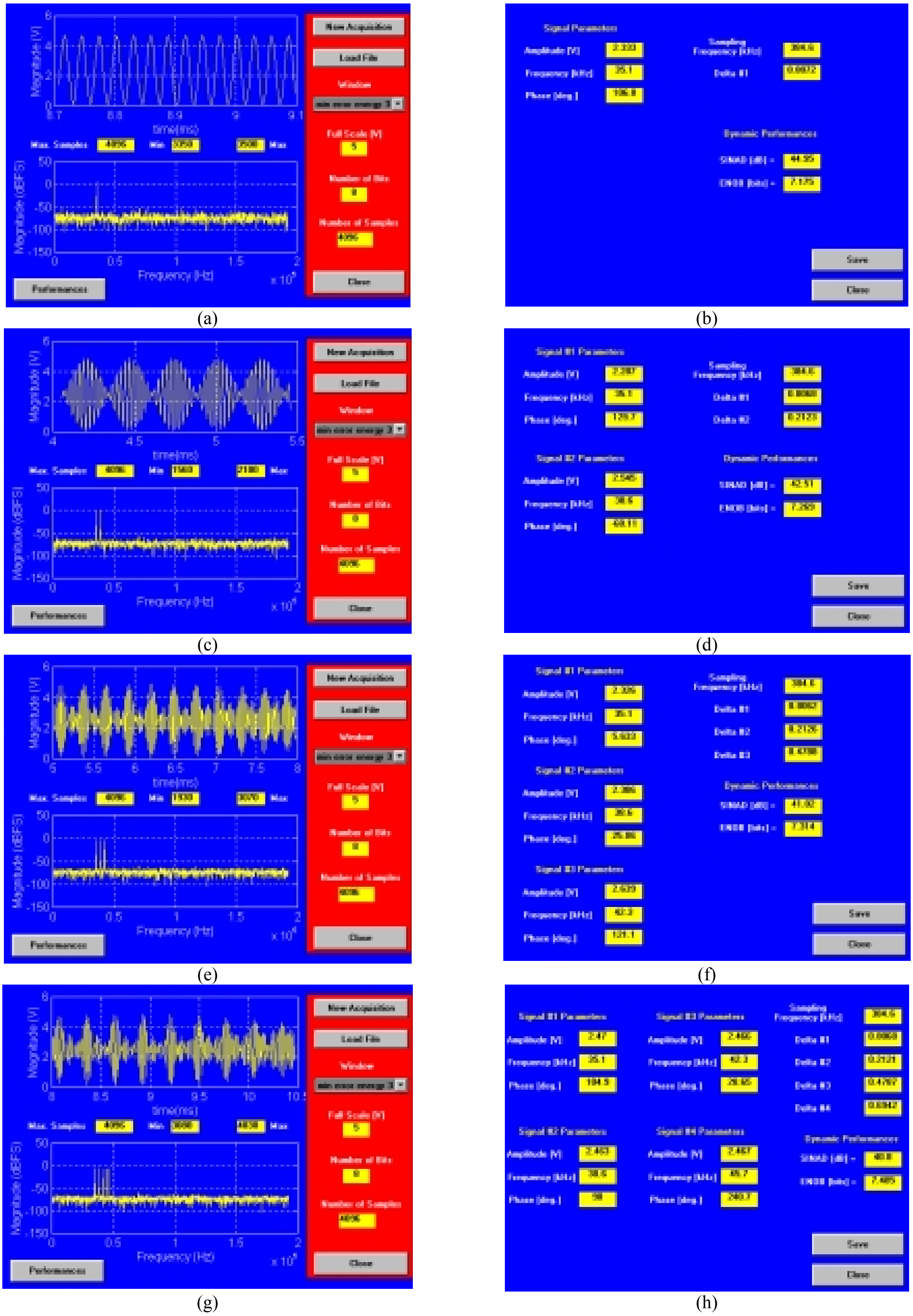


Fig. 4 - The results obtained after the testing of TLC0820 with the third frequency set in:
(a)-(b) single-tone mode; (c)-(d) dual-tone mode; (e)-(f) three-tone mode; (g)-(h) four-tone mode.

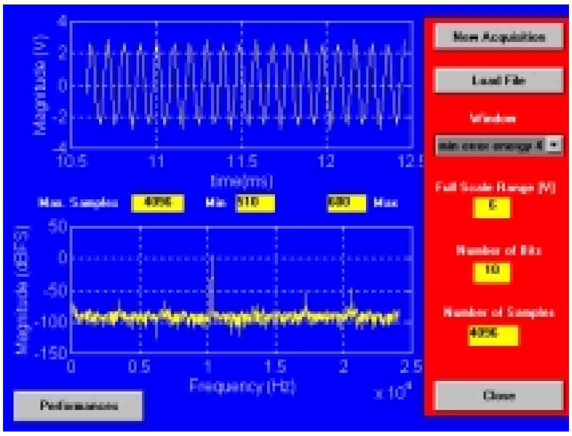
If the first frequency set is employed, the ENOB estimates obtained in the tested modes were practically the same. This behavior is achieved because the distortion performances of the sinewaves at the considered frequencies were much higher than those of the converter. In this situation it is sufficient to use a single-tone signal for the estimation with high precision of the ENOB.

From the results obtained after the testing with the second and the third frequency sets it is obvious that the accuracy in the estimation of ENOB increases when the number of tones increases.

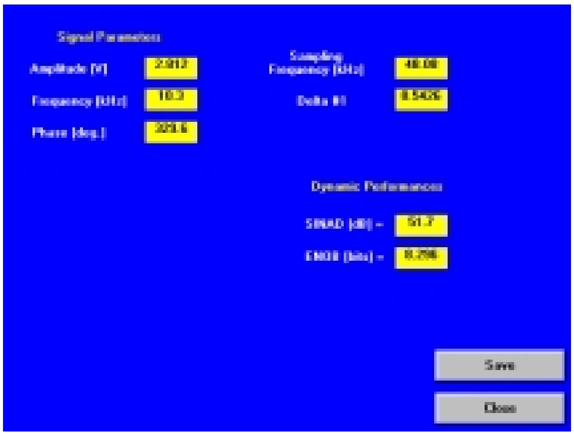
When the second frequency set is used the ENOB estimates obtained in the three and four tone modes are practically the same. Moreover, these ENOB estimates are practically equal with those obtained after testing with the first frequency set. From these results it follows that it is sufficient to test the converter with three-tone signal to obtain high accuracy ENOB estimates. In the analyzed situation the distortion performances of the sinewaves are poorer, but very close to those of the converter.

In the case of testing with the third frequency set the ENOB estimates in the three and four tone modes are not close. From this reason we conclude that is necessary to use a test signal with more than four tones to estimate with high accuracy the ENOB. If the accuracy of the ENOB estimates is the same with that obtained after testing with the first frequency set it is sufficient to use a five-tone signal for estimation with high accuracy the ENOB.

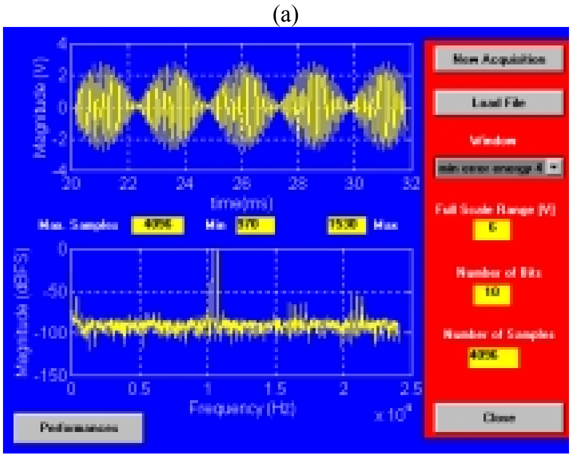
In Figure 5 the results obtained after testing of the TLC32040's ADC are presented. Because the coherent sampling relationships are not met, to eliminate the leakage errors the 4-term minimum error energy window was employed [12].



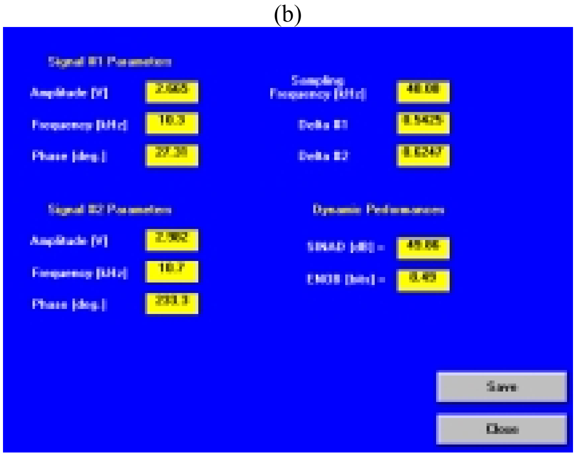
(a)



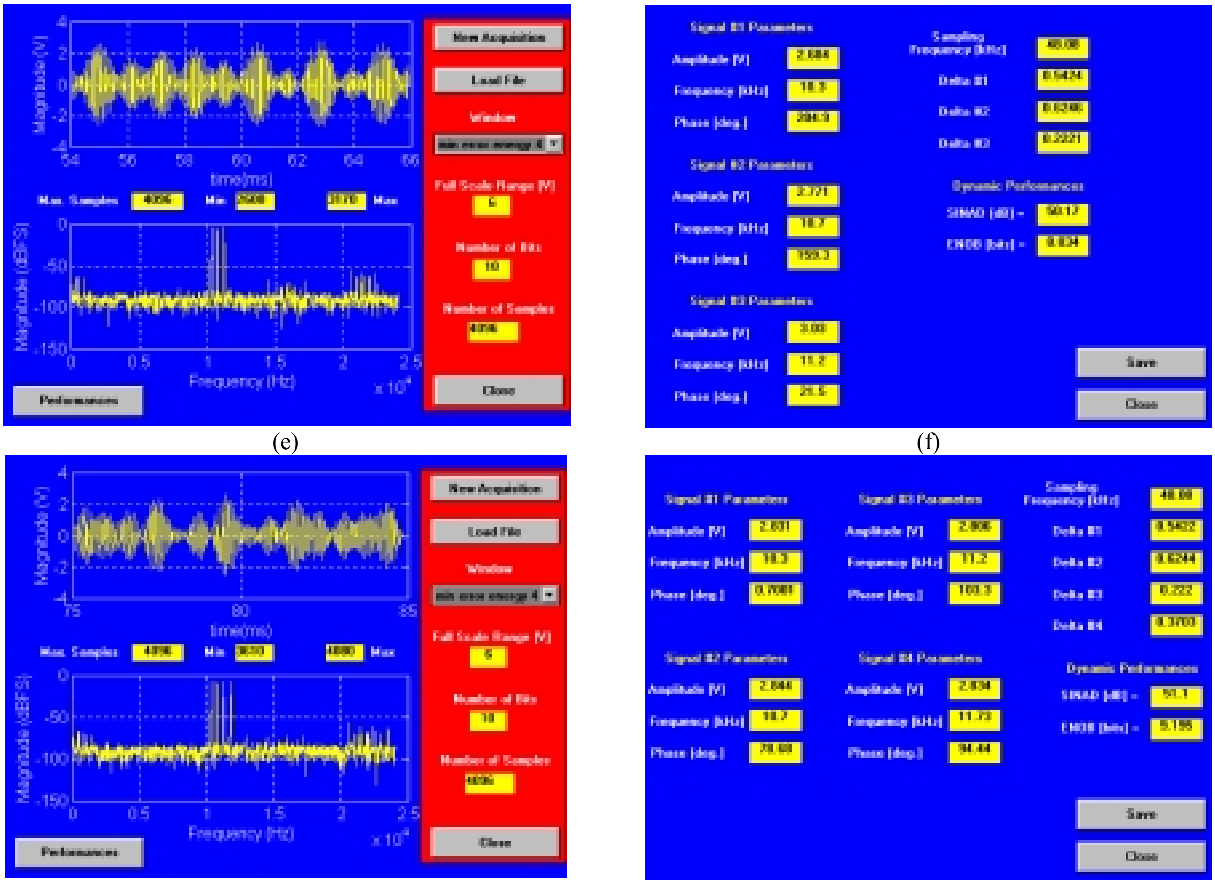
(b)



(c)



(d)



(e) (f) (g) (h)

Fig. 5 - The results obtained after the testing of the TLC32040's ADC in:
(a)-(b) single-tone mode; (c)-(d) dual-tone mode; (e)-(f) three-tone mode; (g)-(h) four-tone mode.

Figure 5 shown that the precision in the estimation of the ENOB increases as the number of tones increases. However, the ENOB estimates are not accurate because the distortion performances of the generators are much lower than those of the converter. For estimating ENOB with high accuracy a test signal with a high number of tones is necessary. This leads to a prohibitive number of sinewave generators. This drawback can be overcome by using sinewave generators with better distortion performances.

4. CONCLUSION

Testing of ADCs by frequency-domain analysis in multi-tone mode has been investigated. The theoretical expressions of the THD, IMD, SINAD and ENOB dynamic parameters were derived. Also, the performance concerning the evaluation of ENOB obtained by testing in multi-tone mode was compared this the one obtained by testing in single-tone mode. For this purpose, the theoretical expression of the difference between the values of ENOB, ΔENOB, obtained by testing in these modes was derived. From the ΔENOB expression it follows that the accuracy in the evaluation of ENOB increases as the number of modes used in the test signal increases. Another important conclusion, based upon the ΔENOB expression, is that the ENOB can be estimated with high accuracy even when using sinewave generators with distortion performances poorer than the ones of the converter under test. The experimental results confirm these conclusions.

Future work will focus on the implementation of accurate *m*-tone signal by numerical methods. This overcomes the necessity to use sinewave generators, and so, lowers significantly the cost of the test system.

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