PUBLISHING HOUSE
OF THE ROMANIAN ACADEMY

PROCEEDINGS OF THE ROMANIAN ACADEMY, Series A,
Volume 5, Number 2/2004, pp.000-000

A THEORETICAL ANALYSIS OF THE ELECTROSTATIC FILTERS

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The paper presents a theoretical analysis of the electrostatic filters considering the DC corona discharge between two conductors, a general case being taken into consideration.

Keywords: theoretical analysis, electrostatic filters.

1. INTRODUCTION

Energy represents a polluting factor of the environment. The thermoelectric power plants are among the most important stationary antropic sources for the pollutant emission in the atmosphere. The combustion gases represent one of the main polluting sources of the thermoelectric power plants. On applying the fossil fuels in the combustion process, in the boilers of the thermoelectric power plants there take place important emissions of CO_2, SO_2, NO_x in the air as well as of suspension particles. Since 39,5% from the electric power necessary in our country is produced in the thermoelectric power plants, the quantity of polluting gas emission is important. For these reasons, the application of the electrostatic filters shows a special interest.

The paper deals with the results of the theoretical researches related to the efficient application of the electrostatic filters.

2. THEORETICAL ANALYSIS OF THE ELECTROSTATIC FILTER PROCESS

From the assembly of the analyzed theoretical problems [1 – 4] we will refer to some aspects largely related to the electrostatic filter functionality.

2.1. Determination of the characteristic of the corona discharge external area for bidimensional fields

2.1.1. DC Corona Discharge for Two Parallel Conductors

The study of the corona discharge in the case of two parallel conductors is confronted with difficulties because of the electric field asymmetry. The calculation hypothesis suggested by Deutsch [5] is not applicable in the case of two parallel conductors. Considering not only the limit conditions but also the electric charge distribution around the conductor, in which the corona discharge occurs, for various value of the applied voltage the corona discharge equation can be integrated through successive approximations [1,3].

Fig. 1 shows the computation diagram; one consider the known equation system:

$$\varepsilon \cdot \operatorname{div} \overline{E} = \rho \qquad (1)$$

$$\overline{E} = -\nabla \varphi \qquad (2)$$

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$$\overline{j} = \rho\mu_i \overline{E} \tag{3}$$

where μ_i is the mobility of the charge carriers.

The equation system (1) – (3) leads to an equation with 3rd partial derivatives for the electric potential:

$$\operatorname{div}\left(\nabla\varphi\Delta\varphi\right) \tag{4}$$

in the conditions:

$$\begin{aligned}&\varphi(\alpha_0, \beta) = u(\beta)\\ &\varphi(0, \beta) = 0\end{aligned} \tag{5}$$

as well as:

$$\frac{\partial(\alpha,0)}{\partial\beta} = \frac{\partial\varphi(\alpha,-\pi)}{\partial\beta} = \frac{\partial\varphi(\alpha,\pi)}{\partial\beta} \tag{6}$$

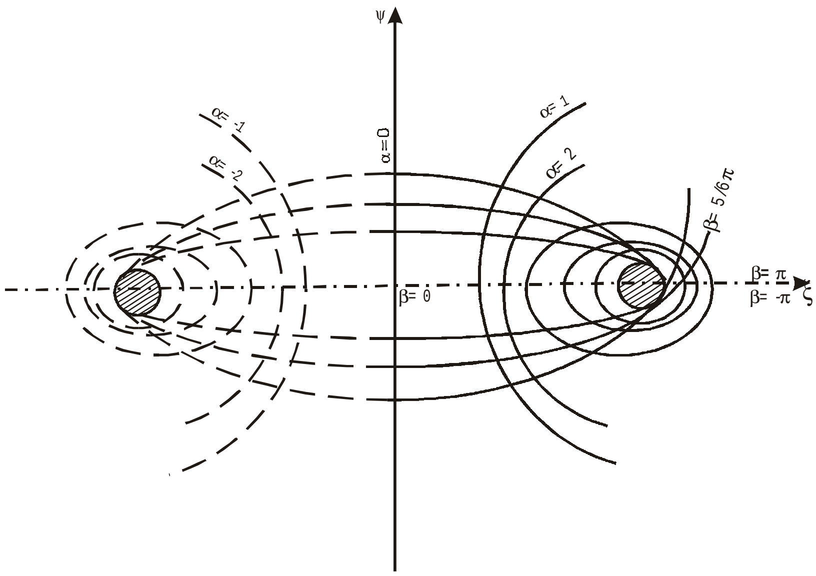

Fig. 1

Eliminating the volume density of the electric charge p from the first two equations (1) and (2) and taking into consideration relation (3) one obtains:

$$(\Delta\varphi)^2 + \frac{\partial\varphi}{\partial\alpha}\left(\frac{\partial^3\varphi}{\partial\alpha^3} + \frac{\partial^3\varphi}{\partial\alpha\partial\beta^2}\right) + \frac{\partial\varphi}{\partial\alpha}\left(\frac{\partial^3\varphi}{\partial\alpha^2\partial\beta} + \frac{\partial^3\varphi}{\partial\beta^3}\right) = 0 \tag{7}$$

After a series of hard operations the calculation relations are obtained:

- for potential

$$\varphi(\alpha, \beta) = \frac{1}{\pi}\int_{-\pi}^{\pi} u(\xi) \sum_{n=0}^{\infty} \frac{sh2(2n+1)\alpha}{sh2(2n+1)\alpha_0} \cos\frac{(2n+1)\xi}{2} \cos\frac{(2n+1)\beta}{2} d\xi \tag{8}$$

- for charge density

$$\rho = \frac{15\varepsilon}{4\pi}\int_{-\pi}^{\pi} u(\xi) \sum_{n=0}^{\infty} (2n+1)^2 \frac{sh2(2n+1)\alpha}{sh2(2n+1)\alpha_0} \cos\frac{(2n+1)\xi}{2} \cos\frac{(2n+1)\beta}{2} d\xi \tag{9}$$

- for the electric field intensity module

$$E = \frac{1}{\pi}\left\{\left[\int_{-\pi}^{\pi} u(\xi)\sum_{n=0}^{\infty}(2n+1)^2 \frac{sh2(2n+1)\alpha}{sh2(2n+1)\alpha_0}\cos\frac{(2n+1)\xi}{2}\cos\frac{(2n+1)\beta}{2}d\xi\right]^2 + \left[\int_{-\pi}^{\pi} u(\xi)\sum_{n=0}^{\infty}\frac{(2n+1)}{2}\frac{sh2(2n+1)\alpha}{sh2(2n+1)\alpha_0}\cos\frac{(2n+1)\xi}{2}\cos\frac{(2n+1)\beta}{2}d\xi\right]\right\}^{1/2} \quad (10)$$

By means of the computer, one can determine the electric charge dependence function the ratio $\frac{U-U_0}{U_0}$ [3].

The deduced relations can be generalized, through succesive approximations, for a series of parallel and coplanar conductors, characteristic of the electrostatic filters.

Fig. 2 shows the dependence of the electrostatic charge multiplication ratio $\frac{n}{n_0}$ function the value if $\varphi = \omega t$, for various percentage values of the ratio $\frac{U-U_0}{U_0}$.

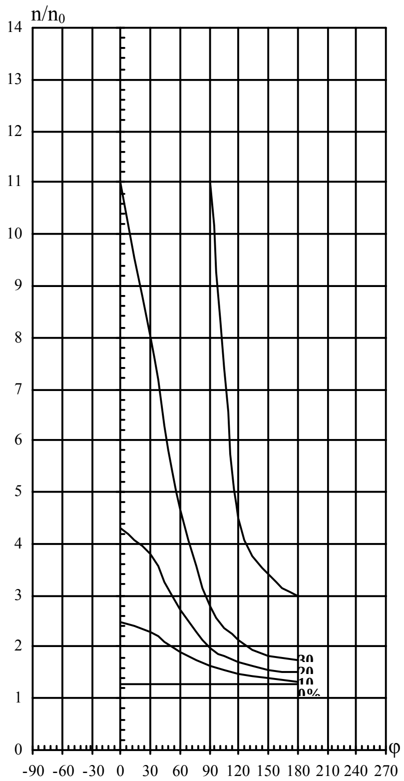

Fig. 2

2.1.2 *Calculation of the Characteristics of the Corona Discharge Area for Bidimensional Fields*

In the electrostatic filters there is used the system of electrodes: plate – conductor series or series of conductors between the plates. The configuration from the electrofilters of unit 3 of the thermoelectic power station in Turceni has been considered as a model of electrode configuration.
plant in Turceni has been considered asa model of electrode configuration.

As relation (8) – (10) require performant calculation means in the case of some complex configurations one has chosen the Deutsch – Popkov calculation relation which are much simpler.

Making use of the isognal representation with the co-ordinates r and θ for the plate conductor series configuration one obtains the relations:

- for θ=0

$$E = E_0 r_0 \frac{\pi}{d} \cdot \frac{\alpha r + 1}{2\alpha r} \sqrt{j_0 \left(\frac{d}{\pi}\right)^2 2\left[\ln(\alpha r + 1) - \frac{\alpha r}{\alpha r + 1}\right] + 1} \tag{11}$$

- for 0< θ<π

$$E = E_0 r_0 \frac{\pi}{d} \cdot \frac{\alpha^2 r^2 + 2\alpha r \cos\theta + 1}{2\alpha r} \sqrt{j_0 \left(\frac{d}{\pi}\right)^2 2\left[\frac{1}{2}\ln\left(\alpha^2 r^2 + 2\alpha r \cos\theta + 1\right) - ctg\theta arctg \frac{\alpha r \sin\theta}{|1 + \alpha r \cos\theta|} + 1\right]} \tag{12}$$

- for θ=0

$$\rho = \frac{j_0}{kE_0 \sqrt{j_0 \left(\frac{d}{\pi}\right)^2 \left[\ln(\alpha r + 1) - \frac{\alpha r}{\alpha r + 1}\right] + 1}} \tag{13}$$

- for 0< θ<π

$$\rho = \frac{j_0}{kE_0} \sqrt{j_0 \left(\frac{d}{\pi}\right)^2 2\left[\frac{1}{2}\ln\left(\alpha^2 r^2 + 2\alpha r \cos\theta + 1\right) - ctg\theta arctg \frac{\alpha r \sin\theta}{|1 + \alpha r \cos\theta|} + 1\right]} \tag{14}$$

where:

$$r^2 = \frac{sh^2 \frac{\pi}{d}(h - y) + \sin^2 \frac{\pi}{d} x}{sh^2 \frac{\pi}{d}(h + y) + \sin^2 \frac{\pi}{d} x}, \tag{15}$$

$$\theta = arctg \frac{2\frac{\pi h}{d} \sin\frac{\pi}{d} x \cos\frac{\pi}{d} y}{sh\frac{\pi}{d}(h - y) sh\frac{\pi}{d}(h + y) - 2\frac{\pi h}{d} + \sin^2 \frac{\pi}{d} x},$$

$$\alpha = 2ch\left(\frac{2\pi}{d}\right)h, \tag{16}$$

$$j_0 = \frac{j_0}{\varepsilon_0 E_0^2 r_0 k}.$$

Fig. 3 shows the electrical field distribution in the plate – conductor series system with x ≤ d/r_0, h = 17,5 cm, d= 30 cm, r_0 =0,75 cm – Isodyn 135.

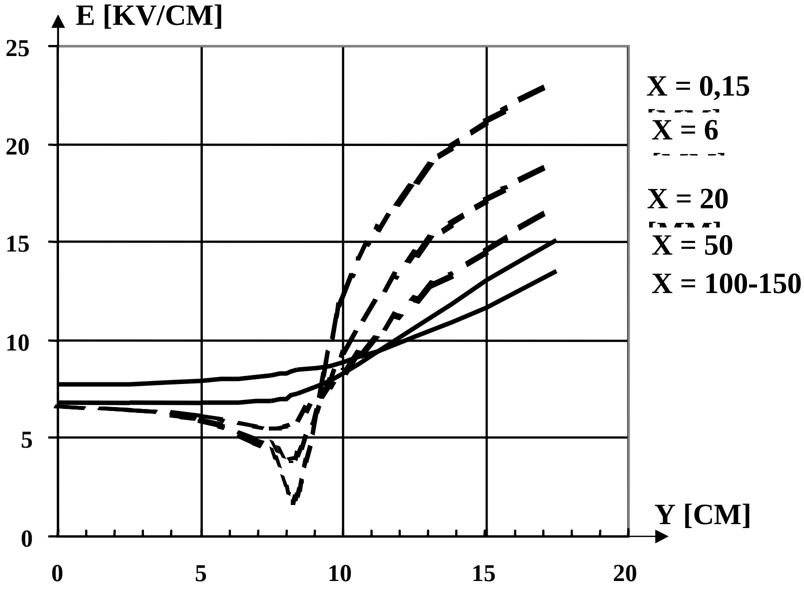

Fig. 3

2.1.3. Calculation of the Corona Discharge Current – Voltage Characteristic

Using the series development method carries out the problem solving. This method is based on the isogonal representation of the initial space on the ring, as well as on the representation of the series – shaped filed potential:

$$\varphi(\mathrm{r},\theta) = \mathrm{f}_0(\mathrm{r}) + \sum_{\mathrm{n}=1}^{\infty} \mathrm{f}_\mathrm{n}\,(\mathrm{r})\cos\mathrm{n}\theta \tag{17}$$

where r and θ are polar coordinates in the ring plane f_n (r); n = 1, 2, …, are unknown functions of the ray r.

The equation of the unipolar corona discharge in the ring plane takes the form:

$$\mathrm{K}(\nabla^2\varphi)^2 + \frac{\partial\varphi}{\partial \mathrm{r}}\frac{\partial(\mathrm{K}\nabla^2\varphi)}{\partial r} + \frac{1}{\mathrm{r}^2}\frac{\partial\varphi}{\partial \mathrm{r}}\frac{\partial(\mathrm{K}\nabla^2\varphi)}{\partial\theta} = 0 \tag{18}$$

On substituting the seri development (17) in the equation 918), the problem reduces itself to the integration of the obtained differential eqaution system function of the unknown quantities f_n (r).

One can prove that the value of the current given by the corona discharge I depends only on the approximation function f_0 (r)

$$I = -\ 2\pi e_0 \mu_i r_0 q_0 U E_i I_0(r_0) \tag{19}$$

where μ_I is the ionic mobility, r_0 – ray of the corona discharge conductor, $I_0(r_0)$ – the operator which is dependent on function $f_n(r)$ (the calculations show that only $I_0(r_0)$ is of interest), E_I – the corona charge initial intensity, q_0 – the electric charge corresponding to E_i.

The approximation function $f_0(r)$ is determinated from the zero approximation equation as well as from the expression for the isogonal transformation coefficient $K(\nabla^2\varphi)$.

After a series of transformations one obtains the zero approximation equation which takes the form:

$$I_0^2 + \frac{\mathrm{d} f_0}{\mathrm{d} r}\frac{\mathrm{d} I_0}{\mathrm{d} r} + I_0\frac{\mathrm{d} f_0}{\mathrm{d} r}\frac{\mathrm{d}\ln k_0}{\mathrm{d} r} = 0 \tag{19}$$

Taking into consideration equations (18) and (20) one obtains:

$$\frac{\mathrm{d}\, f_0}{\mathrm{d} r} = - \frac{\sqrt{C_2 + C_1 \ln \frac{r^2 + 2 - \sqrt{3}}{r^2 + 2 + \sqrt{3}}}}{r} \tag{21}$$

Hence, by determinating the operator $I_0(r)$ when $r = r_0$ by means of relation (19) there results:

$$I = C_1 \pi \sqrt{3} \mu_i \varepsilon_0 \frac{U^2}{h^2} \tag{22}$$

In order to find C_1 one must put down the limit conditions for the zero approximation function $f_0(r)$. After some transformations for the plane – electrode system the quantity C_1 results from relation

$$\int_{r_0}^{1} \frac{\sqrt{(\alpha r_0 + C_1 \ln \frac{r^2 + 2 - \sqrt{3}}{(7 - 4\sqrt{3})(r^2 + 2 + \sqrt{3})}}}{r} dr = 1 \tag{23}$$

The problems concerning the $C_1(\alpha)$ curve approximation (where α is a parameter depending on the electrical field intensity E_l) for various electrode systems are solved by using the method of V.Levitov and S.I. Riaboi [4] and by applying to the series development solution.

4. CONCLUSIONS

The theoretical analysis gives the possibility to analyse the influence of different parameters refering to the functionality of the electrostatic filters.

The electrostatic multiplication ratio B/r_0 depends on the frequency and on ωt, while the dependence of the ratio $(U-U_0)/U_0$ is not so important.

The electric field distribution on the plate – conductor system depends on the ratio d/r_0, consequently on the constructive characteristics of the filters.

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Received March 2, 2004