PUBLISHING HOUSE
OF THE ROMANIAN ACADEMY

PROCEEDINGS OF THE ROMANIAN ACADEMY, Series A,
Volume 5, Number 2/2004, pp.000-000

ON THE ROUGHNESS FRACTAL CHARACTER, THE TRIBOLOGICAL PARAMETERS AND THE ERROR FACTORS

Dan PAVELESCU, Andrei TUDOR

"Politehnica" University of Bucharest, Romania
Corresponding author: Andrei TUDOR, E-mail: tudor@meca.omtr.pub.ro

In this paper, after the fractal conditions knowledge of some rough metallic surface, on can determine her fractal character with a new and more simple method, and the respective tribologycal parameters. We also show the error factors or situations, which can influence the results.

Key words: Fractal geometry, fractal model, roughness parameters, cover compases method.

1. INTRODUCTION

One know that the Tribology out-line was made by H.P. Jost (1966) [1], the fractal geometry, which differs from Newtonian Geometry, practical turn up when B.B. Mandelbroot (1967) asks: "which is the real length of the Britain Coast?" [2]. For this he infers a simple equation:

$$L(\varepsilon) = \varepsilon^{(1-D)} \quad \text{[km]}, \tag{1}$$

where: $L(\varepsilon)$ – the irregular outline length;
ε – the unit length;
D - the fractal parameter, which depends on the irregularity degree of the Coast or frontier:

$$D = 1 - \lim \frac{\log L(\varepsilon)}{\log(\varepsilon)}, \quad \text{when } \varepsilon \to 0 \tag{2}$$

The straight lines, as in Fig. 1, indicates the fractal feature of the irregular out line length [2-4].

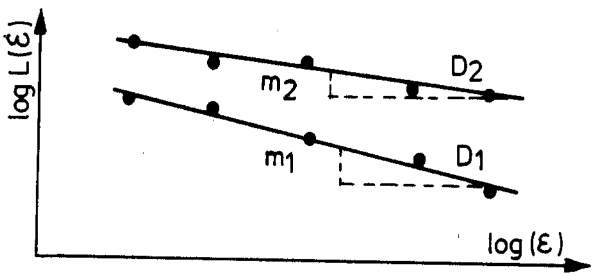

Fig. 1. The coordinate points and the two straight line which indicates:
– the fractality of West coast of the Britain isle, with D_1;
– idem, with D_2 for the frontier between France & Germany.
(the slopes m_1, m_2 to D_1, D_2 fractal parameters).

After he studied many irregularities and shapes in nature named "fractals", Mandelbrot better determined the Fractale Geometry Theory [3] (1982).

In the period 1985 – 1991 some foreign authors accepted his theory and studied only the fractal aspects of nanometric roughness of a magnetic rigid disk surfaces, especially F. F. Ling, A. Majumdar and B.

Recommended by Radu P.VOINEA, member of the Romanian Academy

Bushan [3-6]. In Romania, D. Pavelescu (1992) [7] and then, together with A. Tudor (1994 – 2003) [8- 10], dealt with the micro-metric roughness of the engineering surfaces of some machine elements. For different mechanical processes, we mention also the researches of A. Davidescu (2003) [11-16].

After 1966, the Tribology evolution was considerable. Thus, at the Mondial Tribological Congress from Vienna (2001) K. Holmberg, from Finland [12], imagined the Tribology domain, from molecular and nanometric even to "Universe" (10^{12} m) (Fig. 2).

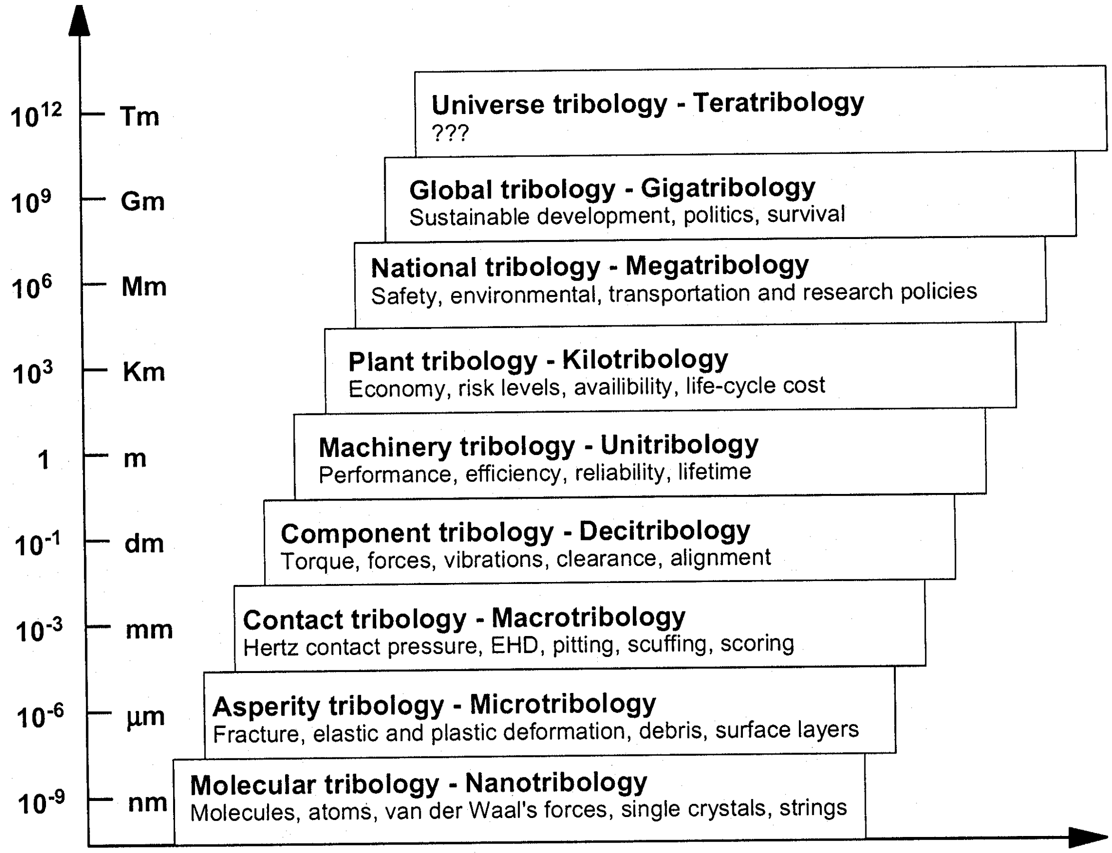

Fig. 2. Scaling up of tribological phenomena from nanotribology to teratribology.

Thus, the mentioned domains (nano and mico-metric roughness) are placed at the low part of this scaling.

We specify too, that are some statistical parameters which are important for Tribological researches but not for the fractal researches! In the first category there are: r.m.s. height (σ or R_q µm), slope (σ'), and curvature (σ''), which depend on the scale length of the roughness, on the measuring instrument and on the magnification on the instrument resolution. But a great error give the stylus magnitude radius of the feeling instrument. It is preferable scanning microscopy therefore without contact [5, 6].

In the second category there are the fractal parameters: D and G which the following advantages: scale independent, and the details of roughness at all scales easily determined. For the micrometrical roughness D has values $1 < D < 2$, for example $D = 1.8806$ for turned surface; in which case $G \cong 4.4591 \times 10^{-2}$ µm [8].

Nevertheless, Majumdar and Bhushan admitted two relations between the classical parameters and the fractal parameters; one is:

$$\sigma = \left[\int_{\omega_l}^{\omega_h} S(\omega) d\omega \right]^{0.5} \tag{3}$$

where: ω_l – the lowest frequency [m^{-1}] related to the length of the sample;

ω_h– the highest frequency which depends on the resolution of the measuring instrument;
$S(\omega)$ – the power of the spectrum of the Weierstrass-Mandelbrot's (W-M) function;
$S(\omega)$ depends of D, G, and γ – scaling parameter of the W-M function ($\gamma = 1.5$).
The second relation (more simple), is:

$$\sigma \approx \omega_l^{(D-2)} \text{ [}\mu\text{m]} \qquad (4)$$

when $\omega_h >> \omega_l$.

2. THE TRUTHFULNESS OF THIS STATISTICAL AND FRACTAL CONNECTION

In order to extend the fractal theory to the roughness surfaces with utility in the Machine Elements domain, we tried to verify the equation (4) in many patterns [7-10]; thus we parallel studied the statistical and fractal methods. The researches are made for a linear contact modeling this contact with a friction pair couple tip TIMKEN: a short rough cylinder and a rigid perfectly smooth plane. The cylinder rough surface appears randomly and multiscale. The profilograme from Fig. 3 is obtained with a stylus type with 5 μm radius. She is mathematically continuous, non-differentiable and statistically self-afine.

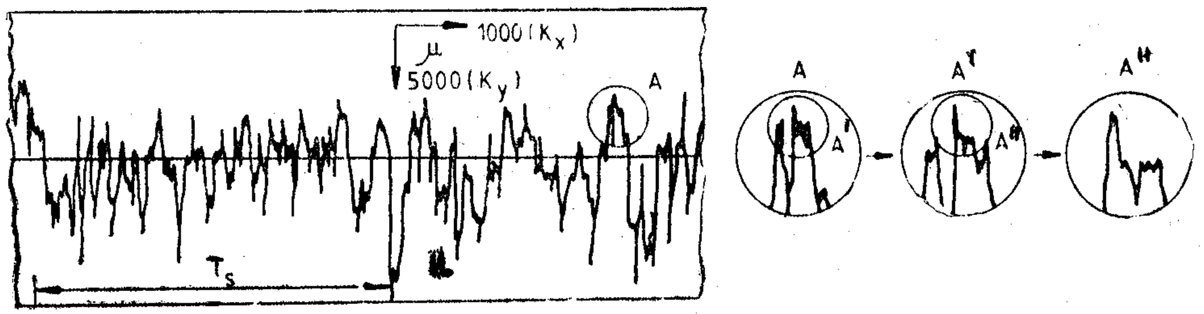

Fig. 3. A partial profilogramme on the generatrice of the short rough cylinder.
The original magnification: $K_y = 5000$, $M_x = 1000$. The repeatedly magnified gives from A the shapes A' and A".

Generally, the results of the mentioned papers show a great difference between statistical and fractal tribological parameters. We appreciate two causes for these differences.

– **The first** is from different composition of the equations for the same tribological statistic or fractal parameter; for example:

Fractal method | **Statistical method**

a) – The power of a profile W-M spectrum

$$S(\omega) = \frac{G^{2(D-1)}}{2\ln\gamma} \times \frac{1}{\omega^{(5-2D)}} \quad [\mu m^3] \qquad (5);$$

$$S(\omega) = \frac{1}{2\pi} \times \frac{1/x'}{(1/x')^2 + \omega^2} \quad [\mu m^3] \qquad (5')$$

$x' = 1/D_0$
D_0 – average profile density (Nr rough./μm)

b) – The mean radius of roughness curvature

$$\overline{\rho_e} = \frac{a^{D/2}}{\pi^2 G^{D-1}} \quad [\mu m] \qquad (6);$$

$$\overline{\rho_e} = \left[1 + (\pi D_0 \sigma)^2\right]^{1.5} / \left(\pi^2 D_0 D_e \sigma\right) \quad [\mu m] \quad (6')$$

a– area of contact spot [μm²]

D_e – density of the extreme min. and max. points (Nr. extrems/μm)

– **The second** cause is due from the appreciation that the shapes A – A" (Fig. 3) in appearance are similar; but are statistical self-afine which means that, the scaling in the vertical and lateral directions of a profile are different, therefore are not self-similar fractal! Consequently, to use this roughness with both methods (fractal and statistical) leads to different results.

In this paper we do not use the approximate equation (4) which does not give accurate values of these and for other parameters as: real area (A_r), the microgeometry complex parameter (Δ_r), plastic and elastic loads (F_p, F_e), friction coefficients (μ_s, μ_a) and friction forces (F_s, F_a). Here we will discuss only the fractal aspects, with a new and more simple method.

3. THE FRACTAL ROUGHNESS AND CALCULUS

To establish the fractal character of the roughness structure M. Hasegawa and al. published a new method [13] (1996) named the cover method (or the compasses method). Thus, if *r* is the Yardstick and *N(r)* the repetitive measurement times, for each *r*, the $N(r)\, x_r$ is the length of the roughness profile. For various ***r*** values, which increase from small scale, between *N(r)* and *r* is a relationship:

$$N(r) \propto r^{-D} \qquad (7)$$

For example, these measurements are made of a conform representation as shown in Fig. 4 for the profilogramme of a turned surface.

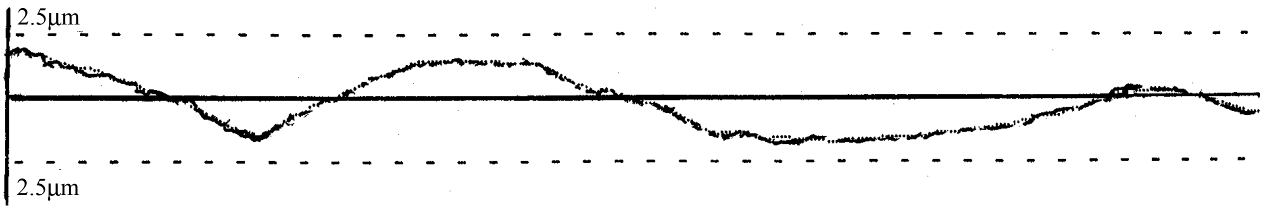

Fig. 4. The turned surface profilogramme: 1400/l, l_s = 120 μm. The conform representation is valid when $K_y = M_x$.

If the points of co-ordonate lg (*r*), lg *N*(*r*) are near straight line as in Fig. 5, the respective roughness with irregular microgeometry fulfills the fractal condition; the slope m of this straight line is even (*–D*).

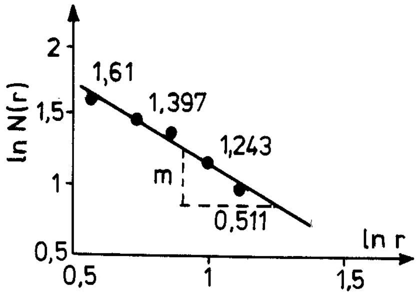

Fig. 5. The fractal condition of the profilogramme from Fig. 4.

For the mechanical operations as: finishing turning, grinding and finishing grinding the R_q or σ values have been between approx. 5 μm and 0.15 μm; the respective *D* values were between approx. 1.02 and 1.1 [11]. We mention that the respective fractal roughness have been without running-in. The surface roughness obtained in the running-in period can become non-fractal. For this, it is necessary the statistical calculation to determine tribological parameters: A_r, $\overline{\rho_e}$, Δ_r, μ_{ap}, μ_e.

The fractal parameter *G* can be obtained from:

$$G = \exp\left\{\frac{1}{2(D-1)} \cdot \ln\left[4(2-D) \cdot R_q^2 \cdot l_s^2 (D-2) \cdot \ln\gamma\right]\right\} \quad [\mu\text{m}] \qquad (1)$$

For *G* the approxim. values are from 9.9 x 10^{-16} to 1.2 x 10^{-2} μm [11].

Other equation for fractal roughness are [8]:

– Real contact area:

$$A_{rf} = \frac{D}{2-D} a_l^2 \quad [\mu\text{m}] \tag{9}$$

where a_l is the highest contact spot area [μm^2].

– The complex parameter of roughness (fractal) [14,15]:

$$\Delta_{rf} = \frac{R_y}{\overline{\rho_{ef}} \cdot b^{1/\nu}} \tag{10}$$

where R_y is the maximum value of roughness [μm]; b and ν are the Abbott – Firstone curve parameters.

Observations:

– The R_y values variation was from approx. 28 μm for finishing turning to 0.8 μm, for finishing grinding; it is somehow similar to R_q variation [11]. Also $\overline{\rho_e}$ values and the pair b, ν have a large spectrum of values depending of the surface procedures.

– These aspects are important for the Δ_{rf} values (10) which can be between approx. 773 to 0.022! The Δ_{rf} values influenced also the fractal friction coefficients for the lubricated contact between the "ideal" plane and the real area (A_{rf}) of the rough cylinder.

– Since $\mu_{ap} >> \mu_{ae}$ [15] we shall use only plastic contact (μ_{ap}) with $(\Delta_{rf})^{0.5}$ in the respective complex situation; it results for μ_{ap} values between 0.926 and 0.085 [11].

In paper [14] Pavelescu and Tudor have been considered the fractal contact time (t_s); when t_s = 0, they obtained μ_{ap} values between approx. 0.711 to 0.322 for Δ_{rf} =538 when $\overline{\rho_e}$ =1.4 x 10^{-3}μm, accepting, according to Kraghelsky [15], the equation:

$$\mu_{ap} = \frac{\tau_0}{c \cdot \sigma_c} + k_{rp} \left(\frac{p_r}{c \cdot \sigma_c} \right)^{\frac{1}{2\nu}} \cdot \left(\Delta_{rf}\right)^{0.5} + \beta_{ad} \tag{11}$$

where: τ_0 = 0.2 MPa for steel; $p_r = \dfrac{F_n}{A_{rf}}$ [N/mm^2]; A_{rf} (eq. 9); Δ_{rf} (eq. 10); β_{ad} = 0.12 is adhesion coefficient for steel/steel; $c \times \sigma_c \approx H$ (hardness); k_{rp} = 0.55.

We mention that the TIMKEN installation have been used from the Machine Elements and Tribology Chair of the Univ. "Politehnica" Bucharest. The triboelements are made of OLC 45 steel with 2000 MPa hardness. The tribological parameters have been: load F_n = 50 N; the relative speed 3.83 m/second; the mineral oil (T90EP2) and constant temperature of 40°C.

CONCLUSIONS

– Not all micrometric roughness are fractals and few after a certain running-in period;
– It is possible that the two equations (3), (4) admitted by Majumdar and Bhushan are proper only for nanometric roughness.
– The new method named the cover method is very laborious but simpler and efficient. Nevertheless [11] remark great differences between $\overline{\rho_{ef}}$, values measured and by calculus.

From this paper it may result also some other error possibilities, as: D estimation, G, A_{rf}, Δ_{rf_1}, μ_{ap} calculus or $\overline{\rho_e}$ measurement or calculus.

ACKNOWLEDGEMENTS

We thank professor I. Iliuc of the I.M.S. (Romanian Academy) for some roughness measurements and dr. A. Davidescu from I.C.B. for some fractal and non-fractal measurements.

REFERENCES

1. JOST, J.P., *A report on the present position and Industry's Needs,* Education and Research Dep. of Educ. and Sci., London, 1966.
2. MANDELBROT, B.B., *How long is the Coast of Britain? Statistical Self-Similiarity and Fractional Dimension*, Science, **155**, pp. 636–639, 1967.
3. MANDELBROT, B.B., *The Fractal Geometry of Nature*, W. H. Freedman, New York, pp. 29-34,1992.
4. LING, F.F., *Engineering surfaces and Tribology*, EUROTRIB'89, Helsinki, **2**, pp. 7–21, 1989.
5. MAJUMDAR, A., BHUSHAN, B., *Role of Fractal geometry in roughness characterization and Contact mechanics of surfaces,* Journal of Tribology, **112**, 1990.
6. MAJUMDAR, A., BHUSHAN, B., *Fractal model of elastic-plastic contact between rough surfaces*, Journal of Tribology, **113**, pp. 1-11, 1991
7. PAVELESCU, D., *On the real contact area for involute gear rough tech., the fractal and statistical concepts*, Rev. Roum. Science Technical-Mechanical Applications, **37**, 4, 1992.
8. PAVELESCU, D., TUDOR, A., *On the fractal and statistical calculus concerning a curved and rough engineering surface contacting a smooth plane*, Rev. Roum. Science, Technical Mechanical Applications, **39**, pp. 432–439, 1994.
9. TUDOR, A., PAVELESCU, D., SAVU, T., *Main aspects and observations concerning the possibility for using the fractal theory in the case of rough engineering surfaces*, ROTRIB'96, **1**, Bucuresti, Ed. Tehnica, 1996.
10. PAVELESCU, D., TUDOR, A., *Several aspects of the fractal theory concerning the metallic engineering surfaces*, SISOM, Romanian Academy, **1**, pp. 19–24, 1999.
11. DAVIDESCU, A., *Contribuţii asupra interdependenţei dintre calitatea suprafeţelor şi performanţele tribologice ale cuplelor neconforme cu alunecare*, Ph. D. Thesis, University "Politehnica" of Bucharest, pp.54-63, 2003.
12. HOLMBERG, K., *Tribology in reability engineering*, The 2nd World Tribology Congress, Vienna, **1**, pp. 13–15, 2001.
13. HASEGAWA, M., et al., *Calculation of the fractal dimensions of machined surfaces profiles*, Wear 192, pp. 40–45, 1996.
14. PAVELESCU, D., TUDOR, A., The contact time in fractal tribological studies, Proceed. of the Romanian Academy Series A, **1**, pp. 55–58, 2000.
15. TUDOR, A., *Frecarea şi uzarea materialelor*, Ed. BREN, Bucureşti, pp.93-96 2002.
16. DAVIDESCU, A., *The Fractal and Non-fractal Roughness of some Differently Machined Steel Surfaces*, EUROTRIB'03 Galati, F VIII, **1**, pp. 413–416, 2003.

Received February 26, 2004