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QUASIRESONANT SCATTERING

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The Quasiresonant Scattering consists from a Single Channel Resonance coupled by Direct Interaction transitions to some competing reaction channels. A description of Quasiresonant Scattering, in terms of generalized reduced *K*-, *R*- and *S*- Matrix, is developed in this work. The Quasiresonance's decay width is, due to channels coupling, smaller than the width of the ancestral Single Channel Resonance (Resonance's Direct Compression).

1. INTRODUCTION

A nuclear reaction develops from initial to final channels via an intermediate Compound System. According to relative number of degrees of freedom involved in the Compound System, the nuclear reactions are considered either as Direct or Resonant ones. The Direct Reactions involve few degrees of freedom, usually those defining initial and final reaction channels as well as those involved in transitions between channels. A direct process does exhibit a monotone energy dependance of the cross-section excitation functions. The Resonant Reactions are on opposite extreme, involving all degrees of freedom of the Compound System. A quasistationary state of the Compound System, called Resonance, takes shape; afterwards it decays with different probabilities – the partial decay widths – in all reaction channels. The Resonance's decay is described by its total decay width, *i.e.* the sum of all channel partial decay widths. The resonant reaction does exhibit sharp change in cross-section excitation functions of all reaction channels.

The multichannel Resonances are, usually, described by poles of the Scattering *S*- Matrix, *e.g.* [1], or within its parameterizations, as poles of the *R*- Matrix, *e.g.* [2], of the *K*- Matrix, *e.g.* [3], or even by Green operators of the Effective Hamiltonians for Compound System, *e.g.* [4]. The Direct Reactions are described either by Distorted Waves Born Approximation or by Coupled Channel Methods, *e.g.* [5].

However there are multichannel reaction phenomena which do share both characteristics of Resonant and Direct processes. They are experimentally evinced as resonant structures in some reaction channels, while other competing reaction channels display a Direct Interaction monotone energy dependance of the excitation functions. Physically they do not involve a genuine multichannel Resonance but rather they correspond to a Single Channel Resonance preceded or/and followed by direct multichannel transitions. Examples of such manifestations in excitation functions do appear in Low Energy Nuclear Physics as, "Coupled Channel Resonances", *e.g.* [6], [7], some "Threshold Anomalies", *e.g.* [7], [8], or "Molecular Resonances" in Heavy Ion Reactions, *e.g.* [9], etc. This type of scattering, involving partial physical features of both Resonant and Direct Reactions, was called Quasiresonant Scattering, [10].

The Quasiresonant Scattering is usually described, within numerical approaches, by the methods of Coupled Channels, *e.g.* [5], [6]. These methods are, however, suitable mainly for descriptions of multichannel direct transitions. One needs a formal framework describing unitary both direct transitions as well as Single Channel Resonance involved in same scattering process. The present approach to Quasiresonant Scattering develops *R*- or *K*- Matrix descriptions, exhibiting how a Single Channel Resonance can induce, via Direct Transitions, "resonant" structures in some competing reaction channels. One obtains an interesting physical result, the Quasiresonance's decay width becomes smaller, due to channel couplings, than that of the ancestral Single Channel Resonance. This effect, exhibiting the Direct Interaction influence on the Resonance Scattering, is named "Direct Compression Effect".

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2. UNITARITY SEPARATION OF DIRECT SCATTERING AND RESONANT STRUCTURES

The Scattering S- or Collision U- Matrices for a multichannel system are parameterized in terms of K- or R- Matrix

$$S = -\mathbf{1} + 2i\{K + i \cdot \mathbf{1}\}^{-1}$$
$$U = -\mathbf{1} + 2i\{P^{1/2}[R^{-1} - S_b]^{-1} P^{1/2} + i \cdot \mathbf{1}\}^{-1}$$

where imaginary P=Im L and real S_b=Re $L - B$ parts of the logarithmic derivative L, (B – boundary condition at channel radius) are Penetration and Shift – Factors. Either of the two parameterizations could be used due to their formal similarity.

The study of Quasiresonant Scattering requires a method able to describe how a Single Channel Resonance (from an invisible or unobserved channel) can induce a resonant structure in some other competing reaction channels. Let divide the set of reaction channels in two groups: the retained (observed) channels, a,b=1,2,…,N and the eliminated (unobserved) channel $n=N+1$. The effect of the eliminated channel $\{n\}$ on retained $\{N\}$ channels is described by Reduced K-, [3], or R-, [2], Matrix

$$K_N^r = K_N - K_{Nn}(K_{nn} + i)^{-1} K_{nN}$$
$$R_N^r = R_N - R_{Nn}(R_{nn} - 1/L_n)^{-1} R_{nN}$$

or by Reduced S- or U- Matrix, [11],

$$S_N = S_N^0 + S_{Nn}(S_{nn} + 1)^{-1} S_{nN}$$
$$U_N = U_N^0 + U_{Nn}(U_{nn} + e^{2i\varphi_n})^{-1} U_{nN}$$

The S_N^0 and U_N^0 Matrices refer to independent N channel system and are constructed with K_N and R_N components of K- and R- Matrix; the phase φ_n is dependent on n – channel logarithmic derivative (L_n-B_n). However both these forms of reduced K-, R-, or S- Matrix do not allow to separate the Resonances from Direct Interaction background. One needs generalized procedures for separation of Resonant Structures and Direct Interaction Scattering, which are based on Unitarity of Scattering Matrix (K-, R- real), [12]. In order to expedite both resonant and direct processes the K- (or R-) Matrix is split into direct – background (β) and resonant (ρ) terms

$$K = K^\beta + K^\rho ; K^\rho = \sum_\lambda \gamma_\lambda \times \gamma_\lambda /(E_\lambda - E)$$

The Reduced K- Matrix becomes

$$K_N^r = K_N^{r\beta} + \sum_{\lambda\mu} \omega_{N\lambda} A_{\lambda\mu} \omega_{\mu N}$$
$$\omega_{N\lambda} = \gamma_{N\lambda} + (S_N^\beta + \mathbf{1}_N)^{-1} S_{Nn}^\beta \gamma_{n\lambda}$$
$$A = (e - E - \xi)^{-1}$$
$$e_{\lambda\mu} = E_\lambda \delta_{\lambda\mu} ; E_{\lambda\mu} = E\delta_{\lambda\mu}$$
$$\xi_{\lambda\mu} = -\gamma_{\lambda n}(K_{nn}^\beta + i)^{-1} \gamma_{n\mu} = \frac{i}{2} \gamma_{\lambda n}(S_{nn}^{0\beta} + 1)\gamma_{n\mu}$$

Observe that ξ – matrix elements are defined in terms of “bare” uncoupled eliminated channel $S_{nn}^{0\beta}$. The Reduced background $K_N^{r\beta}$ - and S_N^β - Matrices refer only to Direct processes. The complete Reduced S-Matrix becomes

$$S_N = S_N^\beta + 2iG_{N\lambda} M_{\lambda\mu} G_{\mu N}$$

where the Level Matrix M and Partial widths $G_{N\lambda}$ are given by

$$M^{-1} = e - E \cdot \mathbf{1} - i\sum_c \gamma_c \times \gamma_c + \sum_{cd} T^{\beta}_{cd} \gamma_c \times \gamma_d$$
$$G_{a\lambda} = \gamma_{a\lambda} + i\sum_c T^{\beta}_{ac} \gamma_{c\lambda}$$

with channel indices c, d extending on both retained and eliminated channels, c, d= 1,2, …,N, n=N+1 and T^{β} is Direct Interaction Transition Matrix defined by S^{β}=1+2iT^{β}.

The "Reduced Operator" procedure has to be extended to the Level Matrix too. A relation between Level Matrix M including all channels and Level Matrix M_0, which does not include the eliminated n=N+1channel,

$$M_0^{-1} = e - E \cdot \mathbf{1} - i\sum_a \gamma_a \times \gamma_a + \sum_{ab} T^{0\beta}_{ab} \gamma_a \times \gamma_b$$

has to be established. Observe that the "bare" Level Matrix M_0 and its counterpart M are related by

$$M^{-1} = M_0^{-1} - 2iG_n (1 + S^{\beta}_{nn})^{-1} G_n = M_0^{-1} - \omega$$

The final results of this procedure are

$$M = M^0 + M \frac{2iG_n \times G_n}{1 + S_{nn}} M$$
$$M = M^0 + M^0 \frac{2iG_n \times G_n}{1 + \tilde{S}_{nn}} M^0$$
$$S_{nn} = S^{\beta}_{nn} + 2i(G_n, MG_n)$$
$$\tilde{S}_{nn} = S^{\beta}_{nn} - 2i(G_n, M^0 G_n)$$

One has to mention that the transformation $G_n \to \tilde{G}_n = iG_n$ results into $\omega \to \tilde{\omega} = -\omega$, $\tilde{M}^{-1} = \tilde{M}_0^{-1} + \omega$, $M \to \tilde{M} = M_0, M_0 \to \tilde{M}_0 = M, S_{nn} \to \tilde{S}_{nn}, \tilde{S}_{nn} \to S_{nn}$, i.e. the reproduction of formulae relating the two Level Matrices.

3. QUASIRESONANT SCATTERING

A Resonance (λ) decays in channel (a) both directly ($\gamma_{\lambda a}$), as in genuine resonant scattering, or via intermediate channels (c) which are coupled to Resonance by Direct Interaction ($T^{\beta}_{ac}\gamma_{c\lambda}$). In R- Matrix terms, the diagonal Penetration Factors Matrix $\| P_a \delta_{ac} \|$ is replaced by a multichannel Direct Transition Matrix, $\| T^{\beta}_{ac} \|$.

Consider now the limit case of a Single Channel Resonance in the eliminated channel n, $\gamma_{\lambda n} \neq 0, \gamma_{\lambda a} = 0.$ The partial width $G_{a\lambda}$ and Level Matrix M become now,

$$G_{a\lambda} = iT^{\beta}_{an} \gamma_{n\lambda}$$

$$M^{-1}_{\lambda\lambda} = E_{\lambda} - E - i\gamma_{\lambda n}\gamma_{n\lambda} + \gamma_{\lambda n} T^{\beta}_{nn} \gamma_{n\lambda}$$

By using the unitarity conditions for background S^{β} – Matrix, $\mathrm{Im}\, T^{\beta}_{nn} = \sum_c | T^{\beta}_{cn} |^2$, one obtains

$$S_{ab} = S^{\beta}_{ab} + 2i\frac{(iT^{\beta}_{an})\gamma^2_{\lambda n}(iT^{\beta}_{nb})}{E_\lambda - E + \mathrm{Re}\,T^{\beta}_{nn}\gamma^2_{nn} - i(1-\sum_c |T^{\beta}_{cn}|^2)\gamma^2_{\lambda n}}$$

The Quasiresonant Scattering consists of Single Channel Resonance coupled by direct transitions to some other reaction channels. The magnitude of the Quasiresonant Process is proportional both to the strength of ancestral (Single Channel) Resonance $\gamma^2_{\lambda n}$ and to multichannel coupling strengths $T^{\beta}_{an}\,T^{\beta}_{nb}$.

One can predict a physical property of the Quasiresoant Scattering, compression of the Quasiresonance's decay width, $\gamma^2_{\lambda n} \rightarrow (1-\sum_c |T^{\beta}_{cn}|^2)\gamma^2_{\lambda n}$: The Quasiresonance's Structure width is narrower than that of the ancestral Single Channel Resonance. This multichannel effect can be named "Direct Compression" of the Resonance. The Compression effect persists even in case of single channel, being related to direct (background) scattering, $S^{\beta}_{aa} = e^{2i\delta_a}$, ($\delta_a$ - single channel background scattering phase shift). For the single channel scattering, the Resonance's width is compressed to $\gamma^2_{\lambda a}\cos^2\delta_a$. For no-background scattering ($\delta_a = 0$) there is no compression; the "strong" – background scattering, (*e.g.* Echo - descending phase shift $\delta \approx \pi/2$), results into Resonance's extinction. Another peculiar property of the background – Resonance's interplay does concern the level shift $\mathrm{Re}\,T^{\beta}_{nn}\gamma^2_{\lambda n}$ which could be non-linear energy dependent; this implies a change of energy scale linearity as stronger as Resonance strength ($\gamma^2_{\lambda n}$) is. Lane discussed such an effect within *R*- Matrix framework, [8].

Interesting physical aspects are exhibited by Level Matrices M and M_0, related by $M^{-1}_{\lambda\lambda} = M^{-1}_{0,\lambda\lambda} - \omega_{\lambda\lambda}, \omega_{\lambda\lambda} = 2iG_{\lambda n}(S^{\beta}_{nn}+1)^{-1}G_{n\lambda}$. If the reduced width $\gamma_{\lambda n}$ becomes imaginary, $G_{\lambda n} \rightarrow \tilde{G}_{\lambda n} = iG_{\lambda n}$, *i.e.* the Resonance is replaced by an Echo, then the relations between the two Level Matrices are inter-changed. The Resonance is related to a scattering phase shift increasing by $\pi/2$ while the Echo is related to a phase shift decreasing by same amount. If, hypothetical, an Echo is coincident with the Resonance then their effects are compensated, *i.e.* Resonance's extinction. The physical implications for connection between Resonances and Echoes, as embodied in relations between the two Level Matrices of the multichannel system, deserve an appropriate insight.

3. CONCLUSIONS

The present study, based on Unitarity Separation of Direct Scattering and Resonant Structures, does provide an adequate framework for description of Quasiresonant Scattering. The Quasiresonance's partial decay widths are product of the Single Channel Resonance strength and of the inter-channel transitions matrix elements. The total width of the Quasiresonant structure is smaller than that of the ancestral Single Channel Resonance. This effect was named "Direct Compression" of the Resonance; it is intimately related to strength of Direct Process. This result can be, probably, related to the Channel Coupling Pole, *e.g.* [7], observed in numerical experiments with Coupled Channels Method for multichannel systems. The Channel Coupling Poles appear for strong channel couplings; they originate in distant poles in energy or wave number complex planes (for no channel coupling) which are driven to physical region (for strong channel coupling), becoming subject of observation.

An alternative approach to Quasiresonant scattering, based on *R*- Matrix, [2], is Bloch's description of Single Particle Resonance, resulting in similar outputs, [13].

A perspective of this formalism should be its extension (to closed channels) below threshold, within Reduced *S*- Matrix framework.

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