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A SPECTRAL CONDITION FOR SPANNING TREES WITH RESTRICTED DEGREES IN BIPARTITE GRAPHS

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Abstract: Let G be a graph and T be a spanning tree of G. We use $Q(G) = D(G)+A(G)$ to denote the signless Laplacian matrix of G, where $D(G)$ is the diagonal degree matrix of G and $A(G)$ is the adjacency matrix of G. The signless Laplacian spectral radius of G is denoted by $q(G)$. A necessary and sufficient condition for a connected bipartite graph G with bipartition (A,B) to have a spanning tree T with $d_T(v) \geq k$ for every $v \in A$ was independently obtained by Frank and Gyárfás (A. Frank, E. Gyárfás, How to orient the edges of a graph?, Colloq. Math. Soc. Janos Bolyai 18 (1976) 353–364), Kaneko and Yoshimoto (A. Kaneko, K. Yoshimoto, On spanning trees with restricted degrees, Inform. Process. Lett. 73 (2000) 163–165). Based on the above result, we establish a lower bound on the signless Laplacian spectral radius $q(G)$ of a connected bipartite graph G with bipartition (A,B), in which the bound guarantees that G has a spanning tree T with $d_T(v) \geq k$ for every $v \in A$.

***Keywords**:* bipartite graph, degree, signless Laplacian spectral radius, spanning tree.

Mathematics Subject Classification (MSC2020): 05C50, 05C05, 05C70

1. INTRODUCTION

Throughout this paper, we only discuss simple, undirected and connected graphs. Let $G = (V(G),E(G))$ be a graph, where $V(G)$ denotes its vertex set and $E(G)$ denotes its edge set. For a vertex $v \in V(G)$, the neighborhood $N_G(v)$ of v in G is defined by $\{u \in V(G) : uv \in E(G)\}$ and the number $d_G(v) = |N_G(v)|$ is the degree of v in G. For a vertex subset $S \subseteq V(G)$, we write $N_G(S) = \bigcup_{v \in S} N_G(v)$.

For a given graph G with vertex set $V(G) = \{v_1, v_2, \ldots, v_n\}$, the adjacency matrix of G is defined by $A(G) = (a_{ij})$, where $a_{ij} = 1$ if two vertices v_i and v_j are adjacent in G, and $a_{ij} = 0$ otherwise. Let $Q(G) = D(G)+A(G)$ denote the signless Laplacian matrix of G, where $D(G) = \mathrm{diag}\{d_G(v_1), d_G(v_2), \ldots, d_G(v_n)\}$ is the diagonal degree matrix of G. Let $\rho_1(G) \geq \rho_2(G) \geq \cdots \geq \rho_n(G)$ and $q_1(G) \geq q_2(G) \geq \cdots \geq q_n(G)$ be the eigenvalues of $A(G)$ and $Q(G)$, respectively. In particular, the largest eigenvalue $\rho_1(G)$ of $A(G)$ is called the adjacency spectral radius of G and denoted by $\rho(G)$, and the largest eigenvalue $q_1(G)$ of $Q(G)$ is called the signless Laplacian spectral radius of G and denoted by $q(G)$. Some properties on spectral radius can be found in [2,14,15,23,24,27,30,31,33].

Let a and b be two integers with $0 \leq a \leq b$. Then a spanning subgraph F of G is called an $[a,b]$-factor of G if $a \leq d_F(v) \leq b$ for every $v \in V(G)$. A spanning tree T of a connected graph G is called a spanning k-tree of G if $d_T(v) \leq k$ for each $v \in V(G)$, that is, the maximum degree of a spanning k-tree of G is at most k, where $k \geq 2$ is an integer. Obviously, a spanning k-tree of G is also a connected $[1,k]$-factor of G. A spanning tree having at most k leaves is called a spanning k-ended tree. Kaneko [7] introduced the concept of leaf degree of a spanning tree. Let T denote a spanning tree of a connected graph G. The number of leaves adjacent to a vertex v in T is called the leaf degree of v. Furthermore, the maximum leaf degree among all the vertices in T is called the leaf degree of T. The minimum of distances between any two leaves in T is called the leaf distance of T.

Lots of scholars investigated the existence of spanning trees under some specified conditions. Ota and Sugiyama [16] posed a sufficient condition for a graph to contain a spanning k-tree via the condition on forbidden subgraphs. Kyaw [13] obtained a degree and neighborhood condition for the existence of a spanning k-tree in a graph. Win [19] showed some results on the existence of a spanning k-tree in a graph. Zhou and Wu [28] provided an upper bound on the distance spectral radius in a graph G to ensure the existence of a spanning k-tree in G. Zhou, Zhang and Liu [32] studied the relation between the spanning k-tree and the distance signless Laplacian spectral radius in a graph and claimed an upper bound on the distance signless Laplacian spectral radius in a graph G to ensure the existence of a spanning k-tree in G. Broersma and Tuinstra [3] presented a degree condition for a graph to contain a spanning k-end tree. Ao, Liu and Yuan [1] obtained tight spectral conditions to guarantee a graph to have a spanning k-end tree, and also posed tight spectral conditions for the existence of a spanning tree with leaf degree at most k in a graph. Zhou, Sun and Liu [26] showed the upper bounds for the distance spectral radius (resp. the distance signless Laplacian spectral radius) of a graph G to guarantee that G has a spanning tree with leaf degree at most k. Wu [20] gave a lower bound on the size of a graph G to guarantee that G has a spanning tree with leaf degree at most k, and established a lower bound on the spectral radius of a graph G to ensure that G has a spanning tree with leaf degree at most k. Kaneko, Kano and Suzuki [8] posed a sufficient condition for a graph to have a spanning tree with leaf distance at least 4. Erbes, Molla, Mousley and Santana [4] investigated the existence of spanning trees with leaf distance at least d, where $d \geq 4$ is an integer. Wang and Zhang [18] showed an A_α-spectral radius condition for the existence of a spanning tree with leaf distance at least 4 in a graph. For more results on spanning subgraphs, we refer the reader to [6, 11, 12, 22, 25, 29].

Let G be a bipartite graph with bipartition (A,B). Let $K_{m,n}$ denote the complete bipartite graph with bipartition (A,B), where $|A| = m$ and $|B| = n$. Given two bipartite graphs $G_1 = (A_1,B_1)$ and $G_2 = (A_2,B_2)$, let $G_1 \nabla G_2$ denote the graph obtained from $G_1 \cup G_2$ by adding all possible edges between A_2 and B_1.

For bipartite graphs, Kano, Matsuda, Tsugaki and Yan [10] provided a degree condition for a connected bipartite graph to contain a spanning k-ended tree. Frank and Gyárfás [5], Kaneko and Yoshimoto [9] independently obtained a necessary and sufficient condition for a connected bipartite graph G with bipartition (A,B) to have a spanning tree T with $d_T(v) \geq k$ for every $v \in A$. Motivated by [5, 9] directly, it is natural and interesting to put forward some sufficient conditions to guarantee that a connected bipartite graph with bipartition (A,B) has a spanning tree T with $d_T(v) \geq k$ for every $v \in A$ with respect to the spectral radius. Our main result is shown as follows.

Theorem 1.1. Let k, m and n be three integers with $k \geq 3$, $m \geq 3$ and $n \geq (k-1)m+1$, and let G be a connected bipartite graph with bipartition $A \cup B$, where $|A| = m$ and $|B| = n$. If

$$q(G) \geq q(K_{1,k-1} \nabla K_{m-1,n-k+1}),$$

then G contains a spanning tree T with $d_T(v) \geq k$ for every $v \in A$ unless $G = K_{1,k-1} \nabla K_{m-1,n-k+1}$, where $q(K_{1,k-1} \nabla K_{m-1,n-k+1})$ is equal to the largest root of $x^4 - (2m+n+k-2)x^3 + (m^2+mn+2km+kn-3m-n-2k+2)x^2 + (km-m+kn-n-km^2+m^2-kmn+mn)x = 0$.

2. SOME PRELIMINARIES

In this section, we introduce some lemmas, which will be used in the proofs of our main results.

Lemma 2.1 (Shen, You, Zhang and Li [17]). Let G be a connected graph, and let H be a subgraph of G. Then

$$q(H) \leq q(G),$$

where the equality holds if and only if $H = G$.

Let M be a real symmetric matrix whose rows and columns are indexed by $V=\{1,2,\cdots,n\}$. Assume that M, with respect to the partition $\pi: V=V_1\cup V_2\cup\cdots\cup V_m$, can be written as

$$M=\begin{pmatrix} M_{11} & M_{12} & \cdots & M_{1m}\\ M_{21} & M_{22} & \cdots & M_{2m}\\ \vdots & \vdots & \ddots & \vdots\\ M_{m1} & M_{m2} & \cdots & M_{mm}\end{pmatrix},$$

where M_{ij} denotes the submatrix (block) of M formed by rows in V_i and columns in V_j. Let q_{ij} denote the average row sum of M_{ij}. Then matrix $M_\pi=(q_{ij})$ is called the quotient matrix of M. If the row sum of each block M_{ij} is a constant, then the partition is equitable.

Lemma 2.2 (You, Yang, So and Xi [21]). Let M be a real symmetric matrix with an equitable partition π, and let M_π be the corresponding quotient matrix. Then every eigenvalue of M_π is an eigenvalue of M. Furthermore, if M is nonnegative, then the largest eigenvalues of M and M_π are equal.

Frank and Gyárfás [5], and Kaneko and Yoshimoto [9] put forward a necessary and sufficient condition for bipartite graphs to have spanning trees with restricted degrees, independently.

Lemma 2.3 (Frank and Gyárfás [5], Kaneko and Yoshimoto [9]). Let G be a connected bipartite simple graph with bipartition $A\cup B$, and $f: A\longrightarrow\{2,3,4,\ldots\}$ be a function. Then G contains a spanning tree T such that $d_T(v)\geq f(v)$ for every $v\in A$ if and only if

$$|N_G(S)|\geq\sum_{v\in S}f(v)-|S|+1$$

for any nonempty subset $S\subseteq A$.

3. THE PROOF OF THEOREM 1.1

Proof of Theorem 1.1. Suppose, to the contrary, that G contains no spanning tree T with $d_T(v)\geq k$ for every $v\in A$. By virtue of Lemma 2.3, we conclude

$$|N_G(S)|\leq(k-1)|S| \tag{1}$$

for some nonempty subset $S\subseteq A$. Choose a connected bipartite graph G with partition $A\cup B$ such that its signless Laplacian spectral radius is as large as possible, where $|A|=m$ and $|B|=n$. We claim that S is a proper subset of A, that is, $S\subset A$. Otherwise, $S=A$. Then it follows from (1) and $S=A$ that $|N_G(A)|\leq(k-1)|A|=(k-1)m$. Combining this with $n\geq(k-1)m+1$, we deduce $n-|N_G(A)|\geq(k-1)m+1-(k-1)m=1$, which is impossible because G is connected. For convenience, we let $|S|=s$ and $|N_G(S)|=r$. Obviously, $1\leq s\leq m-1$ and there are no edges between S and $B-N_G(S)$ in G. In terms of Lemma 2.1 and the choice of G with bipartition $A\cup B$, we conclude $G=K_{s,r}\nabla K_{m-s,n-r}$. It is clear that $G=K_{s,r}\nabla K_{m-s,n-r}$ is a spanning subgraph of $G_1=K_{s,(k-1)s}\nabla K_{m-s,n-(k-1)s}$. Together with Lemma 2.1, we infer

$$q(G)=q(K_{s,r}\nabla K_{m-s,n-r})\leq q(K_{s,(k-1)s}\nabla K_{m-s,n-(k-1)s}), \tag{2}$$

where the second equality holds if and only if $G=K_{s,(k-1)s}\nabla K_{m-s,n-(k-1)s}$. In what follows, we will show that $q(K_{s,(k-1)s}\nabla K_{m-s,n-(k-1)s})\leq q(K_{1,k-1}\nabla K_{m-1,n-k+1})$ with equality if and only if $s=1$.

For the bipartite graph $G_1=K_{s,(k-1)s}\nabla K_{m-s,n-(k-1)s}$, the quotient matrix of the signless Laplacian matrix

$Q(G_1) = Q(K_{s,(k-1)s}\nabla K_{m-s,n-(k-1)s})$ by the partition $V(G_1) = S \cup (A-S) \cup N_{G_1}(S) \cup (B - N_{G_1}(S))$ is equal to

$$B_1 = \begin{pmatrix} (k-1)s & 0 & (k-1)s & 0 \\ 0 & n & (k-1)s & n-(k-1)s \\ s & m-s & m & 0 \\ 0 & m-s & 0 & m-s \end{pmatrix}.$$

Then the characteristic polynomial of B_1 is

$$\begin{aligned}\varphi_{B_1}(x) =& x^4 - (2m+n+ks-2s)x^3 + (m^2+mn+2kms+kns-3ms-ns-2ks^2+2s^2)x^2 \\ &+ (kms^2 - ms^2 + kns^2 - ns^2 - km^2s + m^2s - kmns + mns)x.\end{aligned}$$

Notice that the partition $V(G_1) = S \cup (A-S) \cup N_{G_1}(S) \cup (B - N_{G_1}(S))$ is equitable. In view of Lemma 2.2, the largest root, say q_1, of $\varphi_{B_1}(x) = 0$ satisfies $q_1 = q(G_1) = q(K_{s,(k-1)s}\nabla K_{m-s,n-(k-1)s})$. Note that $K_{m,(k-1)s}$ is a proper subgraph of $G_1 = K_{s,(k-1)s}\nabla K_{m-s,n-(k-1)s}$, and $G_1 = K_{s,(k-1)s}\nabla K_{m-s,n-(k-1)s}$ is a proper subgraph of $K_{m,n}$. According to Lemma 2.1, we have

$$m+n = q(K_{m,n}) > q_1 = q(G_1) > q(K_{m,(k-1)s}) = m + (k-1)s. \tag{3}$$

For the bipartite graph $G_* = K_{1,k-1}\nabla K_{m-1,n-k+1}$, the quotient matrix of $Q(G_*)$ in terms of the partition $V(G_*) = S \cup (A-S) \cup N_{G_*}(S) \cup (B - N_{G_*}(S))$ can be written as

$$B_* = \begin{pmatrix} k-1 & 0 & k-1 & 0 \\ 0 & n & k-1 & n-k+1 \\ 1 & m-1 & m & 0 \\ 0 & m-1 & 0 & m-1 \end{pmatrix},$$

so its characteristic polynomial is

$$\begin{aligned}\varphi_{B_*}(x) =& x^4 - (2m+n+k-2)x^3 + (m^2+mn+2km+kn-3m-n-2k+2)x^2 \\ &+ (km - m + kn - n - km^2 + m^2 - kmn + mn)x.\end{aligned}$$

Note that the partition $V(G_*) = S \cup (A-S) \cup N_{G_*}(S) \cup (B - N_{G_*}(S))$ is equitable. According to Lemma 2.2, the largest root, say q_*, of $\varphi_{B_*}(x) = 0$ satisfies $q_* = q(G_*) = q(K_{1,k-1}\nabla K_{m-1,n-k+1})$.

Let $\psi(x) = (k-2)x^2 - (2km+kn-3m-n-2ks-2k+2s+2)x - kms - km + ms + m - kns - kn + ns + n + km^2 - m^2 + kmn - mn$. Notice that $\varphi_{B_1}(q_1) = 0$. By plugging the value q_1 into x of $\varphi_{B_*}(x) - \varphi_{B_1}(x)$, we obtain

$$\varphi_{B_*}(q_1) = \varphi_{B_*}(q_1) - \varphi_{B_1}(q_1) = q_1(s-1)\psi(q_1). \tag{4}$$

In view of (3) and $k \geq 3$, we easily see

$$\psi(q_1) \leq \max\{\psi(m+n), \psi(m+(k-1)s)\}. \tag{5}$$

Recall that $k \geq 3$ and $2 \leq s \leq m-1$. We deduce

$$\begin{aligned}\psi(m+n) =& (km+kn-m-n)s + km - m - n^2 + kn - n - mn \\ \leq& (km+kn-m-n)(m-1) + km - m - n^2 + kn - n - mn \\ =& -n^2 + (km-2m)n + km^2 - m^2.\end{aligned} \tag{6}$$

Let $f(x) = -x^2 + (km-2m)x + km^2 - m^2$. Then the symmetry axis of $f(x)$ is $x = \frac{km-2m}{2}$, and so $f(x)$ is decreasing in the interval $[\frac{km-2m}{2}, +\infty]$. Note that

$$\frac{km-2m}{2} < (k-1)m < n$$

by $k \geq 3$ and $n \geq (k-1)m+1$. Then we deduce

$$\begin{aligned} f(n) <& f((k-1)m) \\ =& -(k-1)^2m^2 + (km-2m)(k-1)m + km^2 - m^2 \\ =& 0. \end{aligned}$$

Combining this with (6), we obtain

$$\psi(m+n) \leq f(n) < f((k-1)m) = 0. \tag{7}$$

By a direct computation, we get

$$\psi(m+(k-1)s) = (k-1)(k(k-1)s^2 - (kn-2k+2)s + m - n). \tag{8}$$

Let $h(x) = k(k-1)x^2 - (kn-2k+2)x + m - n$. Recall that $2 \leq s \leq m-1$. Then $h(s) \leq \max\{h(2), h(m-1)\}$. By a simple calculation, we have

$$\begin{aligned} h(2) =& 4k(k-1) - 2(kn-2k+2) + m - n \\ =& -(2k+1)n + 4k^2 + m - 4 \\ <& -(2k+1)(k-1)m + 4k^2 + m - 4 \\ =& -(2k^2-k-2)m + 4k^2 - 4 \\ \leq& -3(2k^2-k-2) + 4k^2 - 4 \\ =& -2k^2 + 3k + 2 \\ <& 0 \end{aligned}$$

and

$$\begin{aligned} h(m-1) =& k(k-1)(m-1)^2 - (kn-2k+2)(m-1) + m - n \\ <& k(k-1)(m-1)^2 - (k(k-1)m-2k+2)(m-1) + m - (k-1)m \\ =& -k(k-2)(m-1) - k + 2 \\ <& 0 \end{aligned}$$

due to $k \geq 3$, $m \geq 3$ and $n \geq (k-1)m+1 > (k-1)m$. Thus, $h(s) \leq \max\{h(2), h(m-1)\} < 0$ for $2 \leq s \leq m-1$. Combining this with (8), we infer

$$\psi(m+(k-1)s) = (k-1)h(s) < 0. \tag{9}$$

Using (5), (7) and (9), we conclude

$$\psi(q_1) \leq \max\{\psi(m+n), \psi(m+(k-1)s)\} < 0. \tag{10}$$

It follows from (3), (4), (10), $k \geq 3$ and $1 \leq s \leq m-1$ that

$$\varphi_{B_*}(q_1) = q_1(s-1)\psi(q_1) \leq 0,$$

which yields that $q(K_{s,(k-1)s} \nabla K_{m-s,n-(k-1)s}) = q_1 \leq q_* = q(K_{1,k-1} \nabla K_{m-1,n-k+1})$ with the second equality holding if and only if $s = 1$. Together with (2), we obtain

$$q(G) \leq q(K_{1,k-1} \nabla K_{m-1,n-k+1})$$

with equality if and only if $G = K_{1,k-1} \nabla K_{m-1,n-k+1}$, which is a contradiction to the condition of Theorem 1.1 because $G = K_{1,k-1} \nabla K_{m-1,n-k+1}$ has no spanning tree T with $d_T(v) \geq k$ for every $v \in A$. This completes the proof of Theorem 1.1. □

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