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OPTIMIZATION-DRIVEN CONTROL DESIGN FOR THE NONLINEAR TWO-WHEELED UNSTABLE TRANSPORTER SYSTEM

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Abstract. This paper focuses on integrating optimization algorithms to address practical challenges in nonlinear systems, emphasizing the design of robust and accurate control for a two-wheeled unstable transporter. The laboratory equipment is a research and industrial system designed to test various control algorithms in complex and unstable environments. The system response determined using the default model parameters, provided by the nonlinear mathematical model, was found to differ considerably from real-time experimental results. Therefore, the nonlinear mathematical model had to be modified to be more accurate so that simulation results correspond to the actual behaviour of equipment in real-time. Three optimization algorithms were employed to establish the optimal parameters of the nonlinear mathematical model of the system, considering three key signals: the average rotational velocity of the wheel (θ), the yaw angle from the vertical axis (ψ) and the angle of rotation around the vertical axis (ϕ). A comparative analysis confirms that the proposed methodology can be delivered in practical applications and demonstrates its capability to provide high-performance, flexible, and stable solutions developed for nonlinear systems. The results of this research highlight the need for integrating optimization algorithms in solving real-world engineering problems

Keywords: nonlinear mathematical model, metaheuristic algorithms, control design, two-wheeled unstable transporter.

1. INTRODUCTION

As discussed in many seminal books on mechatronics [1–5], mechatronic systems are the integration of mechanical engineering, electrical engineering, and computer science to have systems that work well and efficiently. The purpose of such intelligent systems is to correlate intelligence, versatility, and malleability to address diverse operational requirements. Mechatronic systems combine sensors, actuators, controllers and software to achieve real-time feedback, allowing for controlled and automated functions in both simple and complex systems. Mechatronics is primarily focused on the integration of hardware and software components. This synergy enables reduced costs, energy efficiency, and enhanced productivity. Moreover, the ability of these systems to be tailored to suit a wide range of environments makes them powerful tools. The importance of the role of mechatronic systems is further emphasized as industries move towards smart and sustainable solutions. Future mechatronic systems are expected to be more autonomous, interconnected and capable of complex operational requirements with emerging technologies such as artificial intelligence (AI), machine learning, and the Internet of Things (IoT) where the innovation will cross regional and sectoral barriers.

The Two-Wheeled Unstable Transporter (Tw-UnTrans), described in [6], is an educational platform that demonstrates real-time control of a mobile vehicle. The control system is also quite complicated and thus two separate control algorithms need to be implemented. A main controller, which usually draws on Linear Quadratic Regulators (LQR), makes sure that the transporter maintains its upright unstable

equilibrium. The secondary algorithm enables the transporter to track a planned path. The Tw-UnTrans is an educational platform that provides live experience with real-time control systems, allowing for different strategies such as Proportional-Integral-Derivative (PID), LQR, Sliding Mode, Fuzzy Logic and Adaptive Control. Real time feedback from this modular design permits research and development to be flexible. It is however an unstable system; the feedback control must be exact and precise. In addition, it is easily disturbed by noise or other external factors, making software-hardware integration a complex and highly resource-consuming process. Taking this into account, the Tw-UnTrans serves as a practical and comprehensive platform for the study and development of advanced control strategies.

This paper proposes a control design methodology based on optimization techniques to improve the performance and efficacy of a Tw-UnTrans system. As shown in [7], regarding the nonlinear mathematical model (MM), the analysis identified several major discrepancies, so the nonlinear MM had to be revised to make the output signals be as close as possible to the real-time simulation of the equipment. Thus, it becomes imperative to find sets of parameters that guarantee that the results from the simulations are as close as possible to the real-time experimental results. In order to optimally tune the performance and efficacy of the model, three metaheuristic algorithms are utilized in the study, such as: Flying Foxes Optimization (FFO) [8–11], Grey Wolf Optimizer (GWO) [9], [12–14] and Particle Swarm Optimization (PSO) [9], [15–17]. These metaheuristic algorithms guarantee that the model parameters are adjusted to stability, robustness, and adaptability requirements. By integrating such advanced optimization techniques, the performance of the whole system can be significantly enhanced even when encountering uncertainties and different operating conditions.

This paper makes the following contributions based on the mathematical modeling and dynamical analysis discussed as follows: (i) a control design methodology based on optimization algorithms for performance and efficiency improvement of the Tw-UnTrans; (ii) a nonlinear MM modification (adjustment) to reduce the differences between the model's simulation and experimental results, allowing for a more realistic real-time equipment representation; (iii) application of three metaheuristic algorithms to optimally tune the model parameters and (iv) a detailed comparative analysis to evaluate and validate the proposed methodology for high performance, flexibility and stability in solutions developed for nonlinear systems.

The use of metaheuristic algorithms is motivated by the idea that an optimal solution can be found for applications for which the search space is complex and nonlinear, since they have achieved great results in multiple domains. Since the authors plan to implement other nonlinear process models based on fuzzy logic and neural networks, to be used in model-based control in the future, the intention is to use the metaheuristic algorithms instead of the classical least-squares regression. Moreover, these algorithms only require evaluations of the cost function, so they yield a continuous-time formulation of the optimization problem which can leverage rich input/output data information and avoid the time treatment specific to discrete input/output data pairs.

This paper is organized as follows: Section 1 outlines the general framework in which this paper is situated, specifying the relevance and motivation for the topic addressed in this study. Details regarding the architecture and functionality of the Tw-UnTrans and the modified nonlinear MM are given in Section 2. Section 3 defines the optimization problem that has been resolved by the three metaheuristic algorithms. In Section 4 a comparison-based validation of the proposed approach was carried out, and in Section 5, conclusions are summarized.

2. TWO-WHEELED UNSTABLE TRANSPORTER

The Tw-UnTrans laboratory equipment is a nonlinear and unstable mobile device, somewhat similar to an inverted pendulum. This equipment is used for practical testing and verification of linear and nonlinear control algorithms, innovative training solutions in the fields of automation, robotics, instrumentation and process control. As pointed out in [6], to make the system stand vertically, it uses two DC motors that can steer the wheels both forward and backward. A real-time control algorithm executes in a single-board computer to determine the appropriate direction and torque produced by these DC motors. An Inertial Measurement Unit (IMU) consisting of encoders, gyroscopes and accelerometers, present data, which is used to control the system input, that is, the controller output determines whether the appropriate direction is

given by these DC motors in real-time or not. Fig. 1 illustrates the functional representation of the Tw-UnTrans device. The Tw-UnTrans parameters, which are derived from [6] are listed in [7] and [18].

Fig. 1 – Tw-UnTranslaboratory equipment [6].

Taking into account the significant discrepancies between the system's response using the default parameters provided by Inteco in [6] and [7], in order to improve the performance of the system, the nonlinear MM was modified, resulting in:

$$\begin{aligned}
\ddot{\theta} &= \frac{c_2\left(F_\theta + f_2\right) - f_1\left(F_{2_\psi} + f_3\right)}{c_1 c_2 - f_1^2}, \\
\ddot{\psi} &= \frac{c_1\left(F_{1_\psi} + f_3\right) - f_1\left(F_\theta + f_2\right)}{c_1 c_2 - f_1^2}, \\
\ddot{\phi} &= \frac{F_\phi - 2ML^2\dot{\phi}\dot{\psi}\sin\psi\cos\psi}{\frac{1}{2}mW^2 + ML^2\sin^2\psi + \frac{W^2}{2R^2}\left(J_w + J_m\right) + J_\phi},
\end{aligned} \tag{1}$$

where

$$\begin{aligned}
&c_1 = \left((2m+M)R^2 + 2J_w + 2J_m\right), c_2 = \left(ML^2 + J_\psi + 2J_m\right), \\
&f_1(\psi) = \left(MRL\cos\psi - 2J_m\right), f_2(\psi,\dot{\psi}) = MRL\dot{\psi}^2\sin\psi, \\
&f_2\left(\psi,\dot{\phi}\right) = ML^2\dot{\phi}^2\sin\psi\cos\psi + MgL\sin\psi,
\end{aligned} \tag{2}$$

and the generalized forces are

$$\begin{aligned}
F_\theta &= d_1(u_r + u_l) + 2d_2\left(\dot{\psi} - \dot{\theta}\right), \\
F_{1_\psi} &= -d_1(u_r + u_l) + 2d_2\left(\dot{\theta} - \dot{\psi}\right), \\
F_{2_\psi} &= d_1(u_r + u_l) + 2d_2\left(\dot{\theta} - \dot{\psi}\right), \\
F_\phi &= d_1\frac{W}{2R}(u_r + u_l) - \frac{W^2}{2R^2}d_2(\dot{\phi} - \dot{\theta}), \\
d_1 &= \frac{K_l}{Rm}, d_2 = \frac{K_l K_b}{Rm} + f_m.
\end{aligned} \tag{3}$$

The measured or calculated parameters of the model are: m and M (kg, kg) denote the weights of the wheel and vehicle, $2R$ (m) denotes the diameter of the wheel, W and L (m) denote the width of the vehicle and the height of the mass center of the vehicle, R_{DC} (Ω) denotes the resistance of the winding of the DC motor, K_l and K_b (Nm/A, Vs/rad) denote the torque and voltage constants of the DC motor, f_m denotes the identified friction coefficient between the vehicle and DC motor, and $J_w=mR^2$, $J_\psi=ML^2/3$, J_ϕ, J_m (kg m^2) denote the moments of inertia of the wheel, of the vehicle tilt axis, of the vehicle related to the axis of rotation and of the DC motor and gearbox taking into account gearbox ratio.

3. OPTIMIZATION PROBLEM AND ALGORITHMS

3.1. Optimization problem

The optimization problem discussed in this paper, needed for the optimal tuning of the parameters in (1)-(3) seeks to minimize the cost function $J(\boldsymbol{\rho}^{(h\text{-}j\text{-}l)})$

$$\begin{aligned}&\boldsymbol{\rho}^{(h-j-l)^*} = \arg\min_{\rho^{(j)}\in D_\rho} J(\boldsymbol{\rho}^{(h-j-l)}),\\ &J(\boldsymbol{\rho}^{(h-j-l)}) = \frac{1}{N}\sum_{k=1}^{N}[y_k^*(\boldsymbol{\rho}^{(h-j-l)}) - y_k(\boldsymbol{\rho}^{(h-j-l)})]^2 = \frac{1}{N}\sum_{k=1}^{N} e_k^2(\boldsymbol{\rho}^{(h-j-l)}),\end{aligned} \tag{4}$$

where h denotes the type of mathematical model of the Tw-UnTrans system (here h =NL, nonlinear MM), j denotes the type of optimization algorithm, here j = {FFO, GWO, PSO}, l denotes the type of signal to be optimized $l = \{\theta, \psi, \phi\}$. $e_k(\boldsymbol{\rho}^{(h-j-l)}) = y_k^*(\boldsymbol{\rho}^{(h-j-l)}) - y_k(\boldsymbol{\rho}^{(h-j-l)})$ indicates the modeling error at the k^{th} sampling interval, $y_k^*(\boldsymbol{\rho}^{(h-j-l)})$ denotes the actual (measured) system output and $y_k(\boldsymbol{\rho}^{(h-j-l)})$ denotes the model output. In this context, $\boldsymbol{\rho}^{(h-j-l)}$ is the vector containing the model parameters, $\boldsymbol{\rho}^{(h-j-l)^*}$ is the vector of the optimal model parameters, D_ρ indicates the feasible range of $\boldsymbol{\rho}^{(h-j-l)}$, and N indicates the length of the time interval. The tunable parameter vector for the model (system) is

$$\boldsymbol{\rho}^{(NL-FFO/GWO/PSO-\theta/\psi/\phi)}) = [\mathrm{m}\ \ \mathrm{R}\ \ \mathrm{M}\ \ \mathrm{W}\ \ \mathrm{L}\ \ \mathrm{J}_\phi\ \ \mathrm{J_m}\ \ \mathrm{K_l}\ \ \mathrm{K_b}\ \ \mathrm{f_m}\ \ \mathrm{k}_\theta\ \ \mathrm{k}_\psi\ \ \mathrm{k}_\phi\ \ \mathrm{k_{l\_r}}]^T. \tag{5}$$

To determine the optimal parameters of the nonlinear MM, three metaheuristic algorithms are used, which are implemented according to the information in Sub-section 3.2.

3.2. Optimization algorithms

Metaheuristic algorithms start from an initial random solution and iteratively generate improved solutions using an operator of specific type to solve optimization problems. They are similar in that they start with one or more random solutions within an acceptable range and use the same approach to find the best solution. For these algorithms, the first set of solutions is referred to as a population, and single solutions are called particles, ants, or chromosomes. New solutions are created through operators and combinations of starting solutions. The cycle continues, choosing from existing solutions until some stopping criterion is reached.

A. Flying Foxes Optimization (FFO)

Inspired by the foraging technique of flying foxes, FFO algorithm has shown to been able to solve complex optimization problems in many problem areas. It is reminiscent of how flying foxes use their excellent sense of smell to detect fruit trees and their echolocation ability to fly towards them. However, perhaps due to its recent rise, it was not investigated or practically applied as much as other well-known optimization methods. In Fig. 2 given in [19], the FFO algorithm flowchart is illustrated. As shown in [8–11], *cool*, *hot* means the best and worst solutions (position vector); $x_i^0 \sim U(x_{min}, x_{max})$, x_{ij}^t indicates the j^{th} element of the i^{th} flying fox at the t iteration; a is a positive attraction constant, *rand~U(0,1), randj* is a random number in (0,1), *pa* is a probability constant, $\mathbf{x}_{\mathbf{R}_1}^t$, $\mathbf{x}_{\mathbf{R}_1}^t$ are two distinct random members of a population, and each population member is uniformly distributed, $x_{i,j}^{t+1}$ is the position of the new flying fox, k is a randomly selected value selected from the group {1,2,...,d} and makes sure that $x_{i,j}^{t+1}$ selects at least one component from $nx_{i,j}^{t+1}$ to stop the creation of the new solution from being identical to the old one, *SL* is the survival list, *NL* is a list with the best distinct solutions known so far, n is a random integer selected in [2, *NL*], while *pD* is a probability constant *pD=(nc−1)/population size*, *nc* is the number of flying foxes that have the same cost function value as the best one discovered, and R_1 and R_2 are randomly selected distinct individuals from a population, whereas L is a random number selected in (0,1).

B. Grey Wolf Optimizer (GWO)

GWO is based on or inspired from the social preferences and hunting mechanism of gray wolves in nature. It mimics how gray wolves align themselves in a hierarchy and work together when hunting. The GWO algorithms are generally known for its simplicity and good balance between exploration and exploitation capability in finding the optimal solution of a broad category of complex optimization problems. GWO has proven to be a successful method in many industries such as economy as well as in computer science and engineering. In Fig. 3 (a) given in [19], the GWO algorithm flowchart is shown, as described in [9] and [12–14], where: $\vec{A} = 2\vec{a}\vec{r}_1 - \vec{a}$, $\vec{C} = 2\vec{r}_2$ represent the coefficient vectors, $\vec{r}_1$, $\vec{r}_2$ represent two random vectors whose element is in [0, 1], $\vec{a}$ presents the linear decrease from 2 to 0 with the number of iterations, and X_α, X_β, X_γ, X_δ represent the wolves positions. Particular attention should be paid to the vector operations that are explained in comprehensible variants and detailed in [13] and [14].

C. Particle Swarm Optimization (PSO)

According to a user-defined quality metric, PSO iteratively improves a candidate solution. This achieves the social dynamics seen among birds or fish. PSO has received attention due to its simplicity and efficiency in tackling complex optimization problems across diverse fields; showing effective application value in the aspect of continuous optimization problems and achieving various successes in engineering design and machine learning. In Fig. 3 (b) given in [19], the PSO algorithm flowchart is presented, as described in [9], [15–17], where: x_i, v_i represents the current positions and velocities, $x_i(t+1)$,$v_i(t+1)$ are position and velocity at iteration $t+1$, p_i, $g(t)$ represents the local best for a particle and global best position and w, c_1, c_2 represents the inertia, cognitive and social coefficients.

4. RESULTS AND DISCUSSION

As detailed in Section 1, in addition to improving the accuracy of our results, the nonlinear MM of the Tw-UnTrans equipment was adapted (modified) to better represent the equipment's real-time responses. The optimal parameters of this model were identified through three different optimization algorithms, as given in Sub-section 3.2. The parameters were determined corresponding to the application of the nonlinear MM as an open-loop system with two additional control signals, u_{rc} and u_{lc}. This represents control signals for right and left motors respectively. These signals were divided into two equal parts, each with a simulation duration of 28 seconds, in order to facilitate testing and validation. Table 1 given in [19] displays the optimal parameters and the corresponding cost function values for the three optimization algorithms in the case of three output signals. For every algorithm as well as for each input signal, ten test runs have been done. The average of the ten runs was then calculated to obtain a single data set. For all three optimization algorithms, the search intervals for the u_{rc} and u_{lc} inputs with the three outputs $\{\theta, \psi, \phi\}$ were

$$\begin{aligned} LB &= [0.3 \quad 0.058 \quad 5.2 \quad 0.39 \quad 0.099 \quad 0.045 \quad 0.0011 \quad 0.014 \quad 0.024 \quad 0.0001 \quad 0.23 \quad 0.07 \quad 0.005 \quad 1], \\ UB &= [0.37 \quad 0.08 \quad 5.42 \quad 0.45 \quad 0.11 \quad 0.05 \quad 0.00151 \quad 0.027 \quad 0.045 \quad 0.0005 \quad 0.3 \quad 0.2 \quad 0.0078 \quad 3]. \end{aligned} \tag{6}$$

Figs. 4 (a)-(f) given in [19] show the output signals after each optimization with FFO (θ signal as a function of $J(\boldsymbol{\rho}^{NL-FFO-\theta})$, $J(\boldsymbol{\rho}^{NL-FFO-\psi})$, $J(\boldsymbol{\rho}^{NL-FFO-\phi})$; ψ signal as a function of $J(\boldsymbol{\rho}^{NL-FFO-\theta})$, $J(\boldsymbol{\rho}^{NL-FFO-\psi})$, $J(\boldsymbol{\rho}^{NL-FFO-\phi})$ and ϕ signal as a function of $J(\boldsymbol{\rho}^{NL-FFO-\theta})$, $J(\boldsymbol{\rho}^{NL-FFO-\psi})$, $J(\boldsymbol{\rho}^{NL-FFO-\phi})$). Fig. 4 (a) and (b) show the θ signal, Fig. 4 (c) and (d) show the ψ signal and Fig. 4 (e) and (f) show the ϕ signal. These are represented as a function of the optimized parameters for each objective function, compared to the experimentally obtained signal. Similar results are found in Figs. 5 (a)-(f) for the GWO algorithm and in Figs. 6 (a)-(f) for the PSO algorithm, figures which are also presented in [19]. Based on Figs. 4-6, best signals appear to be: $J(\boldsymbol{\rho}^{\theta})$ for θ signal, $J(\boldsymbol{\rho}^{\psi})$ for ψ signal and $J(\boldsymbol{\rho}^{\phi})$ for ϕ signal. As seen from the vastly lower deviations and better matching between target and signal behavior in the test and validation stages, these signals align much more closely with the experimental data. This indicates that for every signal, these implementations of the FFO, GWO and PSO algorithms suitably optimize the model parameters for each respective signal.

Another comparison of experimental and simulated signals for testing and validation stages using the three optimization algorithms is illustrated in Fig. 7 for θ signal, Fig. 8 for ψ signal and Fig. 9 for ϕ signal, These figures are also presented in [19]. Additionally, by comparing these four signals, we enable the assessment of relative algorithm performance in specific conditions and discover further information regarding which algorithms find applications in particular engineering domains. To evaluate the performance of the optimization methods for the θ, ψ and ϕ signals, we have examined both optimal parameter values with the cost function values from Tabel 1 and the graphical results illustrated in Figs. 7, 8 and 9.

θ signal: FFO garners a relatively low-cost function value ($J(\boldsymbol{\rho}^{NL\text{-}FFO-\theta})$=0.5457), which indicates that optimizing performance is good. The GWO and PSO performances have the lowest cost function values, indicating the best optimization result with respect to the θ signal for the algorithms compared ($J(\boldsymbol{\rho}^{NL\text{-}GWO-\theta})$=0.5111 and $J(\boldsymbol{\rho}^{NL\text{-}PSO-\theta})$=0.5235). As shown on Fig. 7, it can be seen that the signals extracted by using GWO, PSO and FFO show similar behavior compared with the respective experimental signal, which effectively demonstrates the purpose of reproducing the dynamic process of θ signal. Validation results show that GWO and PSO still continued to remarkably match experimental data and were good enough to extrapolate outside of the training conditions. Here FFO is also good, with only slight deviations.

ψ signal: Out of the three optimization algorithms, GWO achieves the best performance, evidenced by the smallest cost function value $J(\boldsymbol{\rho}^{NL-GWO-\psi})$=5.966·10^{-5}, hence GWO seems to be superior in accurately matching the nonlinear MM with the experimental results. It is followed closely by PSO with marginally higher value of cost function $J(\boldsymbol{\rho}^{NL-PSO-\psi})$=6.5472·10^{-5}, indicating its efficacy in optimally tuning the parameters as well. FFO is also a feasible performer, but not as good as GWO or PSO, having a cost function value of $J(\boldsymbol{\rho}^{NL-FFO-\psi})$=6.551·10^{-5}. Fig. 8 confirms these findings further. During the validation stage, the ψ signal using GWO and PSO closely corresponds with the experimental signal, especially regarding the precision of the phase and amplitude alignment, while FFO shows good agreement with the experimental signal, he only shows minor deviations in its ability to capture the dynamic response.

ϕ signal: At the same time, GWO outperformed all algorithms with the best minimum cost function value $J(\boldsymbol{\rho}^{NL-GWO-\phi})$=4.154·10^{-5}, FFO is the second best with $J(\boldsymbol{\rho}^{NL-FFO-\phi})$=4.514·10^{-5}. PSO did fairly well, albeit with a slightly higher cost function value, $J(\boldsymbol{\rho}^{NL-PSO-\phi})$=4.814·10^{-5}, indicating a competitive optimization. Fig. 9 presents the graphical representations and further corroborates these results. In the testing stage, the signals obtained through GWO, FFO, and PSO closely follow the experimental signal, providing evidence of the model accuracy. But for the validation, again, GWO and FFO are better fitted to the experimental data than PSO.

Overall, GWO achieves superior results across all three signals, followed closely by PSO, whilst FFO gives good results but slightly lower accuracy. To test the three optimization algorithms, Matlab code was used for FFO, GWO and PSO algorithms, the information (m_files) can be found at [20], reproduced in the reference works [8], [9] and [12–18]. Also, detailed descriptions and Matlab implementations of representative metaheuristic algorithms are given in the book [21]. Nevertheless, other metaheuristic algorithms can be successfully applied to solve the optimization problem given in (4); they include the adaptation of the binary anarchic society optimization [22], adaptation of optimization used in cloud-based identification of an evolving systems [23], adaptation of optimization used in fuzzy FMEA-based risk evaluation [24], adaptation of optimization used in object identification and localization [25], hybrid quantum-classical formulations of optimization algorithms [26], adaptations of optimization algorithms used in machine learning [27, 28], adaptation of optimization used in active structures [29], slime mould algorithms [30], and adaptation of optimization used in data-driven control [31, 32]. Additional information on the process, optimal parameters of the nonlinear MM and used algorithms are available from the authors upon request.

5. CONCLUSIONS

This paper proposed a control design methodology based on optimization techniques to improve the performance and efficacy of a Tw-UnTrans system. It was observed that the default nonlinear MM of the system had a large deviation from the real-time experimental results, therefore it was modified to ensure that the model represents more closely the behavior of the real system. With these modifications, the simulation results matched experimental data, reduced errors and improved accuracy.

In order to optimize the nonlinear MM parameters, three metaheuristic algorithms were employed: FFO, GWO and PSO. The model parameters were tuned for θ (the average rotational velocity of the wheel), ψ (the yaw angle from the vertical axis) and ϕ (the angle of rotation around the vertical axis) signals. Among these algorithms, GWO and PSO consistently delivered the most accurate and efficient results, closely matching real-time experimental results. FFO also performed well, although with slightly higher cost function values. The proposed control design methodology was demonstrated through experimental testing and simulations, achieving excellent agreement between the optimized model and real-time experimental results. Graphical comparisons and cost function analyses provided clear evidence of the effectiveness of the approach. This paper establishes a robust and flexible solution for the Tw-UnTrans system, proving the practicality of using metaheuristic optimization for nonlinear control systems.

The study highlights importance of integrating optimization techniques in the design of control systems, especially for systems that are complex or unstable, such as Tw-UnTrans. This methodology can be extended to more general nonlinear systems and has a high potential to be useful in robotics, mechatronics, and control engineering. This paper helps in developing adaptive solutions by creating a relationship between theoretical modeling and practical implementation.

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REFERENCES

[1] Isermann R. Mechatronic Systems: Fundamentals. Berlin, Heidelberg, New York: Springer-Verlag; 2005.
[2] Bishop RH. The Mechatronics Handbook. 2nd ed. Boca Raton, FL: CRC Press; 2007.
[3] Jouaneh M. Fundamentals of Mechatronics. 1st ed. Cengage Learning; 2012.
[4] Bolton W. Mechatronics: Electronic Control Systems in Mechanical and Electrical Engineering. Pearson Education; 2018.
[5] Alciatore DG, Histand MB. Introduction to Mechatronics and Measurement Systems. 5th ed. McGraw-Hill Education; 2019.
[6] Inteco. Two-Wheeled Unstable Transporter, User's Manual. Krakow: Inteco Ltd; 2019.
[7] Gale-Cazan CB, Bojan-Dragos CA, Precup RE, Roman RC, Petriu EM, Szedlak-Stinean AI. GWO-based modeling of an unstable transport system. Proc Comput Sci 2022; 214: 195–202.
[8] Zervoudakis K, Tsafarakis S. A global optimizer inspired from the survival strategies of flying foxes. Eng Comput 2023;39(2):1583–1616.
[9] Szedlak-Stinean AI, Precup RE, Iancu NL. Classical and fuzzy controllers for an optimally tuned model of a strip winding system. In: Proceedings of 2024 International Semiconductor Conference. Sinaia, Romania; 2024. pp. 13–22.
[10] Almakayeel N. Flying foxes optimization with reinforcement learning for vehicle detection in UAV imagery. Sci Rep 2024;14:20616.
[11] Zervoudakis K. Flying Foxes Optimization (Fuzzy self-tuning optimizer), MATLAB Central File Exchange; 2025. [Online]. Available: https://www.mathworks.com/matlabcentral/fileexchange/104810-flying-foxes-optimization-fuzzy-self-tuning-optimizer; [accessed 3 July 2025].
[12] Mirjalili S, Mirjalili SM, Lewis A. Grey wolf optimizer. Adv Eng Softw 2014;69:46–61.
[13] Precup RE, David RC, Petriu EM. Grey wolf optimizer algorithm-based tuning of fuzzy control systems with reduced parametric sensitivity. IEEE Trans Ind Electron 2017;64(1):527–534.
[14] Bojan-Dragos CA, Precup RE, Petriu EM, Roman RC, Hedrea EL, Szedlak-Stinean AI. GWO-based optimal tuning of controllers for shape memory alloy wire actuators. IFAC-PapersOnLine 2022;55(15): 39–44.
[15] Kennedy J, Eberhart RC. Particle swarm optimization. In: Proceedings of 1995 IEEE International Conference on Neural Networks. Perth, Australia; 1995. pp. 1942–1948.
[16] Kennedy J, Eberhart RC. A new optimizer using particle swarm theory. In: Proceedings of 6th International Symposium on Micro Machine and Human Science. Nagoya, Japan; 1995. pp. 39–43.
[17] Tharwat A, Schenck W. A conceptual and practical comparison of PSO-style optimization algorithms. Expert Syst Appl 2021;167:paper 114430.
[18] Ari ZM, Popa SI, Moscal CA. Modeling and Automatic Control of an Unstable Transport System [B.Sc. thesis], Politehnica University of Timișoara, Timișoara, Romania; 2024. (in Romanian).
[19] Szedlak-Stinean AI, Precup RE, Ari ZM, Supplementary material of the paper A.-I. Szedalk-Stinean, R.-E. Precup, Z.-M Ari, "Optimization-Driven Control Design for the Nonlinear Two-Wheeled Unstable Transporter System", Proceedings of the Romanian Academy, Series A: Mathematics, Physics, Technical Sciences, Information Science, 2025, http://www.aut.upt.ro/~rprecup/Supplementary_material_PRA_2025.doc, accessed July 12, 2025.

[20] Nature-inspired Optimization Algorithms in MATLAB Central; Jan. 2024. [Online]. Available: https://www.mathworks.com/support/search.html?q=Nature-inspired%20optimization&page=1; [accessed 3 July 2025].
[21] Precup RE, Roman RC, Hedrea EL, Szedlak-Stinean A-I, Zamfirache IA. Classical and Modern Optimization Techniques Applied to Control and Modeling, 1st ed. Boca Raton, FL, Abingdon: CRC Press; 2025.
[22] Kilic U, Sarac Essiz E, Kaya Keles M. Binary anarchic society optimization for feature selection. Rom J Inform Sci Technol 2023;26(3–4):351–364.
[23] Blažič S, Dovžan D, Škrjanc I. Cloud-based identification of an evolving system with supervisory mechanisms. In: Proceedings of 2014 IEEE International Symposium on Intelligent Control. Antibes, France; 2014. pp. 1906–1911.
[24] Johanyák ZC, Pásztor A. Fuzzy FMEA-based risk evaluation of student software projects. Acta Polyt Hung 2024; 21(10): 301–316.
[25] Hvizdoš J, Vaščák J, Březina A. Object identification and localization by smart floors. In: Proceedings of IEEE 19th International Conference on Intelligent Engineering Systems. Bratislava, Slovakia; 2015. pp. 113–117.
[26] Romero SV, Osaba E, Villar-Rodriguez E, Asla A. Solving logistic-oriented bin packing problems through a hybrid quantum-classical approach. In: Proceedings of 2023 IEEE 26th International Conference on Intelligent Transportation Systems. Bilbao, Spain; 2023. pp. 2239–2245.
[27] Lu ZL, Lok UH. Dimension-reduced modeling for local volatility surface via unsupervised learning. Rom J Inform Sci Technol 2024;27(3–4):255–266.
[28] Essiz UE, Aci C, Sarac E, Aci M. Deep learning-based prediction models for the detection of vitamin D deficiency and 25-hydroxyvitamin D levels using complete blood count tests. Rom J Inform Sci Technol 2024;27(3–4):295–309.
[29] Milić P, Marinković D, Klinge S, Ćojbašić Ž. Reissner-Mindlin based isogeometric finite element formulation for piezoelectric active laminated shells. The Vjesn 2023; 30(2):416-425.
[30] Zamfirache IA, Precup RE, Petriu EM. Adaptive reinforcement learning-based control using proximal policy optimization and slime mould algorithm with experimental tower crane system validation. Appl Soft Comput 2024; 160: paper 111687.
[31] Formentin S, Campi MC, Caré A, Savaresi SM. Deterministic continuous-time Virtual Reference Feedback Tuning (VRFT) with application to PID design. Syst Control Lett 2019;127:25–34.
[32] Join C, Jovellar DB, Delaleau E, Fliess M. Detection and suppression of epileptiform seizures via model-free control and derivatives in a noisy environment. In: Proceedings of 12th International Conference on Systems and Control. Batna, Algeria; 2024. pp. 79–84.

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