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INVESTIGATION OF NONLINEAR DYNAMIC RESPONSE OF ENGINEERING ROPEWAY WITH A MOVING PENDULUM MODEL

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Abstract. Engineering ropeway has the characteristics of convenient transportation, low cost and strong environmental adaptability, which is widely used in production and life. However, strong nonlinear vibration behavior will occur due to complex coupled vibration during operation, which threatens the safety of the whole system. This kind of nonlinear vibration behavior is difficult to express in the form of analytical solution. For the comprehensive consideration of safety and efficiency in the operation of engineering ropeway, it is necessary not only to express the nonlinear vibration dynamic response of engineering ropeway quickly, but also to clarify the influence of ropeway parameters on the nonlinear vibration response. In this paper, a typical form of ropeway -- bridge crane is selected for constructing a moving pendulum model, and the corresponding nonlinear vibration equation is derived by Lagrange method. By using McLaughlin series expansion and orthogonal Chebyshev polynomials, the differential equation with both sinusoidal and cosine nonlinearities is reduced to polynomial equations. The analytical approximate solution of nonlinear vibration equation of engineering ropeway is obtained by combining Newton linearization with harmonic balance method. The results show that the analytical approximation solutions are highly consistent with the exact solutions obtained by the shooting method. The influences of parameters such as rope length, hanging weight and amplitude, on the nonlinear vibration response are analyzed. It is found that the length and amplitude of the rope are positively correlated with the period, while the hanging weight is the opposite. The present results have certain guiding significance for engineering ropeway design, optimization, and application.

Keywords*:* engineering ropeway, nonlinear vibration, analytical approximation, Newton linearizaiton, harmonic balance method.

1. INTRODUCTION

The engineering ropeway is a special means of transportation, which has the advantage of being able to transport both vertically and horizontally. Moreover, the transportation span is large, and the work can be carried out in the complex terrain and mountain appearance as well as the severe wind, rain, snow and fog weather. Compared with other modes of transportation, ropeways have lower infrastructure costs, lower maintenance costs, and electric power is more environmentally friendly. In summary, ropeway is a kind of efficient and fast transportation means, and its application fields cover machinery, forestry, agriculture, mining, electricity, civil construction, tourism, aerospace and so on. Therefore, the majority of scholars and researchers are concerned about this, and have obtained fruitful academic results and applied them to the practice to improve the engineering ropeway [1–5].

When the ropeway is working, the whole system is in dynamic change. The carrier vehicle runs with a heavy load, which has a strong dynamic effect on the carrying rope. At the same time, there are external incentives, such as wind load will affect the carrier vehicle and load rope. As early as 2007, Petrova [6] considered the dynamic response of the ropeway under the action of crosswind and established the corresponding mechanical model. Through the use of MATLAB to calculate and analyze the vibration behavior of the heavy weight suspended in the model (namely the pod), the obtained data results have certain significance for improving the safety of the ropeway. Bryja [7] first derived the nonlinear differential equation of ropeway vibration based on Lagrange equation. Then, considering the linear and nonlinear interaction forces between the ropeway and the pendulum, the equations governing the motion of the pendulum are derived. Finally, a numerical method for simulating the nonlinear vibration of double ropeway under operating load is proposed. Knawa-Hawryszków [8] used numerical analysis method to study the initial tension of the load rope in the double rope channel, considering the nonlinear effect of the load rope. The influence of this method on displacement state and internal force is analyzed, and the recommended initial tension value of rope and the limit value of transverse displacement of rope are given. Xiao [9] proposes a nonlinear model for vehicle-ropeway system. An incremental nonlinear equation of motion is established by using the updated Lagrange expression and the principle of virtual work, which develops a new iterative algorithm for finding the response solution.

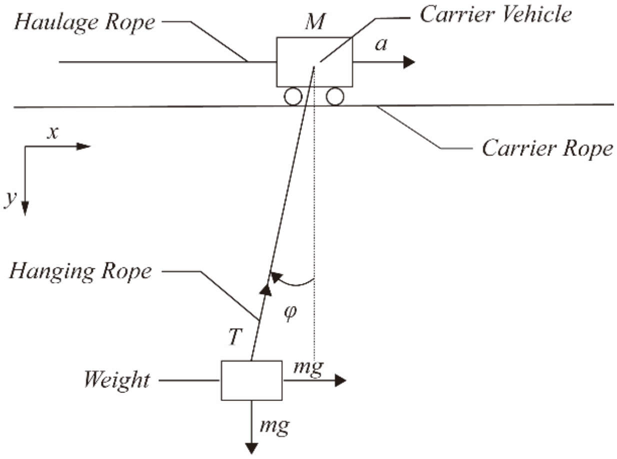
If multiple factors are coupled together and the coupled vibration equation is further established, the vibration characteristics of ropeway hoisting work can be expressed more accurately [10–14]. Therefore, it is very important to analyze the dynamic behavior of engineering ropeway from shallow to deep. Bridge crane is a kind of engineering ropeway, because of the low cost and complete functions, suitable for most of the need to control the cost but also to improve the efficiency of work, expand the scale of small and medium-sized enterprises [15] .At the end of last century, Lee [16] first applied the Lagrange dynamic equation to the problem of crane load swing, and established the mathematical model of bridge crane on this basis, which laid the foundation for the subsequent related research on crane anti-swing. Dai, et al. [17] established a 3D crane system coordinate system for the change of rope length and the control idea, and used Lagrange equation to establish a dynamic mathematical model and MATLAB for calculation, which not only simplified the modeling process, but also more intuitively reflected the crane swing response. Ouyang [18] analyzed the linearization of the simple pendulum model and obtained the second-order oscillation link according to the linearization results. Sun [19] added the consideration of coupling effects of other external disturbances on the basis of considering friction, so that the model can better simulate the real operating environment of bridge crane. In the study of nonlinear vibration, Mori and Tagawa [20] developed a linear parameter varying acceleration limit model considering the nonlinear acceleration of cranes, and combined the dynamic model of cranes with the speed limit model to establish a controlled object model considering the acceleration limit characteristics. Colic, et al. [21] describes the dynamic behavior of bridge crane under non-stationary operation state by using the Lagrange method and the small oscillation theory is used to analyze the system oscillation amplitude.

The above researchers have done a lot of work on the dynamic response of ropeway vibration, but most of the results are calculated by numerical simulation, because the nonlinear vibration response of ropeway is difficult to be expressed in the form of analytical solution. Based on the theory of bridge crane and the corresponding research results of vibration theory and vibration mechanics of single-span ropeway system, the hoisting and transportation stage of ropeway system is analyzed in this paper. Based on the Lagrange dynamics modeling method, the nonlinear swing equation of the mobile operation of the loaded carrier vehicle is derived. By using Maclaurin expansion and Chebyshev polynomial acceleration, the nonlinear governing equation including sine and cosine functions is reduced to polynomial equations, and reasonable initial approximations satisfying the boundary conditions are given. Then combining Newton method and harmonic balance method, the analytical approximation solution of the bridge crane pendulum model is obtained. The analytical approximation solution obtained is simple in form and has high approximation accuracy compared with the exact solution.

2. SIMPLE PENDULUM MODEL AND EQUATION OF MOTION

When the carrier vehicle starts to run, it goes through the process of starting acceleration, running at a constant speed and braking deceleration in an operation cycle time. However, due to the particularity of lifting wire rope material and time displacement hysteresis, there will be heavy objects and lifting wire rope around the running carrier vehicle swing. And the swing mainly appears in the direction of its movement. Yang [22] found in their research that the brake swing angle of the vehicle was approximately 0 when the weight was vertically lifted, so this paper only studied the horizontal movement and did not consider the lifting process.

Therefore, the following assumptions are made for the simple pendulum model: i) Friction and air resistance are ignored; ii) The weight and deformation of the rope are ignored; iii) The swing only appears in the movement direction of the vehicle, that is, only the plane motion trajectory is studied, and the horizontal running state is maintained. The simplified moving simple pendulum system is shown in Fig. 1, where the mass of the carrier vehicle is M, the mass of the hanging weight is m, the length of the rope is l, and the swing angle is φ.



(a) (b)

Fig. 1 – Schematic diagram of weight swing of a simple pendulum system.

The force analysis is carried out on the moving simple pendulum model. According to the geometric relationship between the carrier vehicle and the hanging weight, the displacement components of the hanging weight can be obtained as follows

$$\begin{cases} X = x - l\sin\varphi \\ Y = l\cos\varphi \end{cases} \tag{1}$$

Then, the velocity components of the hanging weight in the horizontal and vertical directions are

$$\begin{cases} \dot{X} = \dot{x} - l\dot{\varphi}\cos\varphi \\ \dot{Y} = -l\dot{\varphi}\sin\varphi \end{cases} \tag{2}$$

The kinetic energy of the simple pendulum system is

$$T = \frac{1}{2}(M+m)\dot{x}^2 + \frac{1}{2}m(l^2\dot{\varphi}^2 - 2\dot{x}l\dot{\varphi}\cos\varphi) \tag{3}$$

The potential energy of the simple pendulum system is

$$V = -mgl\cos\varphi \tag{4}$$

Therefore, the Lagrange function of the system is

$$L = T - V = \frac{1}{2}(M+m)\dot{x}^2 + \frac{1}{2}m(l^2\dot{\varphi}^2 - 2\dot{x}l\dot{\varphi}\cos\varphi) + mgl\cos\varphi \tag{5}$$

The Lagrange equation under the conservative force system is

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L}{\partial \dot{q}_\alpha}\right) - \frac{\partial L}{\partial q_\alpha} = 0 \tag{6}$$

Therefore, the Lagrange equation of the system in generalized coordinates x and φ is

$$\begin{cases} \dfrac{\mathrm{d}}{\mathrm{d}t}\left(\dfrac{\partial L}{\partial \dot{x}}\right) - \dfrac{\partial L}{\partial x} = 0 \\ \dfrac{\mathrm{d}}{\mathrm{d}t}\left(\dfrac{\partial L}{\partial \dot{\varphi}}\right) - \dfrac{\partial L}{\partial \varphi} = 0 \end{cases} \tag{7}$$

According to Eq. (6), we can obtain, by substituting Eq. (5) into Eq. (7), the nonlinear dynamic equation of the simple pendulum model can be obtained.

$$\begin{cases} (M+m)\ddot{x} - ml\ddot{\varphi}\cos\varphi + ml\dot{\varphi}^2\sin\varphi = 0 \\ l\ddot{\varphi} - \ddot{x}\cos\varphi + g\sin\varphi = 0 \end{cases} \tag{8}$$

The $(\dot{\ })$ here is the derivative of the variable t.

3. ESTABLISH ANALYTICAL APPROXIMATION SOLUTIONS

In this section, Eq. (8) will be solved by using the harmonic balance method and Newton's linearization method [23–25].

Firstly, Eq. (8) is simplified as follows

$$(2M + m - m\cos 2\varphi)l\ddot{\varphi} + ml\sin 2\varphi\,\dot{\varphi}^2 + 2g\,(M + m)\sin\varphi = 0 \tag{9}$$

The corresponding boundary conditions are

$$\varphi(0) = A, \quad \dot{\varphi}(0) = 0 \tag{10}$$

Introduce a new variable $\tau = \omega t$, where ω is the corresponding nonlinear vibration frequency, rewrite Eq. (10), then the equation and its boundary conditions are as follows,

$$\begin{gathered}(2M + m - m\cos 2\,\varphi)l\Omega\varphi'' + ml\sin 2\,\varphi\,\Omega\,\varphi'^{\,2} + 2g(M + m)\sin\varphi = 0 \\ \varphi(0) = A, \quad \varphi'(0) = 0\end{gathered} \tag{11}$$

The ($'$) here is the derivative of the variable τ, $\Omega = \omega^2$. This variable is introduced so that the solution of Eq. (11) is a periodic function of 2π with respect to variable τ, and the corresponding nonlinear vibration frequency is $\omega = \sqrt{\Omega}$.

Let $\varphi = Au$, Eq. (11) can be written as

$$\begin{gathered}[2M + m - m\cos(2Au)]l\Omega\, u'' + ml\sin(2Au)\,\Omega u'^{\,2} + 2g(M + m)\sin(Au) = 0 \\ u(0) = 1, \quad u'(0) = 0\end{gathered} \tag{12}$$

The trig-functions in Eq. (11) were expanded by Maclaurin series and accelerated by Chebyshev polynomial, and then constructed into a new nonlinear equation without trigonometric functions [26]

$$\begin{aligned}\cos(2Au) &= B_1 + B_2u^2 + B_3u^4 \\ \sin(2Au) &= C_1u + C_2u^3 \\ \sin(Au) &= D_1u + D_2u^3\end{aligned} \tag{13}$$

where

$$\begin{aligned}B_1 &= 1 - \frac{A^6}{360} + \frac{A^8}{2880} \\ B_2 &= -2A^2 + \frac{A^6}{20} - \frac{A^8}{180} \\ B_3 &= \frac{2}{3}A^4 - \frac{2}{15}A^6 + \frac{A^8}{90} \\ C_1 &= 2A - \frac{A^5}{12} + \frac{A^7}{90} - \frac{A^9}{1440} \\ C_2 &= -\frac{4}{3}A^3 + \frac{A^5}{3} - \frac{A^7}{30} + \frac{A^9}{540} \\ D_1 &= A - \frac{A^5}{384} + \frac{A^7}{11520} - \frac{A^9}{737280} \\ D_2 &= -\frac{A^3}{6} + \frac{A^5}{96} - \frac{A^7}{3840} + \frac{A^9}{276480}\end{aligned}$$

Substituting Eq. (13) into Eq. (12), this nonlinear equation can be expressed as

$$\begin{gathered}\Omega f_1(u)u'' + \Omega f_2(u)u'^2 + f_3(u) = 0 \\ u(0) = 1, \quad u'(0) = 0\end{gathered} \tag{14}$$

where

$$\begin{aligned}&f_1(u) = [2M + m - m(B_1 + B_2u^2 + B_3u^4)]lA^2 \\ &f_2(u) = ml(C_1u + C_2u^3)A^2 \\ &f_3(u) = 2g(M + m)(D_1u + D_2u^3)\end{aligned}$$

The initial analytical approximation solution satisfying the boundary conditions is approximated as follows

$$u_1 = \cos\tau \tag{15}$$

Substitute Eq. (15) into Eq. (14) and expand into Fourier series. To eliminate the secular term, the coefficient of $\cos\tau$, must be zero, such that

$$\frac{1}{32}\pi\{16D_1g(M+m)+12D_2g(M+m)+A^2l[(-8+8B_1+6B_2+5B_3+2C_1+C_2)m-16M]\Omega\}=0 \tag{16}$$

The first analytical approximation solution of Ω can be obtained by solving Eq. (16).

$$\Omega_1 = -\frac{[4(4D_1gm+3D_2gm+4D_1gM+3D_2gM)]}{[A^2l(-8m+8B_1m}+6B_2m+5B_3m+2C_1m+C_2m-16M)] \tag{17}$$

Thus, the first analytic approximation period and periodic solution of the simple pendulum moving system are obtained.

$$\begin{gathered} T_1(A)=\frac{2\pi}{\omega_1(A)},\quad \omega_1(A)=\sqrt{\Omega_1(A)} \\ u_1(t)=\cos\tau\,,\quad \tau=\sqrt{\Omega_1(A)}t,\quad \varphi_1(t)=Au_1(t) \end{gathered} \tag{18}$$

Next, Newton's method and harmonic balance method are combined to further solve the analytical approximation solution of Eq. (14). The first step is Newton's method, the periodic solution of Eq. (14) and the square of the frequency can be expressed as

$$u = u_1+\Delta u_1,\ \ \Omega=\Omega_1+\Delta\Omega_1 \tag{19}$$

Substitute Eq. (19) into Eq. (14), linearize correction terms Δu_1 and $\Delta\Omega_1$, and get

$$\begin{aligned} &2D_1gmu_1+2D_1gMu_1+2D_2gmu_1{}^3+2D_2gMu_1{}^3+2D_1gm\Delta u_1+2D_1gM\Delta u_1 \\ &\quad+6D_2gmu_1{}^2\Delta u_1+6D_2gMu_1{}^2\Delta u_1+A^2C_1lm\Omega_1u_1(u_1'^2+2u_1'\Delta u_1') \\ &\quad+A^2C_2lm\Omega_1u_1{}^3(u_1'^2+2u_1'\Delta u_1')+A^2C_1lmu_1\Delta\Omega_1u_1'^2+A^2C_2lmu_1{}^3\Delta\Omega_1u_1'^2 \\ &\quad+A^2C_1lm\Omega_1\Delta u_1u_1'^2+3A^2C_2lm\Omega_1u_1{}^2\Delta u_1u_1'^2+A^2lm\Omega_1(u_1''+\Delta u_1'') \\ &\quad-A^2B_1lm\Omega_1(u_1''+\Delta u_1'')+2A^2lM\Omega_1(u_1''+\Delta u_1'')-A^2B_2lm_1\Omega_1u_1{}^2(u_1''+\Delta u_1'') \\ &\quad-A^2B_3lm\Omega_1u_1{}^4(u_1''+\Delta u_1'')+A^2lm\Delta\Omega_1u_1''-A^2B_1lm\Delta\Omega_1u_1''+2A^2lM\Delta\Omega_1u_1'' \\ &\quad-A^2B_2lmu_1{}^2\Delta\Omega_1u_1''-A^2B_3lmu_1{}^4\Delta\Omega_1u_1''-2A^2B_2lm\Omega_1u_1\Delta u_1u_1'' \\ &\quad-4A^2B_3lm\Omega_1u_1{}^3\Delta u_1u_1''=0 \end{aligned} \tag{20}$$

$$\Delta u_1(0)=0,\ \ \Delta u_1'(0)=0 \tag{21}$$

Δu_1 is a periodic function of period 2π with respect to τ, and both Δu_1, $\Delta\Omega_1$ are unknowns.

To solve the unknowns Δu_1 and $\Delta\Omega_1$ in the linearized Eq. (20) by HB method, let

$$\Delta u_1 = x_1(\cos\tau-\cos 3\tau) \tag{22}$$

This assumption satisfies the initial conditions of Eq. (21).

Substituting Eq. (15) and Eq. (22) into Eq. (20), the obtained expression was expanded into trigonometric series, and the coefficients of $\cos\tau$ and $\cos 3\tau$ in the equation were set to 0 [26], respectively, to obtain

$$\begin{gathered} g_1(A)x_1+g_2(A)\Delta\Omega_1=0 \\ h_1(A)x_1+h_2(A)\Delta\Omega_1+h_3(A)=0, \end{gathered} \tag{23}$$

where

$$g_1(A)=g(M+m)\cdot$$

$$\cdot\frac{[4(20B_2+25B_3+12C_1+7C_2)D_1m+3(-16+16B_1+32B_2+35B_3+16C_1+9C_2)D_2m-96D_2M)]}{[16(-8+8B_1+6B_2+5B_3+2C_1+C_2)m-256M]}$$

$$g_2(A)=\frac{1}{32}A^2l[(-8+8B_1+6B_2+5B_3+2C_1+C_2)m-16M]\pi$$

$$h_1(A)=\frac{\{(4D_1+3D_2)g[(128B_1+64B_2+43B_3-128+16C_1+9C_2)m-256M](M+m)\pi\}}{[16(-8+8B_1+6B_2+5B_3+2C_1+C_2)m-256M]}$$

$$h_2(A) = \frac{1}{64}A^2(4B_2 + 5B_3 - 4C_1 - C_2)lm\pi$$

$$h_3(A) = g(M+m)\cdot$$
$$\cdot\frac{\{[-16B_2D_1 + 4(4C_1 + C_2)D_1 + 16(-1 + B_1 + C_1)D_2 + 5C_2D_2 - 5B_3(4D_1 + D_2)]m - 32D_2M\}}{[16(-8 + 8B_1 + 6B_2 + 5B_3 + 2C_1 + C_2)m - 256M]}$$

By solving Eq. (23), we can obtain x_1 and $\Delta\Omega_1$, respectively.

$$x_1 = \frac{g_2h_3}{-g_2h_1 + g_1h_2},\quad \Delta\Omega_1 = \frac{g_1h_3}{g_2h_1 - g_1h_2} \tag{24}$$

Therefore, the second analytic approximation period and the periodic solution of the simple pendulum moving system are

$$\begin{aligned} &T_2(A) = 2\pi/\omega_2(A),\quad \omega_2(A) = \sqrt{\Omega_2(A)}, \Omega_2(A) = \Omega_1(A) + \Delta\Omega_1(A) \\ &u_2(t) = u_1(t) + \Delta u_1(t) = X(A)\cos\tau + Y(A)\cos 3\tau,\quad \tau = \sqrt{\Omega_2(A)}t,\ \ \varphi_2(t) = Au_2(t) \end{aligned} \tag{25}$$

where

$$X(A) = 1 + x_1(A),\ \ Y(A) = -x_1(A). \tag{26}$$

4. RESULTS AND DISCUSSIONS

In this section, the accuracy of the analytical approximation solution derived in Section above is verified by comparison with the numerical results obtained for the nonlinear vibration equations. The influence of different parameters on the periodic is analyzed, and the applicability of the analytical approximation solution is verified.

4.1. Comparison of exact period and approximate period

In order to describe the dynamic vibration behavior of the single pendulum model of the bridge crane and verify the accuracy of the analytical approximation solution, the value of the obtained analytical approximation solution is compared with that of the exact solution. Let the initial values be $l = 5$m, $M = 450$ kg, $m = 3\,000$ kg, $g = 9.8\ \mathrm{m/s^2}$. The exact period T_e that be obtained by using the shooting method [25], compared with the approximation periods T_1 and T_2 obtained through Eqs. (18) and (25). In the case of different initial values, the list comparison is shown in Table 1. The table shows that the period accuracy obtained by the analytical approximation solution is good, and the percentage error has been significantly reduced.

Table 1

Comparison of approximate period and exact period

A	T_1 (error%)	T_2 (error%)	T_e
0.1	0.482153(1.257644)	0.484883(0.698555)	0.488294
0.2	0.677375(2.520122)	0.690499(0.63147)	0.694887
0.3	0.871892(3.330414)	0.897157(0.529198)	0.90193
0.4	1.10232(3.068887)	1.13196(0.462531)	1.13722
0.5	1.37742(2.579409)	1.39987(0.991591)	1.41389
1	3.27208(–1.74862)	3.19828(0.546264)	3.215847
2	7.30936 (3.011163)	7.48863(0.632407)	7.53629

4.2. Parameter sensitivity analysis

In this subsection, the parameter sensitivity analysis of the period of the model is carried out. Set the crane model as QD32/5, the rated lifting weight of the main hook of the crane is 32 t, and the highest lifting height is 12 m. According to the reference table of crane parameters, the weight of the carrier vehicle is 11 500 kg. Next, sensitivity analysis is carried out on rope length l, hanging weight mass m and amplitude A. That is, the numerical values of each parameter are substituted into the approximate period $T_2(t)$ formula obtained by Eq. (25), and comparative analysis is made by drawing, as shown in Fig. 2. The range of the rope length l is 5~10 m, the range of the hanging weight mass m is 10 000~30 000 kg, and the range of amplitude A is 0.5~2 rad.

Figure 2a represents the relationship between the period T, the mass of the hanging weight m and the length of the rope l when $A = 1$ rad. It can be seen that m and l have little mutual influence, and l has a greater correlation with period T than m. Figure 2b represents the relationship between the period T, the mass of the hanging weight m and the amplitude A when $l = 5$ m. At this time, A has a greater correlation with period T than l. Figure 2c represents the relationship between the period T, the mass of the length of the rope l and the amplitude A when $m = 25\ 000$ kg. Different from the previous two figures, l and A influence each other. As shown in Fig. 2, when A is small, T increases slowly as l increases. But as A gradually increases, the relationship between T and l gradually reverses into inverse proportion. And the rate of change between T and A is just increasing a little bit as l increases.

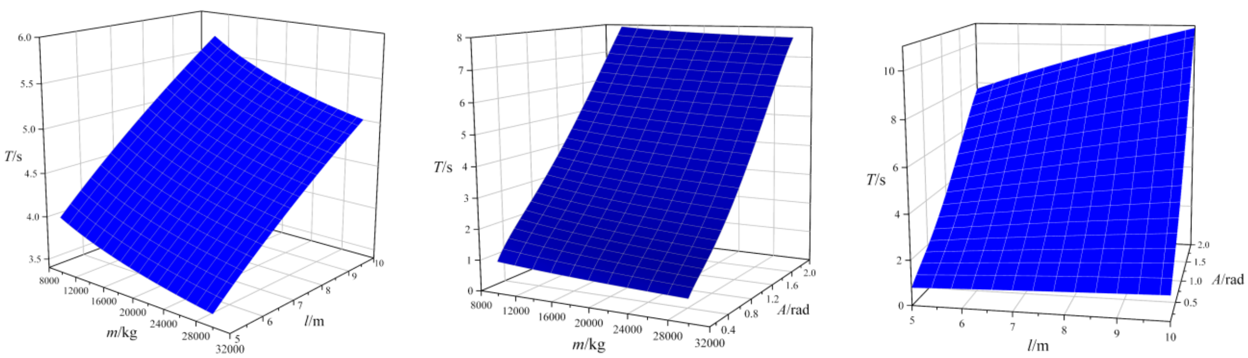


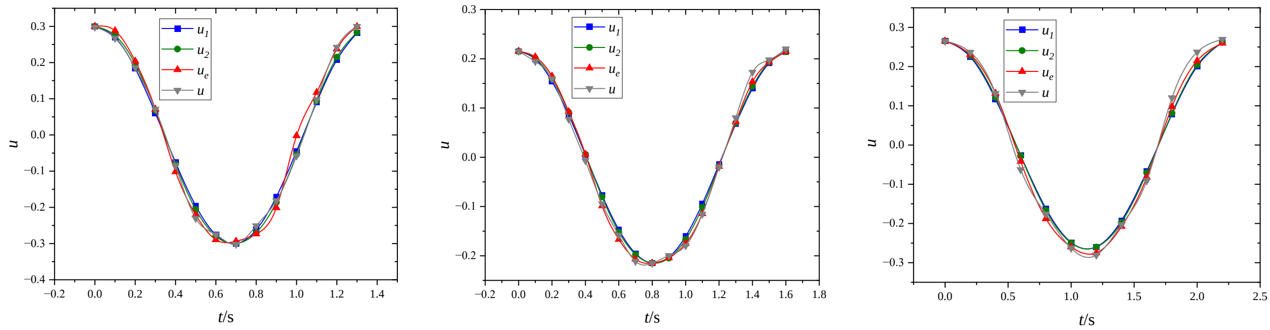
Fig. 2 – Effect of different parameters on the period T: a) $A = 1$rad, $m = 10\ 000$~$30\ 000$ kg, $l = 5$~10 m; b) $l = 5$ m, $m = 10\ 000$~$30\ 000$ kg, $A = 0.5$~2 rad; c) $m = 25\ 000$ kg, $l = 5$~10 m, $A = 0.5$~2 rad.

As shown in Figs. 2a, 2b and 2c, when $A = 1$ rad, the mutual influence between m and l is small, and between l and m, T is more sensitive to l. When $l = 5$ m, T is more sensitive to A. When $m = 25\ 000$ kg, unlike the first two figures, l and A interact. When A is small, T increases slowly as l increases. But as A gradually increases, the relationship between T and l gradually reverses into inverse proportion. However, the rate of change between T and A increases slightly with the increase of l.

4.3. An engineering example

In this section, the data in engineering examples are substituted into the equation for calculation, and the results are compared with the actual data. The example object is the QD32/5 crane [27], where the carrier vehicle mass is 11.5 t ($M = 11\ 500$ kg), the carrier vehicle running motor is the YZR160m-1, the brake is the JWZ9-200/E23. There are three working conditions as follows: (a) $l = 5$ m, $m = 25\ 000$ kg, $A = 0.3$ rad; (b) $l = 10$ m, $m = 25\ 000$ kg, $A = 0.215$ rad; (c) $l = 10$ m, $m = 12\ 500$ kg, $A = 0.265$ rad. The effects of different rope lengths and hanging weights on the model vibration are respectively considered.

The parameters were substituted into exact periodic solution $u_e(t)$ and approximate periodic solution $u_1(t), u_2(t)$, and the results were graphically compared with the example data, as shown in Fig. 3. As can be seen from the figure, the approximate periodic solution obtained by the equation has high accuracy and is close to the actual situation. The described motion situation of the crane carrier vehicle lifting the goods smoothly runs provides the foundation for the system's lifting weight swing control, and has certain reference value for the application in practical engineering.



(a) (b) (c)

Fig. 3 – Comparison of exact periodic solution, approximate periodic solution and actual engineering data at 3 conditions: a) $l = 5$ m, $m = 25\ 000$ kg, $A = 0.3$ rad; b) $l = 10$ m, $m = 25\ 000$ kg, $A = 0.215$ rad; c) $l = 10$ m, $m = 12\ 500$ kg, $A = 0.265$ rad.

5. CONCLUSIONS

Taking bridge crane as an example, this paper analyzes the response characteristics of engineering ropeway under nonlinear vibration, and uses an alternative analytical solution to construct an analytical approximation solution with high accuracy. The main conclusions are as follows:

1. By means of harmonic balance method and Newton linearization method, the analytic approximation period and periodic solution of nonlinear vibration of a simple pendulum model of engineering ropeway are obtained.

2. The analytical approximation solution is compared with the exact solution, and the error of the two solutions is small. The accuracy of the analytical approximation solution is verified by a practical engineering example, and it is found that the approximation solution is in good agreement with the engineering data. That is, the approximate solution has good accuracy for the nonlinear vibration of the model. By applying the analytical formula, the nonlinear frequency and periodic responses of the simple pendulum model of the ropeway can be analyzed without numerical integration.

3. Based on the analytical approximation solution, the dependence of oscillation period and response on the parameters (rope length, hanging weight mass and amplitude) is studied. It can be found that the rope length and amplitude are positively related to the period, while the hanging weight is the opposite. Moreover, the amplitude can affect the effect of rope length on the period variation. When the amplitude reaches a certain degree, the rope length is negatively correlated with the period, which should be paid attention to in practical application.

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