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LONGITUDINAL STABILITY ANALYSIS OF LARGE AMPHIBIOUS AIRCRAFT GLIDING ON STILL WATER SURFACE

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Abstract. Stability of large amphibious aircraft is the basis of its operating and control where longitudinal stability should be most focused on. In order to study the longitudinal stability of large amphibious aircraft gliding on still water surface, the longitudinal dynamic model was built and the linearization equation was established with small-perturbation method. On the basis, the longitudinal static stability was researched and the longitudinal dynamic stability was analyzed with stability criterion. Stability test was conducted in a high-velocity hydrodynamic test pool. The theoretical result was consistent with the test result. The result shows that the derivation of hydraulic pitching moment with respect to pitch angle changes to a positive number with the increase of the pitch angle and then the static stability is disrupted. The pitch angle should be in the range of 4°~7° to maintain stable motion when large amphibious aircraft glides on still water surface.

***Keywords**:* large amphibious aircraft, longitudinal stability, static stability, dynamic stability, still water surface.

1. INTRODUCTION

Large amphibious aircraft are fixed-wing aircraft capable of water and land takeoff and landing [1]. Due to the unique mission capabilities, large amphibious aircraft can be widely used in forest fire fighting, water rescue, maritime patrol, ocean environment exploration, and island reef material transportation, among other tasks [2–3]. Research has shown that most accidents occur when large amphibious aircraft is operated on water surface, and stability is the critical influencing factor [4]. The most concerning instability phenomenon is the “porpoise motion”, which occurs when the parameters or attitude of amphibious aircraft are unreasonable, leading to intense coupled heave-pitch motions [5]. The longitudinal stability of amphibious aircraft is clearly defined in the standards “Flying Qualities of Piloted Aircraft (Fixed Wing) ” [6] and “Airworthiness Standards for Transport Aircraft” [7]: “There shall be no dangerous uncontrollable porpoise motion”.

Current research on amphibious aircraft mostly focus on numerical simulations and tests to study the t responses when amphibious aircraft is gliding on the surface of water. Huang *et al*. [8] used scaled model tests of amphibious aircraft in water tank to study the relationship between heave/pitch motion response and environment parameters and the stability of aircraft gliding on water surface is analyzed. Lian *et al*. [9–10] conducted a scaled model tank experiment in waves of an amphibious aircraft. The responses of pitch angle and vertical displacement were tested and stability of amphibious aircraft in regular waves was analyzed. Lu *et al*. [11] researched the effects of the initial conditions including the incident angle, the descent velocity and the horizontal velocity on the landing stability using CFD methods by solving the unsteady Reynolds-averaged Navier-Stokes equations coupled with the standard *k*-*ω* turbulence model. Duang *et al*. [12] studied the dynamic characteristics of an amphibious aircraft with four turboprop engines during taxiing at high velocity on water with the actuator disk method using OpenFOAM. The hydrodynamics and aerodynamics of high-velocity amphibious aircraft during water taxiing are examined through a coupled solver approach, incorporating the influence of propulsion thrust and slipstream effects. Sun *et al*. [13] employed both simulation and experimental methods to investigate the influence of pitch angles on the water-entry performance of an amphibious aircraft's hull. The study focuses on analyzing the impact of initial pitch angles on the water loads and motion responses during the water-entry process. Chu *et al*. [14] utilized numerical simulation and model testing to analyze the water load on amphibious aircraft hulls, incorporating an air field into the finite element model. Local slamming pressure variations and distributions across different hull bottom cross-sections at various structural weights and entry velocities were calculated, comparing these numerical findings with empirical test data to validate the results and inform hull configuration design.

As can be seen from the above research, studies have largely concentrated on the response and performance of amphibious aircraft sliding on waves or calm water using CFD method and experiment, but there is less research in the field of the stability analysis of amphibious aircraft using theory of stability. Therefore, the stability of amphibious aircraft is further studied and this paper proposed a method to analyze the static and dynamic stability. The main contribution of this paper can be described as follows:

(1) Static stability of amphibious aircraft was derived and the influence factors were researched.
(2) Dynamic stability of amphibious aircraft was analyzed and the stable conditions for gliding on still water surface were studied.
(3) Stability test was conducted and theoretical and experimental stable conditions were compared to prove the validity of the analysis.

The main content of the paper is described as follows: Section 2 establishes the dynamic equation of the longitudinal motion of large amphibious aircraft gliding on still water surface. Section 3 analyzes the static stability under different sliding velocities and attitudes. Section4 studies the dynamic stability and conducted stability test. Finally, Section 5 gives some conclusions.

2. DYNAMIC MODEL OF LARGE AMPHIBIOUS AIRCRAFT

2.1. Reference coordinate systems and force analysis

To describe the motion state of a large amphibious aircraft, three sets of coordinate systems are required: the earth coordinate system, the aircraft body coordinate system and the velocity coordinate system, as shown in Fig. 1. The forces acting on large amphibious aircraft gliding on the water surface including gravity force G, engine thrust T, aerodynamic force, hydrodynamic force, residual buoyancy and added force. Aerodynamic force includes aerodynamic drag D, aerodynamic lateral force C, aerodynamic lift L, as well as aerodynamic rolling moment l, aerodynamic pitching moment M and aerodynamic yawing moment N. Hydrodynamic force includes hydrodynamic drag D_w, hydrodynamic lateral force C_w, hydrodynamic lift L_w, and hydrodynamic rolling moment l_w, hydrodynamic pitching moment M_w, and hydrodynamic yawing moment N_w. Residual buoyancy refers to buoyancy after the attitude of hull of aircraft is changed, where residual drag D_{dw}, residual lift L_{dw} and residual pitching moment M_{dw} are involved. Added forces are the forces of water to the hull due to the changes of acceleration, including added drag D_{add}, added lateral force C_{add}, added lift L_{add}, added rolling moment l_{add}, added pitching moment M_{add}, and added yawing moment N_{add}.

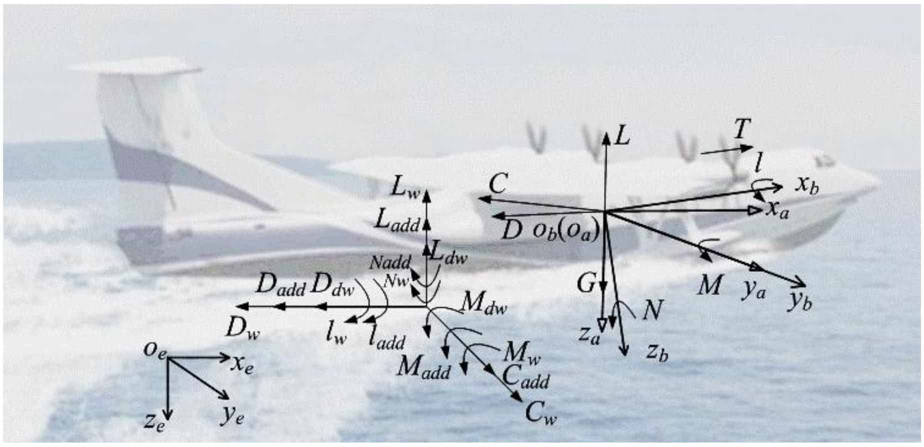

Fig. 1 – Reference coordinates system and diagram of force analysis of the large amphibious aircraft.

2.2. Longitudinal dynamic model of large amphibious aircraft

In this paper, the longitudinal movement is only considered, and the lateral movement is neglected, in other words, the lateral movement parameters should be set to zero. Similar to the equations of aircraft in the air [15], according to the force analysis in Fig. 1, the equations of large amphibious aircraft gliding on still water surface can be expressed as:

In the $o_b x_b$ direction, the force balance equation for the surge motion is:

$$m(\dot{u}+wq)=T\cos\varphi_T-D\cos\alpha+L\sin\alpha+D_w+D_{dw}+D_{add}-G\sin\theta \tag{1}$$

where m is the mass, u is the velocity in the $o_b x_b$ axis, w is the vertical velocity in the $o_b z_b$ axis, q is the rate of pitch angle, φ_T is the mounting angle of engine, which is 0 in this paper, α is the angle of attack, G is the gravity of aircraft, θ is the pitch angle.

In the $o_b z_b$ direction, the force balance equation for the sinking motion is:

$$m(\dot{w}-uq)=-T\sin\varphi_T-D\sin\alpha-L\cos\alpha+L_w+L_{dw}+L_{add}+G\cos\theta \tag{2}$$

The moment balance equation around the $o_b y_b$ axis for the pitching motion is:

$$\dot{q}I_y=M+M_w+M_{dw}+M_{add}+Tz_T \tag{3}$$

where z_T is the distance from gravity centre to the line of thrust direction, which is set to be 0 in the paper.

Aerodynamic drag D, aerodynamic lift L, and aerodynamic pitching moment M are calculated using the following equations [15]:

$$D=\frac{1}{2}\rho V^2 S_w C_D(V,\alpha,H,\delta_e), L=\frac{1}{2}\rho V^2 S_w C_L(V,\alpha,\dot{\alpha},q,H,\delta_e), M=\frac{1}{2}\rho V^2 S_w c_A C_M(V,\alpha,\dot{\alpha},q,H,\delta_e) \tag{4}$$

where ρ is the air density, S_w is the wing area, c_A is the mean aerodynamic chord, C_D is the drag coefficient, C_L is the lift coefficient, C_m is the pitching moment coefficient, H is the altitude, which is taken as 0 for the aircraft glides on the water surface, δ_e is the deflection angle of elevator.

Hydrodynamic forces including hydrodynamic drag D_w, hydrodynamic lift L_w, and hydrodynamic pitching moment M_w, can be expressed as [16]:

$$D_w=\frac{1}{2}\rho V^2 L_b^2 C_{Dw}(\theta,\xi,u,w,q), L_w=\frac{1}{2}\rho V^2 L_b^2 C_{Lw}(\theta,\xi,u,w,q), M_w=\frac{1}{2}\rho V^2 L_b^2 C_{Mw}(\theta,\xi,u,w,q) \tag{5}$$

where C_{Dw} is the hydrodynamic drag coefficient, C_{Lw} is the hydrodynamic lift coefficient, C_{Mw} is the hydrodynamic pitching moment coefficient, L_b is the length of the hull, ξ is the draft of the hull.

Added forces include added drag D_{add}, added lift L_{add}, and added pitching moment M_{add}, which can be calculated as follows [16]:

$$D_{add}=\frac{1}{2}\rho V^2 L_b^3\left[D_{w\dot{u}}\dot{u}+D_{w\dot{w}}\dot{w}+L_b D_{w\dot{q}}\dot{q}\right],\ \ L_{add}=\frac{1}{2}\rho V^2 L_b^3\left[L_{w\dot{u}}\dot{u}+L_{w\dot{w}}\dot{w}+L_b L_{w\dot{q}}\dot{q}\right],$$

$$M_{add}=\frac{1}{2}\rho V^2 L_b^3\left[M_{w\dot{u}}\dot{u}+M_{w\dot{w}}\dot{w}+L_b M_{w\dot{q}}\dot{q}\right] \tag{6}$$

where $D_{w\dot{u}}$ is the added mass of water on the hull caused by $\dot{u}$, $D_{w\dot{w}}$ is the added mass caused by $\dot{w}$, $D_{w\dot{q}}$ is the added mass caused by $\dot{q}$, $L_{w\dot{u}}$ is the added mass caused by $\dot{u}$, $L_{w\dot{w}}$ is the added mass caused by $\dot{w}$, $L_{w\dot{q}}$ is the added mass caused by $\dot{q}$, $M_{w\dot{u}}$ is the added inertia caused by $\dot{u}$, $M_{w\dot{w}}$ is the added inertia caused by $\dot{w}$, $M_{w\dot{q}}$ is the added inertia caused by $\dot{q}$.

Buoyancy including residual drag D_{dw}, residual lift L_{dw} and residual pitching moment M_{dw} is the function of θ and ξ, which can be shown as [16]:

$$D_{dw}=D_{dw}(\theta,\xi),\ \ L_{dw}=L_{dw}(\theta,\xi),\ \ M_{dw}=M_{dw}(\theta,\xi) \tag{7}$$

Hydrodynamic forces and moments and added inertial masses and inertias are measured through hydrodynamic tests. Hydrodynamic tests are conducted in a high-velocity towing tank. Hydrodynamic forces and moments are measured with different aircraft velocities, pitch angles, and drafts, which can then be used to calculate hydrostatic force coefficients. Added mass and related derivatives for hydrodynamic forces can be obtained using pure heave, pure pitch, and pure roll tests methods [17].

2.3. Linear equations

To obtain the linear equations, the Eqs. (1)–(3) should be trimmed at different velocities and pitch angles to get the steady flight condition at trimming conditions as follows:

$$q_0 = 0,\ \dot{V}_0 = 0,\ \gamma_0 = 0 \tag{8}$$

$$\phi_0 = 0,\ \psi_0 = 0,\ \beta_0 = 0,\ p_0 = 0,\ r_0 = 0,\ \chi_0 = 0,\ \mu_0 = 0 \tag{9}$$

where Eq. (8) is the longitudinal trimming conditions, Eq. (9) is the lateral trimming conditions, the subscript 0 represents the trimming conditions, q is the rate of pitch, V is the velocity of the aircraft, γ is the flight path angle, ϕ is the roll angel, ψ is the yaw angle, β is the sideslip angle, p is the rate of roll, r is the rate of yaw, χ is the flight path azimuth angle, μ is the flight path roll angle.

The longitudinal dynamic equations of the large amphibious aircraft gliding on still water surface are assumed to be linear near the trimming conditions in Eqs. (8) and (9) and the linear equations can be obtained using perturbation method. The perturbation motion is assumed as:

$$V = V_0 + \Delta V,\ u = u_0 + \Delta u,\ w = w_0 + \Delta w,\ \alpha = \alpha_0 + \Delta\alpha,\ q = q_0 + \Delta q,\ \theta = \theta_0 + \Delta\theta$$
$$D = D_0 + \Delta D,\ L = L_0 + \Delta L,\ M = M_0 + \Delta M$$
$$D_W = D_{W0} + \Delta D_W,\ L_W = L_{W0} + \Delta L_W,\ M_W = M_{W0} + \Delta M_W$$
$$D_{add} = D_{add0} + \Delta D_{add},\ L_{add} = L_{add0} + \Delta L_{add},\ M_{add} = M_{add0} + \Delta M_{add}$$
$$D_{dw} = D_{dw0} + \Delta D_{dw},\ L_{dw} = L_{dw0} + \Delta L_{dw},\ M_{dw} = M_{dw0} + \Delta M_{dw}$$

where the subscript 0 denotes the steady flight condition, the prefix Δ denotes the perturbation method.

The surge motion in the $o_b x_b$ direction is a long-period motion. For convenience in analysis, it is assumed that the aircraft only undergoes heave and pitch motion, and the surge motion can be balanced by controlling the engine thrust, and meanwhile lateral motion is neglected. The flight control of true aircraft demonstrates the reasonableness of this assumption. Therefore, the longitudinal perturbation motion can be get through the Eqs. (2)–(9). The vertical perturbation motion equation is

$$m(\Delta\dot{w} - u_0\Delta q) = -\Delta L + \Delta L_w + \Delta L_{dw} - mg\sin\theta_0\Delta\theta \tag{10}$$

The moment perturbation equation around the $o_b y_b$ axis for the pitching motion is:

$$\Delta\dot{q} I_y = \Delta M + \Delta M_w + \Delta M_{dw} \tag{11}$$

The perturbation of aerodynamic according to the Equation (4) is

$$\begin{aligned} \Delta D &= D_\alpha\Delta\alpha + D_q\Delta q + D_{\delta_e}\Delta\delta_e \\ \Delta L &= L_\alpha\Delta\alpha + L_{\dot{\alpha}}\Delta\dot{\alpha} + L_q\Delta q + L_{\delta_e}\Delta\delta_e \\ \Delta M &= M_\alpha\Delta\alpha + M_{\dot{\alpha}}\Delta\dot{\alpha} + M_q\Delta q + M_{\delta_e}\Delta\delta_e \end{aligned} \tag{12}$$

where L_α is derivative of aerodynamic lift with respect to angle of attack, $L_{\dot{\alpha}}$ is derivative of aerodynamic lift with respect to rate of angle of attack, L_q is derivative of aerodynamic lift with respect to rate of pitch, L_{δ_e} is derivative of aerodynamic lift with respect to angle of elevator, M_α is derivative of aerodynamic pitching moment with respect to angle of attack, $M_{\dot{\alpha}}$ is derivative of aerodynamic pitching moment with respect to rate of angle of attack, M_q is derivative of aerodynamic pitching moment with respect to rate of pitch, M_{δ_e} is derivative of aerodynamic pitching moment with respect to angle of elevator.

The derivative of aerodynamic can be expressed as

$$\begin{gathered} D_\alpha = \frac{1}{2}\rho_a V^2 S_w C_{D\alpha},\ D_q = \frac{1}{2}\rho_a V^2 S_w C_{Dq},\ L_{\delta_e} = \frac{1}{2}\rho_a V^2 S_w C_{D\delta_e} \\ L_\alpha = \frac{1}{2}\rho_a V^2 S_w C_{L\alpha},\ L_{\dot{\alpha}} = \frac{1}{2}\rho_a V^2 S_w C_{L\dot{\alpha}},\ L_q = \frac{1}{2}\rho_a V^2 S_w C_{Lq},\ L_{\delta_e} = \frac{1}{2}\rho_a V^2 S_w C_{L\delta_e} \\ M_\alpha = \frac{1}{2}\rho_a V^2 S_w c_A C_{M\alpha},\ M_{\dot{\alpha}} = \frac{1}{2}\rho_a V^2 S_w c_A C_{M\dot{\alpha}},\ M_q = \frac{1}{2}\rho_a V^2 S_w c_A C_{Mq},\ M_{\delta_e} = \frac{1}{2}\rho_a V^2 S_w c_A C_{M\delta_e} \end{gathered} \tag{13}$$

where ρ_a is the density of air, $C_{L\alpha}$ is the derivative of aerodynamic lift coefficient respect to angle of attack, $C_{L\dot{\alpha}}$ is the derivative of aerodynamic lift coefficient respect to rate of angle of attack, C_{Lq} is the derivative of aerodynamic lift coefficient respect to rate of pitch, $C_{L\delta_e}$ is the derivative of aerodynamic lift coefficient respect to angle of elevator, $C_{M\alpha}$ is the derivative of aerodynamic pitching moment coefficient respect to angle of attack, $C_{M\dot{\alpha}}$ is the derivative of aerodynamic pitching moment coefficient respect to rate of angle of attack, C_{Mq} is the derivative of aerodynamic pitching moment coefficient respect to rate of pitch, $C_{M\delta_e}$ is the derivative of aerodynamic pitching moment coefficient respect to angle of elevator.

The perturbation of hydrodynamic according to the Eq. (5) is

$$\begin{aligned} \Delta D_w &= D_{w\theta}\Delta\theta + D_{w\xi}\Delta\xi + D_{wq}\Delta q + D_{ww}\Delta w \\ \Delta L_w &= L_{w\theta}\Delta\theta + L_{w\xi}\Delta\xi + L_{wq}\Delta q + L_{ww}\Delta w \\ \Delta M_w &= M_{w\theta}\Delta\theta + M_{w\xi}\Delta\xi + M_{wq}\Delta q + M_{ww}\Delta w \end{aligned} \tag{14}$$

where $L_{w\theta}$ is derivative of hydrodynamic lift with respect to pitch angle, $L_{w\xi}$ is derivative of hydrodynamic lift with respect to draft, L_{wq} is derivative of hydrodynamic lift with respect to pitch rate, L_{ww} is derivative of hydrodynamic lift with respect to vertical velocity, $M_{w\theta}$ is derivative of hydrodynamic pitching moment with respect to pitch angle, $M_{w\xi}$ is derivative of hydrodynamic pitching moment with respect to draft, M_{wq} is derivative of hydrodynamic pitching moment with respect to pitch rate, M_{ww} is derivative of hydrodynamic pitching moment with respect to vertical velocity.

The derivative of hydrodynamic can be expressed as

$$\begin{aligned} &\mathrm{D}_{w\theta} = \frac{1}{2}\rho_w V^2 L_b^2 C_{Dw\theta},\ D_{w\xi} = \frac{1}{2}\rho_w V^2 L_b^2 C_{Dw\xi},\ D_{wq} = \frac{1}{2}\rho_w V^2 L_b^2 C_{Dwq},\ D_{\mathrm{ww}} = \frac{1}{2}\rho_w V^2 L_b^2 C_{Dww} \\ &L_{w\theta} = \frac{1}{2}\rho_w V^2 L_b^2 C_{Lw\theta},\ L_{w\xi} = \frac{1}{2}\rho_w V^2 L_b^2 C_{Lw\xi},\ L_{wq} = \frac{1}{2}\rho_w V^2 L_b^2 C_{Lwq},\ L_{ww} = \frac{1}{2}\rho_w V^2 L_b^2 C_{Lww} \\ &M_{w\theta} = \frac{1}{2}\rho_w V^2 L_b^3 C_{Mw\theta},\ M_{w\xi} = \frac{1}{2}\rho_w V^2 L_b^3 C_{Mw\xi},\ M_{wq} = \frac{1}{2}\rho_w V^2 L_b^3 C_{Mwq},\ M_{ww} = \frac{1}{2}\rho_w V^2 L_b^3 C_{M\mathrm{ww}} \end{aligned} \tag{15}$$

where ρ_w is the density of water, $C_{Lw\theta}$ is the derivative of hydrodynamic lift coefficient respect to pitch angle, $C_{L\xi}$ is the derivative of hydrodynamic lift coefficient respect to draft, C_{Lwq} is the derivative of hydrodynamic lift coefficient respect to rate of pitch, $C_{L\mathrm{ww}}$ is the hydrodynamic of aerodynamic lift coefficient respect to vertical velocity, $C_{Mw\theta}$ is the derivative of hydrodynamic pitching moment coefficient respect to pitch angle, $C_{M\xi}$ is the derivative of hydrodynamic pitching moment coefficient respect to draft, C_{Mwq} is the derivative of hydrodynamic pitching moment coefficient respect to rate of pitch, C_{Mww} is the derivative of hydrodynamic pitching moment coefficient respect to vertical velocity.

The perturbation of added forces according to the Eq. (6) is

$$\Delta L_{add} = \frac{1}{2}\rho_w V^2 L_b^3\left[L_{w\dot{w}}\Delta\dot{w} + L_b L_{w\dot{q}}\Delta\dot{q}\right],\ \Delta M_{add} = \frac{1}{2}\rho_w V^2 L_b^3\left[M_{w\dot{w}}\Delta\dot{w} + L_b M_{w\dot{q}}\Delta\dot{q}\right] \tag{16}$$

The perturbation of buoyancy according to the Eq. (7) is

$$\begin{aligned} \Delta D_{dw} &= D_{dw\theta}\Delta\theta + D_{dw\xi}\Delta\xi \\ \Delta L_{dw} &= L_{dw\theta}\Delta\theta + L_{dw\xi}\Delta\xi \\ \Delta M_{dw} &= M_{dw\theta}\Delta\theta + M_{dw\xi}\Delta\xi \end{aligned} \tag{17}$$

According to Eqs. (10)–(17), the longitudinal linear equation can be obtained as follows:

$$\dot{X}_g = A_g X_g + B_g u_g \tag{18}$$

where X_g is the longitudinal motion variables, $X_g = [\Delta\alpha, \Delta q, \Delta\theta, \Delta\xi]^{\mathrm{T}}$, Δ represents the perturbation for variables, $\Delta\xi$ is the perturbation for draft, A_g is the longitudinal state matrix, B_g is the longitudinal control matrix, u_g is the longitudinal input, $u_g = \delta_e$.

The state matrix can be described as:

$$A_g = \begin{bmatrix} \bar{Z}_\alpha & \bar{Z}_q & \bar{Z}_\theta & \bar{Z}_\xi \\ \bar{M}_\alpha & \bar{M}_q & \bar{M}_\theta & \bar{M}_\xi \\ 0 & 1 & 0 & 0 \\ 1 & 0 & -V_0 & 0 \end{bmatrix} \tag{19}$$

where $\bar{Z}_q = (a_{11}b_{12} + a_{12}b_{22})/(V_0 den)$, $\bar{Z}_\theta = (a_{11}b_{13} + a_{12}b_{23})/(V_0 den)$, $\bar{Z}_\xi = (a_{11}b_{14} + a_{12}b_{24})/(V_0 den)$, $\bar{Z}_\alpha = (a_{11}b_{11} + a_{12}b_{21})/(V_0 den)$, $\bar{M}_\alpha = (a_{21}b_{11} + a_{22}b_{21})V_0/den$, $\bar{M}_q = (a_{21}b_{12} + a_{22}b_{22})/den$, $\bar{M}_\theta = (a_{21}b_{13} + a_{22}b_{23})/den$, $\bar{M}_\xi = (a_{21}b_{14} + a_{22}b_{24})/den$, $a_{11} = I_y - M_{w\dot{q}}$, $a_{12} = L_{w\dot{q}}$, $a_{21} = M_{\dot{\alpha}}/V_0 + M_{\mathrm{w}\dot{\mathrm{w}}}$, $a_{22} = m + L_{\dot{\alpha}}/V_0 - L_{w\dot{w}}$, den is an intermediate variable, $den = a_{22}a_{11} - a_{21}b_{12}$, $b_{11} = -L_\alpha/V_0 + L_{ww}$, $b_{21} = -M_\alpha/V_0 + M_{ww}$, $b_{12} = -L_q + L_{wq}$, $b_{22} = M_q + M_{wq}$, $b_{13} = L_{w\theta} + L_{wq}$, $b_{23} = M_{w\theta} + M_{dw\theta}$, $b_{14} = L_{w\xi} + L_{dw\xi}$, $b_{24} = M_{w\xi} + M_{dw\xi}$.

3. LONGITUDINAL STABILITY ANALYSIS

3.1. Longitudinal static stability

Static stability refers to that, there are static restoring forces or moments when the aircraft deviates from its equilibrium state due to a disturbance, allowing the aircraft to return to its original equilibrium state after the disturbance disappears. According to the longitudinal dynamic equations of the large amphibious aircraft gliding on still water surface as shown in Eq. (19), there are three position parameters in the longitudinal motion: ξ, α and θ, where ξ is the vertical position parameter, and α and θ are the pitch position parameters. The lift force and pitching moment acting on the aircraft are the functions of the three parameters ξ, α, and θ, which is described as

$$\begin{gathered}\sum X = \Delta D + \Delta D_w + \Delta D_{dw} + \Delta D_{add} \\ \sum Z = \Delta L + \Delta L_w + \Delta L_{dw} + \Delta L_{add} \\ \sum M = \Delta M + \Delta M_w + \Delta M_{dw} + \Delta M_{add}\end{gathered} \tag{20}$$

where ΣZ is the disturbance of the total force on the aircraft in the vertical direction, ΣM is the disturbance of the total pitching moment on the aircraft, ΣX is the disturbance of the total force on the aircraft in $o_b x_b$ direction.

The velocity of aircraft is assumed to be constant, total force in $o_b x_b$ direction is set to be 0, that is $\Sigma X=0$. The disturbance of the total force balance in the vertical direction firstly influences ξ and then influences the total pitching moment balance. The disturbance of the total pitching moment firstly influences α and θ and then influences the total force balances in the vertical direction. The change in aerodynamic lift due to change of vertical position can be neglected. The partial derivative of total force in the vertical direction ΣZ with respect to ξ represents the static stability of ξ, which can be expressed as

$$\frac{\partial \sum Z}{\partial \xi}\Big|_{\substack{\sum X=0 \\ \sum M=0}} = \frac{\partial \Delta L}{\partial \alpha}\frac{\partial \alpha}{\partial \xi} + \frac{\partial \Delta L_w}{\partial \xi} + \frac{\partial \Delta L_w}{\partial \theta}\frac{\partial \theta}{\partial \xi} + \frac{\partial \Delta L_{dw}}{\partial \xi} + \frac{\partial \Delta L_{dw}}{\partial \theta}\frac{\partial \theta}{\partial \xi} \tag{21}$$

The partial derivatives of total pitching moment ΣM with respect to α and θ represent the static stability of α and θ, which can be written as

$$\frac{\partial \sum M}{\partial \theta}\Big|_{\substack{\sum X=0 \\ \sum Z=0}} = \frac{\partial \Delta M}{\partial \alpha}\frac{\partial \alpha}{\partial \theta} + \frac{\partial \Delta M_w}{\partial \theta} + \frac{\partial \Delta M_w}{\partial \xi}\frac{\partial \xi}{\partial \theta} + \frac{\partial \Delta M_{dw}}{\partial \theta} + \frac{\partial \Delta M_{dw}}{\partial \xi}\frac{\partial \xi}{\partial \theta} \tag{22}$$

$$\frac{\partial \sum M}{\partial \alpha}\Big|_{\substack{\sum X=0 \\ \sum Z=0}} = \frac{\partial \Delta M}{\partial \alpha} + \frac{\partial \Delta M_w}{\partial \theta}\frac{\partial \theta}{\partial \alpha} + \frac{\partial \Delta M_w}{\partial \xi}\frac{\partial \xi}{\partial \theta} + \frac{\partial \Delta M_{dw}}{\partial \theta}\frac{\partial \theta}{\partial \alpha} + \frac{\partial \Delta M_{dw}}{\partial \xi}\frac{\partial \xi}{\partial \theta} \tag{23}$$

The static stable conditions are the partial derivative is negative [15]. The longitudinal static stable conditions can be expressed by the following equations:

$$\frac{\partial \sum Z}{\partial \xi}\Big|_{\substack{\sum X=0 \\ \sum M=0}} < 0 \tag{24}$$

$$\frac{\partial \sum M}{\partial \alpha} < 0, \qquad \frac{\partial \sum M}{\partial \theta}\Big|_{\substack{\sum X=0 \\ \sum Z=0}} < 0\,, \qquad \frac{\partial \sum M}{\partial \alpha}\Big|_{\substack{\sum X=0 \\ \sum Z=0}} < 0 \tag{25}$$

Equation (24) represents the static stability of draft and Eq. (25) represents the static stability of pitch position, which needs to be satisfied simultaneously. The first term meets the pitch static stability for the aircraft in flight, and the second and third terms meet the pitch static stability for the aircraft gliding on water. The second term is the static stability of the pitch angle, and the third term is the static stability of the angle of attack.

Substitute Eqs. (12)–(17) into Eq. (21), the static stability of ξ can be described as:

$$\frac{\partial \sum Z}{\partial \xi}\Big|_{\substack{\sum X=0 \\ \sum M=0}} = \frac{1}{2}\rho_w V^2 L_b^2 \hat{Z}_\xi = \frac{1}{2}\rho_w V^2 L_b^2 \left(C_{Lw\xi} + C_{Lw\theta}\frac{\partial \theta}{\partial \xi} + \frac{L_{dw\xi}}{\frac{1}{2}\rho_w V^2 L_b^2} + \frac{L_{dw\theta}}{\frac{1}{2}\rho_w V^2 L_b^2}\frac{\partial \theta}{\partial \xi} + \frac{\rho_a}{\rho_w} C_{L\alpha}\frac{S_W}{L_b^2}\frac{\partial \alpha}{\partial \xi} \right) \tag{26}$$

where $\hat{Z}_\xi$ is the dimensionless static stability of draft.

By neglecting the effect of thrust disturbance on static stability and taking the total differential of ΣX and ΣM, the expression as follows can be obtained through solving the equations $\sum X = 0, \sum M = 0$

$$\begin{bmatrix} \dfrac{\partial\theta}{\partial\xi} \\ \dfrac{\partial\alpha}{\partial\xi} \end{bmatrix} = \begin{bmatrix} L_b^2(D_{w\theta} + D_{dw\theta}) & S_w D_\alpha \\ L_b^3(M_{w\theta} + M_{dw\theta}) & S_w c_A M_\alpha \end{bmatrix}^{-1} \begin{bmatrix} -L_b^2(D_{w\xi} + D_{dw\xi}) \\ -L_b^3(M_{w\xi} + M_{dw\xi}) \end{bmatrix} \tag{27}$$

For the second term of Eq. (25), it can be expressed as:

$$\frac{\partial \sum M}{\partial\theta}\Big|_{\substack{\sum X=0 \\ \sum Z=0}} = \frac{1}{2}\rho_w V^2 L_b^3 \widehat{M}_\theta =$$

$$= \frac{1}{2}\rho_w V^2 L_b^3 \left(C_{Mw\theta} + C_{Mw\xi}\frac{\partial\xi}{\partial\theta} + \frac{M_{dw\theta}}{\frac{1}{2}\rho_w V^2 L_b^3} + \frac{M_{dw\xi}}{\frac{1}{2}\rho_w V^2 L_b^3}\frac{\partial\xi}{\partial\theta} + \frac{\rho_a}{\rho_w} C_{M\alpha} \frac{S_W c_A}{L_b^3}\frac{\partial\alpha}{\partial\theta} \right) \tag{28}$$

where $\widehat{M}_\theta$ is the dimensionless static stability of pitch angle.

By neglecting the effect of thrust disturbance on static stability and taking the total differential of ΣX and ΣZ, the expression as follows can be obtained through solving the equations $\sum X = 0, \sum Z = 0$

$$\begin{bmatrix} \dfrac{\partial\xi}{\partial\theta} \\ \dfrac{\partial\alpha}{\partial\theta} \end{bmatrix} = \begin{bmatrix} L_b^2(D_{w\xi} + D_{dw\xi}) & S_w D_\alpha \\ L_b^2(L_{w\xi} + L_{dw\xi}) & S_w L_\alpha \end{bmatrix}^{-1} \begin{bmatrix} -L_b^2(D_{w\theta} + D_{dw\theta}) \\ -L_b^2(D_{w\theta} + D_{dw\theta}) \end{bmatrix} \tag{29}$$

For the third term of Eq. (10), it can be expressed as:

$$\frac{\partial \sum M}{\partial\alpha}\Big|_{\substack{\sum X=0 \\ \sum Z=0}} = \frac{1}{2}\rho_w V^2 L_b^3 \widehat{M}_\alpha =$$

$$= \frac{1}{2}\rho_w V^2 L_b^3 \left(C_{Mw\theta}\frac{\partial\theta}{\partial\alpha} + \frac{M_{dw\theta}}{\frac{1}{2}\rho_w V^2 L_b^3}\frac{\partial\theta}{\partial\alpha} + C_{Mw\xi}\frac{\partial\xi}{\partial\alpha} + \frac{M_{dw\xi}}{\frac{1}{2}\rho_w V^2 L_b^3}\frac{\partial\xi}{\partial\alpha} + \frac{\rho_a}{\rho_w} C_{M\alpha}\frac{S_W c_A}{L_b^3} \right) \tag{30}$$

where $\widehat{M}_\alpha$ is the dimensionless static stability of angle of attack

By neglecting the effect of thrust disturbance on static stability and taking the total differential of ΣX and ΣZ, the expression can be obtained as through solving the equations $\sum X = 0,\ \sum Z = 0$

$$\begin{bmatrix} \dfrac{\partial\xi}{\partial\alpha} \\ \dfrac{\partial\theta}{\partial\alpha} \end{bmatrix} = \begin{bmatrix} L_b^2(D_{w\xi} + D_{dw\xi}) & L_b^2(D_{w\theta} + D_{dw\theta}) \\ L_b^2(L_{w\xi} + L_{dw\xi}) & L_b^2(D_{w\theta} + D_{dw\theta}) \end{bmatrix}^{-1} \begin{bmatrix} -S_w D_\alpha \\ -S_w L_\alpha \end{bmatrix} \tag{31}$$

3.2. Longitudinal dynamic stability

From Eq. (19), the characteristic equation of matrix Ag is:

$$S^4 + a_3 S^3 + a_2 S^2 + a_1 S + a_0 = 0 \tag{32}$$

where

$$\begin{cases} a_3 = -\bar{Z}_\alpha - \bar{M}_q \\ a_2 = \bar{Z}_\alpha \bar{M}_q - \bar{M}_\theta - V_0 \bar{Z}_\xi - \bar{Z}_q \bar{M}_\alpha \\ a_1 = V_0 \bar{M}_\xi + \bar{Z}_\alpha \bar{M}_\theta - \bar{Z}_\theta \bar{M}_\alpha - V_0 \bar{Z}_q \bar{M}_\xi + V_0 \bar{Z}_\xi \bar{M}_q \\ a_0 = V_0 \bar{Z}_\xi \bar{M}_\theta - V_0 \bar{Z}_\theta \bar{M}_\xi - V_0 \bar{Z}_\alpha \bar{M}_\xi + V_0 \bar{Z}_\xi \bar{M}_\alpha \end{cases} \tag{33}$$

According to the Lienard-Chipart stability criterion, the necessary and sufficient conditions for the dynamic stability are:

(1) $a_3, a_2, a_1, a_0 > 0$; (2) $a_3 a_2 a_1 - a_3^2 a_0 - a_1^2 > 0$.

4. RESULTS AND DISCUSSION

This section is to analyze the static and dynamic stability of large amphibious aircraft gliding on still water surface with the factors including velocity and pitch angle. The main analysis parameters are $m = 60000$ kg, $L_b = 37$ m, $S_w = 172.35$ m^2, maximum velocity on water surface is 180 km/h. The derivatives are shown in Tables 1–5.

Table1

Aerodynamic derivative (Mach number Ma = 0.2)

$C_{L\alpha}$	$C_{L\dot{\alpha}}$	$C_{LM\mathrm{a}}$	$C_{D\alpha}$	$C_{M\alpha}$	$C_{M\dot{\alpha}}$	C_{Mq}
5.48	–6.4	–1.1	1.12	–1.37	–3.3	–21.4

Table 2

Derivatives of hydrodynamic respect with ξ

θ (°)	$C_{Dw\xi}$ (10^{-4})			$C_{Lw\xi}$ (10^{-4})			$C_{Mw\xi}$ (10^{-4})		
	50%V_{TO}	70%V_{TO}	90%V_{TO}	50%V_{TO}	70%V_{TO}	90%V_{TO}	50%V_{TO}	70%V_{TO}	90%V_{TO}
2	–2.13	–1.99	–2.03	–5.16	–4.99	–4.29	–0.72	–0.48	–0.46
4	–1.62	–1.45	–1.23	–4.71	–6.35	–2.98	–0.88	–0.76	–0.64
6	–0.62	–0.64	–0.67	–9.49	–10.33	–10.49	–1.98	–1.59	–1.51
8	–0.59	–0.73	–0.59	–10.13	–12.10	–11.77	–1.08	–0.94	–0.93

Table 3

Derivatives of hydrodynamic respect with θ

ξ (m)	$C_{Dw\theta}$ (10^{-4}/°)			$C_{Lw\theta}$ (10^{-4}/°)		
	50%V_{TO}	70%V_{TO}	90%V_{TO}	50%V_{TO}	70%V_{TO}	90%V_{TO}
0.8	0.015	0.019	0.022	–0.44	–0.29	–0.32
0.6	0.059	0.067	0.066	–0.71	–0.62	–0.73
0.3	0.16	0.14	0.15	–0.95	–0.94	–1.08

Table 4

Derivatives of buoyancy respect with ξ

θ (°)	$D_{dw\xi}$ (10^4 N/m)	$L_{dw\xi}$ (10^4 N/m)	$M_{dw\xi}$ (10^4 N)
2	1.411	–40.426	27.152
4	3.187	–45.565	–103.246
6	52.681	–50.185	–226.471
8	73.034	–51.973	–290.999

Table 5

Derivatives of buoyancy respect with θ

ξ (m)	$D_{dw\theta}$ (10^4 N/°)	$L_{dw\theta}$ (10^4 N/°)	$M_{dw\theta}$ (10^4 Nm/°)
0.8	0.462	–2.129	–35.755
0.6	0.283	–1.543	–23.242
0.3	0.0145	–0.664	–4.474

4.1. Simulation of longitudinal static stability

It is clear that by calculating Eq. (26), the static stability of draft can be obtained. The static stability of draft is shown in Fig. 2, where the vertical axis is represented as a percentage of the take-off velocity V_{TO} on the surface of water. From the figure, it can be seen that within the gliding velocity and pitch angle range defined in this paper, the aircraft has static stability of draft when it is gliding on still water surface.

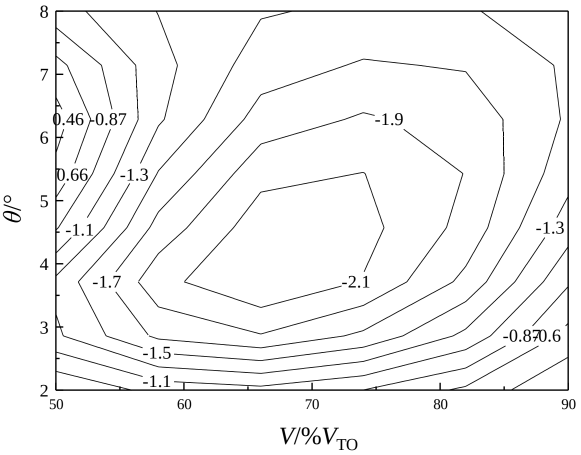

Fig. 2 – Static stability of draft.

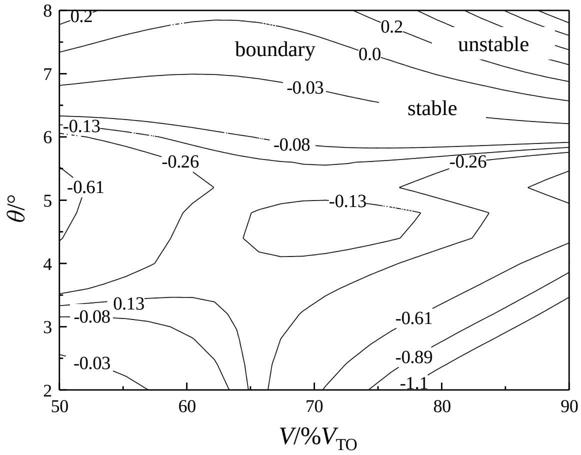

Fig. 3 – Static stability of pitch angle.

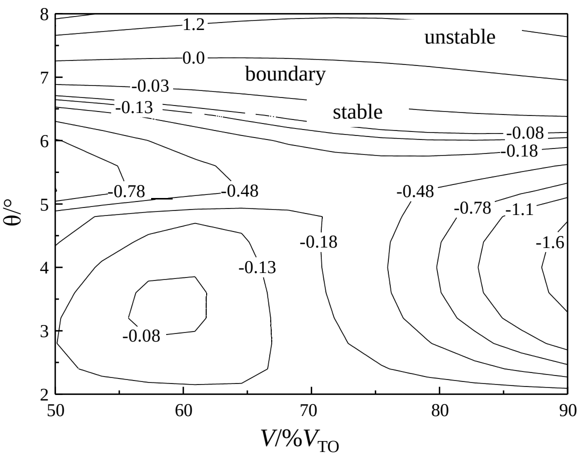

Fig. 4 – Static stability of angle of attack.

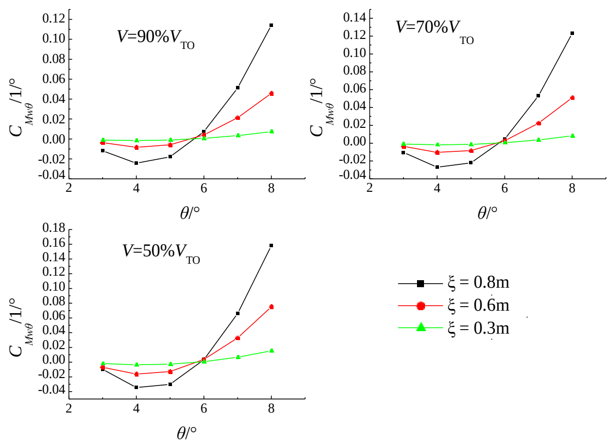

Fig. 5 – Tested derivation of hydraulic pitching moment to pitch angle with pitch angle.

The dimensionless static stability of pitch angle and angle of attack can be obtained through Eqs. (28) and (30), and the result is shown in Figs. 3 and 4. It can be seen from the figure that with the variation of velocity, there is a boundary of pitch angle for the static stability of pitch angle and angle of attack. When the pitch angle is less than the boundary, the static stability is satisfied, otherwise the static stability cannot be satisfied.

This is caused by the derivative of hydrodynamic pitching moment coefficient with respect to pitch angle. The curve of tested derivative of hydrodynamic pitching moment coefficient with respect to pitch angle is shown in Fig. 5. When the pitch angle is less than about 6°, the derivative of hydrodynamic pitching moment coefficient with respect to pitch angle is negative. When the pitch angle is greater than about 6°, the derivative of hydrodynamic pitching moment coefficient with respect to pitch angle quickly becomes positive and increases more rapidly, affecting the static stability of pitch angle and angle of attack heavier and heavier. According to the Eqs. (28) and (30), when the derivative of hydrodynamic pitching moment coefficient with respect to pitch angle increases to a certain extent, the static stability of pitch angle and angle of attack will become unstable.

4.2. Simulation and test of longitudinal dynamic stability

The test object is a scaled model of large amphibious aircraft, and the test location is a high-velocity hydrodynamic test pool. The test equipment is shown in Fig. 6. The pulley can move within a certain range along the heading direction on the carriage, with the front end connected to a resistance meter. The heave rod passes through the carriage and is connected to the model at the centre of gravity of the scaled model, allowing the model to move vertically around the centre of gravity as the pivot point. The model can rotate in the pitch direction, and the upper end of the heave rod is connected to a linear displacement sensor. The yaw restrictor limits the model's yaw motion and, together with the carriage and heave rod system, restricts the model's lateral movement and roll. Before the test, the elevator deflection angle is adjusted. During the test, the model is accelerated to a specified velocity, and after loading the perturbation, the stability of motion is judged, and the pitch angle boundary for stable motion at different velocities is measured.

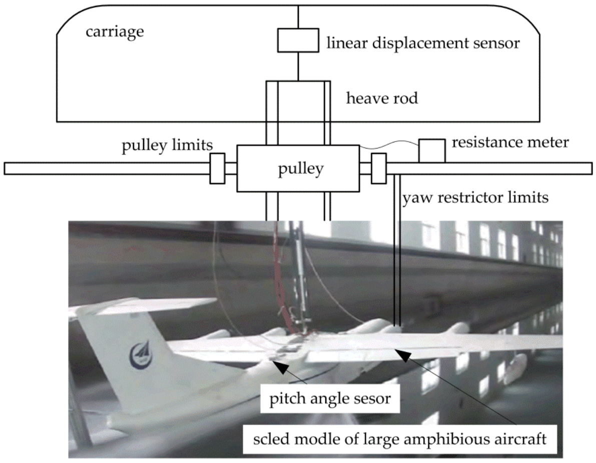

Fig. 6 – Test equipment [8].

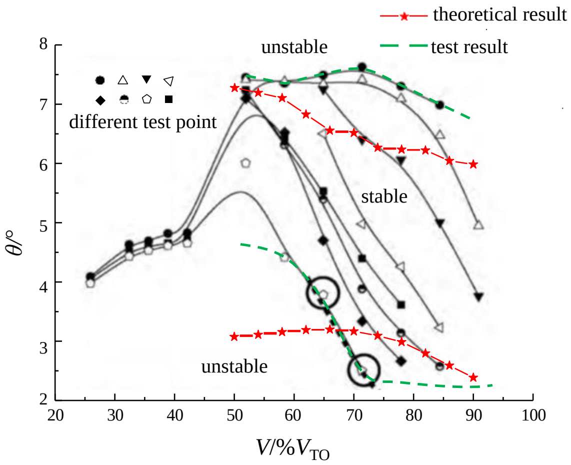

Fig. 7– Comparison of theoretical and test results [8].

Figure 7 shows the comparison between theoretical results calculated in chapter 3.2 and test results. From the figure, it can be seen that large amphibious aircraft has stable upper and lower boundaries when it glides on still water surface. This is because when the pitch angle is small, the hydrodynamic force point changes sharply after a disturbance. As the pitch angle decreases further, the hydrodynamic force point moves forward the centre of gravity, making the aircraft unstable. When the pitch angle is large, the aircraft becomes unstable after a disturbance due to the increase of pitch angle, which increases the aero-dynamic force, allowing the aircraft to take off. However, when the pitch angle returns, the aerodynamic force is insufficient to support the aircraft's weight, causing the aircraft to touch the water again, resulting in porpoise motion. On the other hand, if the pitch angle is too large, large amphibious aircraft becomes lack of static stability, which contributes to the instability. The stable boundary obtained through theoretical calculations and experiments have some differences. This is because the test and calculation processes of aerodynamic and hydrodynamic coefficients may introduce errors and inconsistencies. However, the stable boundary areas of theoretical and test results are consistent. Through comprehensive analysis of the theoretical and test results, the stable pitch angle boundary can be determined as 4°~7°. The area outside the boundary is the unstable area, and large amphibious aircraft should avoid these pitch angles during the gliding process.

5. CONCLUSION

In this paper, the longitudinal static and dynamic stabilities of large amphibious aircraft gliding on still water surface was studied which can provide an analysis method to guide the design of large amphibious aircraft. A design criterion for the longitudinal static stability of large amphibious aircraft was proposed, which includes static stability of draft, static stability of pitch angle, and static stability of angle of attack. The stable boundary of pitch angle for large amphibious aircraft gliding on still water surface is 4°~7°. When the pitch angle is less than 4° and the perturbation of pitch angle occurs, the hydrodynamic force point moves forward the centre of gravity, making the aircraft unstable. When the pitch angle is more than 7° and the perturbation of pitch angle occurs, the aerodynamic force is increased, allowing the aircraft to take off and then the pitch angle returns, leading to that the aerodynamic force are insufficient to support the air-craft's weight and the aircraft touches the water again. If the pitch angle is too large, the large amphibious aircraft becomes lack of static stability, which contributes to the instability.

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