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TOPOLOGICAL QUANTUM PHASES FOR AN ELECTRON OR SYSTEM OF ELECTRONS IN BRAIDED ENERGY FIELD FLUID CONFIGURATIONS

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Abstract. The energy eigenvalues of 'quantum fluctuations' as a manifestation of 'quantum phases' for electrons with stretching, twisting, and twigging in topological space-time we obtained. Electrons are treated as energy field fluids in braided configurations instead of point particles. The quantization of events in 'quasi–particles or time crystals occurs because of the stretching of eigenfunctions, which are manifestations of twisting and twigging with relativistic quantum electrodynamics (QED) behavior. The mathematical expression or energy eigenvalues are $\pm\hbar t_{n_f} \upsilon\alpha(t_{n_f})$, where ± signs indicate forward and backward helical responses of "quantum phases", $0.1 \le n_f \le 0.9$, $0.17 \le \alpha(t_{n_f}) \le 1.53$ rad and υ the frequency at Twiggs with whirlpools and potential barriers.

Keywords: quasi–particles, topology, quantum phases, twigging, braided configurations.

1. INTRODUCTION

The study of topological quantum phases in electron systems has become an exciting new frontier of condensed matter physics, the underlying protection against environmental perturbations revealing much about the fundamental features of quantum states. Topological phases are so different from conventional ones, which symmetry-breaking order parameters can identify, that they must instead be defined via topological invariants, global quantities invariant under a local perturbation. Such fault tolerance is undoubtedly vital for specific application areas such as cryptography, quantum computing, and spintronics [1] where it is desired to have a tunable but robust quantum state of matter in an error-resilient way [2]. A natural way to achieve this protection is to construct topologically distinct quantum systems that will be immune to the harmful energy of noise and decoherence [3, 4].

According to Haldane [5], the strength of topological quantum matter is that a topological invariant can give rise to stable quantum states embedded in materials under specific protection. Topological phases thus identify braid-like arrangements of quasi-particles that can have an effective liquid behavior, and lead to fractional quantization and non-local entanglement [5]. One such study, by Tang *et al.* [6], has shown that topologically protected states in such configurations are exceptionally easily immune to low-level environmental interference such as static disorder or vibrations as a defined suit for applications a couple of many lengthy coherence long-term coherence quantum body systems. These results highlight the significance of these braided energy field configurations, where the specific individual electrons appear to have lost their identity and unite with each other, appearing as point particles. They can be considered energy field fluids, quantum phases from stretching, twisting, and twigging degrees of freedom interactions.

This article contributes to a comprehensive theoretical framework regarding braided field configurations in topological quantum phases. Considering electrons as quasi-particles in topological spaces with respect to the stability of quantum phases, we analyzed the influence of quantum fluctuations and interactions like twigging, stretching, and twisting. Despite the importance of theory, possessing such inherent features, for example, a time-reversal-breaking superconductor may be topological in 2D but non-topological in 3D [7], and Weyl metal is a metallic material with a nontrivial topology of the electronic structure in momentum space, which distinguishes it from simple metals [8]. Such materials will assist upcoming quantum technologies, which exploit the coherence and stability of their operation in configurations that benefit from such properties.

We demonstrate via a mathematical analysis that yields energy eigenvalues, the topological invariants thus established determine which configurations are energetically stable to temporal evolution. Depending on these topological invariants, we demonstrate that such configurations can be stable or unstable. This paper studies quantum fluctuations due to quantum phases induced via braiding within a space-time topological quantum electrodynamics (QED) formulation. We assess the energy eigenvalues and characteristics of wave function profiles, exemplifying how these analogous stretch, twist, and twig interactions supplement and confer quantum state robustness to behaviors within a topological phase. The stable and genuinely coherent states are considered a quantum resource for various implementations of quantum computation and its cryptographic protocols; thus, this work can provide leads in the field of quantum information science.

2. THEORETICAL FRAMEWORK AND RESULTS

The study of topological quantum phases relies on a rigorous mathematical framework that explains the underlying properties of quantum systems [1, 9]. Examining the wavefunctions, energy eigenvalues, and topological invariants can help understand the mechanisms that dictate the stability or instability of braided quantum configurations. The wavefunctions and energy quantization that describe the quasi-particles in these systems, as well as the role of topological invariants in governing the robustness of these configurations against perturbations [3, 10], are crucial for understanding.

2.1. Quasi-particles and energy eigenvalues

The wavefunction describing the quasi-particles in braided systems is given by

$$\psi = \mathrm{e}^{-\mathrm{i}(kx-\omega t)} H_n(\xi) \tag{1}$$

where, $H_n(\xi)$ represents the Hermite polynomials, k is the wavevector, ω is the angular frequency, t is time, and ξ is the normalized position.

This wavefunction abridges the oscillatory and localized nature of the quantum states. The exponential term governs the oscillatory behavior over space and time, while the Hermite polynomials ensure spatial localization. Such localization is essential for achieving stable quantum phases [9, 10].

Quasi–particles or time crystals of an electron or many electrons in a braided configuration are associated with stretching, twisting, and twigging [11, 12]

$$\psi\left(r_{op}, \alpha, t_{n_f}\right) = \frac{\mathrm{i}\hbar t_{n_f}}{m_{n_f}} \left(\frac{0.17 \le \alpha \le 1.53\, rad}{\sqrt{\pi}\, 2^{n_f}}\right)^{\frac{1}{2}} \tag{2}$$

where, $r_{op} \equiv \delta_{\mathrm{Dirac}}(r - r_o) = \hbar,\ \hbar = \frac{h}{2\pi}$ is Planck's constant, $0.1 \le n_f \le 0.9,\ 0.17 \le \alpha \le 1.53$ rad and $H_{n_f} = 2^{n_f}$ is the Hermite polynomial for quantized beaded fractional harmonic oscillators for corresponding fractional charges and masses, respectively.

$$e_{n_f} = \sum_{n_f=0.1}^{0.9} n_f \cdot e \quad \text{and} \quad m_{n_f} = \sum_{n_f=0.1}^{0.9} n_f \cdot m_e \tag{3}$$

where e is the electron's charge, m_e is the mass of the electron, e_{n_f} the fractional charge of the electron and m_{n_f} the fractional mass of the electron. Quasi-particles or time crystals of an electron cannot be treated as point particles; the corresponding energy field fluids of a braided configuration are the only option where quantum field theory (QFT) is applicable. The eigenfunction $\psi(\text{stretching})$ is interpreted as the "quantization of events", $\hbar t_{n_f}$ with twigging and twisting, however, with the relativistic behavior of the fractional masses.

The energy eigenvalues corresponding to the wavefunction are quantized and expressed as

$$E_n = \hbar\omega\left(n + \frac{1}{2}\right) \tag{4}$$

where n is the quantum number. These eigenvalues represent discrete energy levels of the quasi-particles in braided configurations. The term $\hbar\omega$ defines the energy scale, while $n + \frac{1}{2}$ represents the harmonic oscillator's quantization [10].

Let the eigenfunction for the "quantum fluctuations" be defined as manifestations of "quantum phases" in terms of stretching, that is,

$$\psi(\text{quantum fluctuations}) = \mathrm{e}^{\pm \mathrm{i}\hbar\cdot\omega\cdot t_{n_f}} = \sqrt{\frac{\mathrm{i}\hbar t_{n_f}}{m_{n_f}}} = \psi^{\frac{1}{2}}(r_{op}, \alpha, t_{n_f}) \tag{5}$$

where $\psi\left(r_{op}, \alpha, t_{n_f}\right) \equiv \psi(\text{stretching}) = \frac{\mathrm{i}\hbar t_{n_f}}{m_{n_f}}$, in Eq. (2).

The probability density, derived from the wavefunction, is expressed as:

$$|\Psi(\alpha, \mathrm{t})|^2 = \left|\mathrm{e}^{-\mathrm{i}(kx-\omega t)} H_n(\xi)\right|^2 \tag{6}$$

The probability density describes the localization of quantum states in space and time. Peaks in $|\Psi(\alpha, \mathrm{t})|^2$ indicate areas of maximum stability and robustness of quantum configurations. These localized states ensure that the quantum system remains resilient against environmental perturbations, which is a critical feature of topologically stable phases [1, 3].

Equations (2), (3), and (5) comply with the Schrödinger and Dirac quantum wave mechanical equations.

$$\left.\begin{array}{r} H_{op}\psi = \left[\dfrac{\hbar^2\nabla_r^2}{2m} + < |V(r,t)| > \right]\psi = E_{op}\psi \\ \text{and} \quad \left(\beta mc^2 - \mathrm{i}m\hbar c\left(\sum_{k=1}^{3} a_k \dfrac{\partial}{\partial r_k}\right)\right)\psi = \mathrm{i}\hbar\dfrac{\partial}{\partial t}\psi \end{array}\right\} \tag{7}$$

where $\beta = \frac{1}{\sqrt{1-\frac{\upsilon^2}{c^2}}}$, $\hbar = \frac{h}{2\pi}$, c is the velocity of light, $p = \hbar k$ the quantum momentum, H_{op} the Hermitian Hamiltonian operator, ψ the eigenfunction, $p_{op} = -\hbar\nabla_r$, $E_{op} = \mathrm{i}\hbar\frac{\partial}{\partial t}$, m the mass of an electron, and $< |V(r,t)| >$ the potential energy of quantization.

Figure 1 shows the spatial-temporal probability density of electron states in a topological phase based on Eqs. (2) and (3). This wave function has peaks representing the localization in the electron's topological phase. This localization, as noted by Haldane [5], this localization is critical for quantum states that are braided to protect against errors because they need to be coherent over long periods and not just local dephasing.

Research by Tang *et al*. [6] supports topologically protected states, which are known to be very attractive due to the robustness of these states against any perturbations, that is, as this property of the topologically protected states is paramount for quantum memory and computation. The probability density distribution shown in demonstrated in the diagram shows how topological protection results in such quantum protection, which is required for fault-tolerant systems [6]. This simulation is consistent with the study of Weng *et al*. [13] and Huang *et al*. [14], demonstrating that a topological state can persist in real-world computing environments.

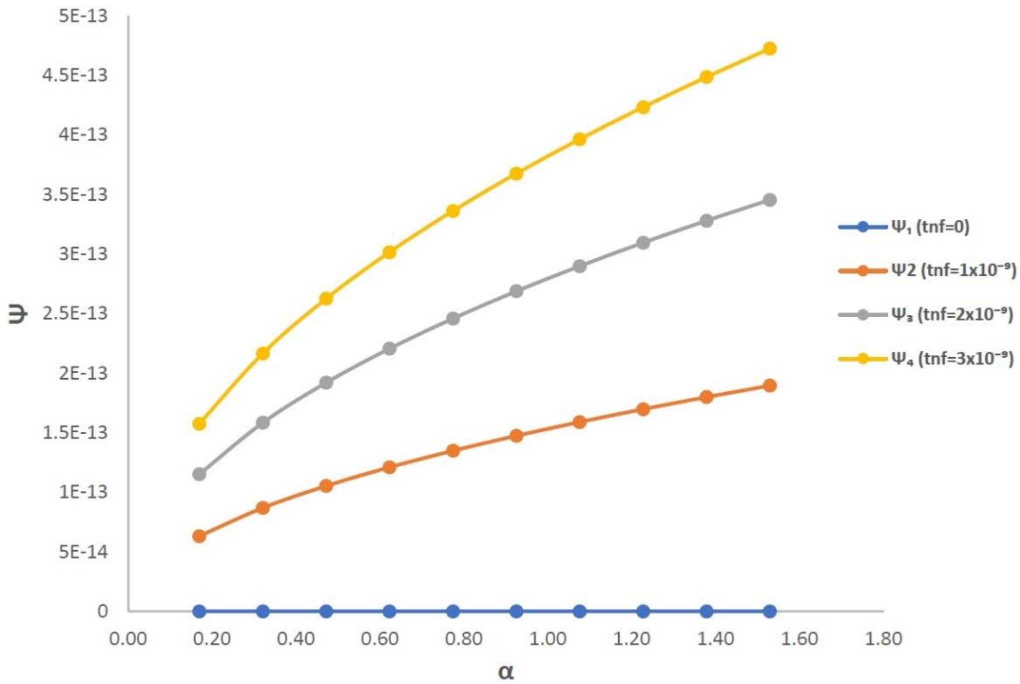

Fig. 1 – Spatial-temporal probability density: Variation of Ψ with α showing localized quantum states in braided topological configurations.

2.2. Role of topological invariants in stability and instability

We consider the eigenfunction in Eq. (5) for topological phases in topological space-time due to the stretching, twisting, and twigging of an electron or many systems of electrons in a braided energy field fluid configuration. We will use the quantum electrodynamic (QED) theory [15] in topological space-time.

$$\left.\begin{aligned} \bar{\emptyset}(\bar{r},t) = 0 \quad &; \quad \bar{A}(\bar{r},t) = -\hbar n_f \alpha(t_{n_f}) \\ \bar{E}(\bar{r},t) \approx \sqrt{2}\, n_f \alpha\left(t_{n_f}\right) \omega\hbar \, ; \quad &\bar{B}(\bar{r},t) \cong -\sqrt{2}\, n_f \hbar \theta \sin\theta\, \bar{\nabla}_r \alpha(t_{n_f}) \end{aligned}\right\} \tag{8}$$

$$\left.\begin{array}{c} \theta \sin\theta\, \bar{\nabla}_r \alpha(t_{n_f}) \text{ is geometric representation for helical pattern of} \\ \text{dipole radiations due to “quantum fluctuations" or "quantum phases"} \\ \text{of energy field fluids in topological space} - \text{time} \end{array}\right\} \tag{9}$$

The “oscillatory effects” or “quantum fluctuations” or “quantum phases” in time-crystals with QED behavior in topological space-time is considered [15].

Topological invariants, such as the Chern number [16] and Berry curvature [17, 18], play a pivotal role in determining the stability or instability of quantum configurations. These invariants provide a global measure of the quantum system’s topology [19], securing it to be stable under local perturbations.

The Chern number C, a key topological invariant, is defined as:

$$\begin{gathered} C = \frac{1}{2\pi} \int_{BZ} F(k) \mathrm{d}^2 k \, , \\ F(k) = \nabla \times A(k) \end{gathered} \tag{10}$$

where BZ denotes the Brillouin zone, $F(k)$ is the Berry curvature, and $A(k)$ is the Berry connection. The Chern number quantifies the topological nature of quantum states. A nonzero C ensures that the system supports robust, topologically protected states immune to small perturbations while $C = 0$ indicates the absence of such protection [1, 3].

The oscillatory components in the magnetic vector potential $\bar{A}(\bar{r},t)$, and in an electric field, $\bar{E}(\bar{r},t)$, are defined [15]

$$\left.\begin{array}{c}\left\{\dfrac{-\hbar}{2|a_{n_f}(k)|^2}\right\}\alpha(t_{n_f}) \quad ; \\ -\omega_0^2\left[\bar{A}_0\, e^{2i(\bar{k}\bar{r}-\omega t)}+\bar{A}_0^{\dagger}\, e^{-2i(\bar{k}\bar{r}-\omega t)}\right]\end{array} \text{and}\right\} \tag{11}$$

where,

$$\bar{A}(\bar{r},t_{n_f})=\hbar n_f\alpha(t_{n_f}),\ \ \bar{A}_0^{\dagger}(\bar{r},t_{n_f})=-\hbar n_f\alpha(t_{n_f}) \tag{12}$$

And ω_0 is the resonant angular frequency of “quantum phases”.

The stability of braided configurations is characterized by their oscillatory energy behavior:

$$E_{\text{osc}}=\hbar\omega\cos(\alpha t) \tag{13}$$

where α is the twisting parameter. This equation describes how the system dynamically stabilizes itself by restoring equilibrium after perturbations. Oscillations in energy ensure that the system maintains coherence over time, a hallmark of stable topological phases [20].

Substitution of E. (12) in Eq. (11) for $\bar{E}^2(\bar{r},t)$, we obtain

$$\bar{E}_{\text{oscill}}^{\,2}\cong 2i\hbar n_f\alpha(t_{n_f})\omega_0^2\left|\sin2(\bar{k}\cdot\bar{r}-\omega t_{n_f})\right|^2 \tag{14}$$

where,

$$\left|\partial_{n_f}(k)\right|\cong\left|\sin2(\bar{k}\cdot\ \bar{r}-\omega t_{n_f})\right|^2 \tag{15}$$

Equation (15) is also applicable to the oscillatory component of the magnetic vector potential, $\bar{A}(\bar{r},t_{n_f})$ in Eq. (11), and Eq. (8) describes the helical pattern for dipole radiations due to twisted magnetic fields which if multiplied by $\nabla_r\propto(t_{n_f})$ will correspond to “quantum phase” charges.

The stability or instability of a configuration is determined by the curvature of the potential energy perspective:

$$\begin{gathered}\frac{\partial^2E}{\partial q^2}>0\quad(\text{Stability}),\\ \frac{\partial^2E}{\partial q^2}<0\quad(\text{Instability})\end{gathered} \tag{16}$$

Positive curvature corresponds to stable configurations, where the system resists perturbations. Negative curvature indicates unstable configurations, where small deviations amplify, leading to dephasing and loss of coherence [21, 22].

Let us calculate the energy eigenvalues of “quantum phases” in topological space-time for time-crystals or quasiparticles with stretching, twisting, and twigging.

$$E_{op}\psi\left(r_{op},\alpha,t_{n_f}\right)=i\hbar\frac{\partial}{\partial t}\psi\left(r_{op},\alpha,t_{n_f}\right)$$

where E_{op} is equivalent to the Dirac equation that is in equation (5). Let the argument of the exponential function in Eq. (5) be made “congruent” to Eqs. (8) and (9), that is, $\pm i\hbar\omega t_{n_f}\equiv\pm2i\hbar n_f\alpha(t_{n_f})\omega_o^2a_{n_f}^2(k)$ and so the case would be for their corresponding eigenfunctions.

$$\psi(\text{quantum phases})=e^{\pm i\hbar\cdot\omega\cdot t_{n_f}}\equiv\ e^{\pm2i\hbar\cdot n_f\cdot\alpha\,(t_{n_f})\omega_o^2a_{n_f}^2(k)} \tag{17}$$

Here, we consider $\bar{E}^2(\bar{r},t)$ instead of $\bar{\emptyset}(\bar{r},t)$. The scalar electric field potential is $\bar{\emptyset}(\bar{r},t)=0$ in topological space-time. $\bar{E}^2(\bar{r},t)$ refers to $\bar{E}(\bar{r},t)\times\bar{E}(\bar{r},t)=\bar{E}^2(\bar{r},t)\sin\theta$ which is indicative of the helical responses of electric fields in forward and backward directions, respectively. In this case, we have $\theta\equiv0.17\leq\alpha(t_{n_f})\leq1.53$ rad.

$$\left.\begin{aligned} E_{op}\psi = \mathrm{i}\hbar\frac{\partial}{\partial t}\,\mathrm{e}^{\pm 2\mathrm{i}\hbar\cdot n_f\cdot\alpha(t_{n_f})\omega_o^2 a_{n_f}^2(k)} = \left[\left(\beta mc^2 - \mathrm{i}m\hbar c\left(\sum_{k=1}^{3} a_k\frac{\partial}{\partial r_k}\right)\right)\psi\right] \\ = \pm 2\hbar\cdot n_f\cdot\omega_o^2 a_{n_f}^2(k)\frac{\partial}{\partial t}\alpha(t_{n_f})\mathrm{e}^{\pm 2\mathrm{i}\hbar\cdot n_f\cdot\alpha(t_{n_f})\omega_o^2 a_{n_f}^2(k)} = E\psi \end{aligned}\right\} \tag{18}$$

where the energy eigenvalues in Eq. (18) are obtained.

$$E_{\text{oscillatory}}(\text{energy eigenvalues}) = \pm 2\hbar\cdot n_f\cdot\omega_o^2\cdot a_{n_f}^2(k)\frac{\partial}{\partial t}\alpha(t_{n_f}) \tag{19}$$

For $0.1 \le n_f \le 0.9$ and $0.17 \le \alpha \le 1.53$ rad Eq. (19) represents energy eigenvalues of 'quantum phases' or 'quantum fluctuations' with QED (quantum optics) behavior for electrons in braided field fluids configurations (following QFT). The positive and negative signs in Eq. (19) correspond to travel of 'quantum phases' in forward and backward directions, respectively.

The forward and backward helical responses of braided configurations are represented by

$$E_{\pm} = \pm\hbar\omega\left(n + \frac{1}{2}\right) \tag{20}$$

These eigenvalues quantify the stability of quantum states under twisting and twigging interactions. Forward and backward responses stabilize the energy levels, ensuring robustness in topological configurations [3, 23].

The twisting angles, $0.17 \le \alpha \le 1.53$ rad are manifestations of $\psi_{\text{stretching}}(\alpha)$ as in Eq. (5) and or consequences of which, we obtain $\frac{\mathrm{d}}{\mathrm{d}t}\left(\alpha_{t_{n_f}}\right)$ for Eq. (19) that is [24],

$$\left.\begin{aligned} E_{op}\psi_{\text{stretchimg}}(\alpha) = E_{op}\alpha \\ E_{op}\alpha = E_{op}\psi_{\text{stretchimg}}(\alpha) = \frac{-\hbar}{\mathrm{i}}\frac{\partial}{\partial t}\sqrt{\frac{\mathrm{i}\hbar t_{n_f}}{m_{n_f}}} = \frac{-\hbar}{\mathrm{i}}\frac{\partial}{\partial t}(\alpha) \\ = \frac{-\hbar}{\mathrm{i}}\frac{1}{2}\left(\frac{\mathrm{i}\hbar t_{n_f}}{m_{n_f}}\right)^{-1/2}\frac{\mathrm{i}\hbar}{m_{n_f}} = \frac{-\mathrm{i}\hbar}{\mathrm{i}}\frac{\partial}{\partial t}(\alpha) \\ \frac{\mathrm{d}}{\mathrm{d}t}(\alpha) = \left(\frac{\mathrm{i}\hbar}{4t^4 m_{n_f}}\right)^{1/2} = \frac{1}{2t}\left(\frac{\mathrm{i}\hbar}{m_{n_f}}\right)^{1/2} = \frac{\nu}{2}(\alpha) \end{aligned}\right\} \tag{21}$$

where ν is the frequency at twigs, $0.1 \le n_f \le 0.9$ and $0.17 \le \alpha \le 1.53$ rad [25] with substitution of Eq. (21) in Eq. (19), we obtain the oscillatory energy eigenvalues for 'quantum phases' as

$$\pm 2\hbar\cdot n_f\cdot\omega_o^2.\, a_{n_f}^2(k)\nu\,\alpha(t_{n_f}) \tag{22}$$

The term $\omega_o^2\cdot a_{n_f}^2(k)$ in Eqs. (18) and (19) refers to Fermi Golden transition probability with a unitary operator. It would be considered 'unity', with a maximum probability of one in the Hilbert space for topological space-time.

$$\left|\omega_o^2\cdot a_{n_f}(k)\right|^2 \cong 1 \tag{23}$$

Equation (23) is valid only for any significant physical event not under consideration. Hence, we obtain energy eigenvalues for 'quantum phases' in topological space–time as manifestations of continuously changing configurations of an electron or many electron systems in braided field fluid configurations.

$$E = \pm\hbar\cdot n_f\cdot\nu\cdot\alpha(t_{n_f}) \tag{24}$$

where $0.1 \le n_f \le 0.9,\ 0.17 \le \alpha(t_{n_f}) \le 1.53$ rad and $\hbar = \frac{h}{2\pi}$ is the reduced Planck's constant.

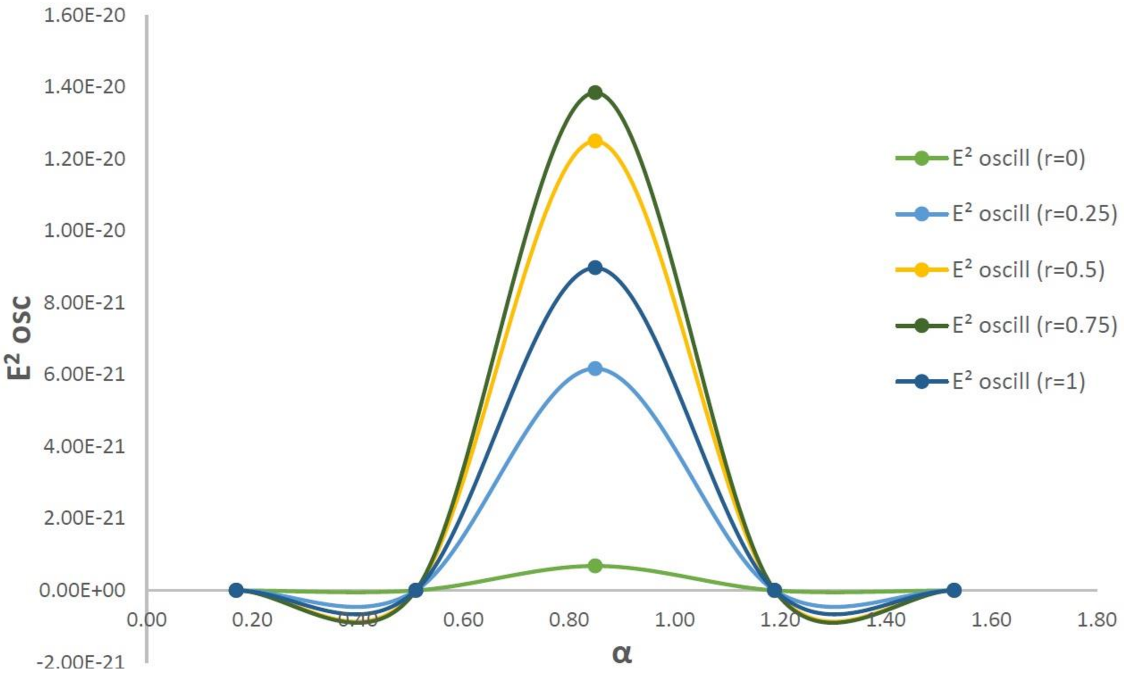

Fig. 2 – Energy oscillations in topological phases: Oscillatory behavior of E_{osc}^2 as a function of α indicating stable phase transitions in topological states.

From Eq. (14), the graph in Fig. 2 is obtained. It indicates energy oscillation corresponding to angular momentum and frequency in topological phases. Such oscillations are related to the robustness of quantum states in topological phases, especially in non-Abelian configurations based on quantum statistics. Within these phases, the role of braiding mechanisms gives localization of energy states and is a crucial mechanism for the stability of coherent quantum states [26]. The stability of such configurations is governed by topological invariants, such as the Chern number, and the energy oscillations derived from the wavefunctions. These insights align with experimental observations of fractional quantum Hall states and topological insulators [1, 3, 27].

As Wang *et al*. [20] proposed, such oscillatory behaviors serve as signatures of stable phase transitions in topological materials because energy coherence stabilizes quantum systems. This robustness is critical for quantum computing, where the ability to do accurate computations by changing qubit states relies upon precise energies [28]. These results suggest that topological quantum phases may maintain coherence, a key feature for effective quantum information processing.

3. CONCLUSION

Topological quantum systems, such as in braided energy field configurations, is a pivotal development of condensed matter physics that can find propitious applications in quantum information science. This framework allows us to conceptualize electrons as quasi-particles in these patterns. It provides a means for stretching, twisting, and twigging interactions, which prepare quantum states characterized by robustness against decoherence and environmental variations. Those states feature topological protection necessary for further quantum computation and cryptography development, demanding high stability and coherence for errorless functioning.

In this research article, we propose a theoretical understanding of how these braided energy field configurations affect the quantum phases of electron systems. We established the role of interactions like stretch, twist, and twig in promoting stability and coherence of quantum states using concepts such as topological invariants and energy eigenvalues. The work demonstrates stable quantum phases that resist perturbation when electrons are treated as quasi-particles comprising braided energy field fluids. The oscillatory energy behaviors and the spatial-temporal probability density distribution analyses corroborate that these topological configurations provide robustness.

Recent studies, including those by Tang *et al*. [6] and Burkov [8], underscores topologically protected states that imply increasing stability of quantum properties, forming the basis for robust quantum architectures. Such results align with the foundational work of [21], indicating that braided features are one of the most important aspects of next-generation quantum technologies. The integration of topological invariants in combination with quantum dynamics provides a relatively robust approach to arranging stable quantum systems, making it one of the most important components we require for future use in quantum applications.

However, the theoretical exploration of the braided field-like configurations is part of a more established conceptual framework for topological quantum phases that remains to be implemented in advanced quantum technologies. This work is essential to construct stable, fault-tolerant systems to form the basis for a new class of quantum computing and cryptography protocols.

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