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MALMQUIST ORTHOGONAL FUNCTIONS BASED TUBE MODEL PREDICTIVE CONTROL WITH SLIDING MODE FOR DC MOTOR SERVO SYSTEM

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Abstract. This paper introduces a novel approach to the conventional model predictive control design within a tube model predictive control framework for a DC motor servo system that is an important component of various control systems in process industries. Tube model predictive control proves to be an effective method in the formulation, analysis, and implementation of robust control strategies. The objective is to include all possible trajectories of an uncertain system within a tube of the nominal system trajectories. Therefore, the proposed method incorporates discrete-time generalized Malmquist orthogonal functions for nominal model predictive controller design, which is the first time that these functions are used for this purpose in combination with the auxiliary sliding mode controller. The sliding mode controller has a crucial role in determining the robust dynamics of a closed-loop system in the presence of disturbances and plant nonlinearities. Experimental results of DC servo motor angular position control are presented and discussed for two different sliding mode control algorithms. The analysis shows improved performance in terms of fast reference tracking and disturbance rejection.

***Keywords**:* tube model predictive control, discrete-time Malmquist orthogonal functions, sliding mode control, DC motor servo system.

1. INTRODUCTION

Due to its good control characteristics, a DC motor servo system is widely used in process industries, where precise control of speed and position is required in a large range. On the other hand, it is originally a nonlinear system with several non-electrical parameters changing with the time, motor temperature and working environment, which are difficult to measure directly. These parameter variations affect the static and dynamic control system performance by deteriorating them. The linear model of the DC servo motor is still predominant in system design both with the classical PI and PID control algorithms. To attain the desired system behavior in the presence of external disturbances, parameter perturbations and plant nonlinearities, some robust modifications of latter control laws should be considered. Moreover, the reduced models of DC servo motors are often used in the controller design process, as well as in this manuscript. In [1], it is presented how to derive one such reduced model and use it to tune the PI controller parameters to alleviate the effects of closed-loop zeros on servo system performance. Under set-point change and during load disturbances, PI controllers obtained from reduced model-based design fail to keep the servo system overshoot and/or undershoot within the acceptable limit. As set-point filtering and weighting methods give acceptable responses to set-point changes, but not to load ones, an online dynamic set-point weighting technique in PI controller design is shown in [2], demonstrating its effectiveness for DC servo motor position control. The robust stability of the closed-loop servo system concerning parametric variations of the DC motor, represented by the first and second-order model, as well as PI and PID controller robustness are considered in [3]. PI and PID parametric conditions, which provide robust stability to plant parameter perturbations, are derived first from Kharitonov's theorem. Thereafter, other parametric conditions are obtained to provide PI and PID controller robustness assuming that the DC servo motor parameters are constant, and the controller parameters are variable within the given intervals. These conditions are applied to an extended symmetrical optimum method for tuning PI and PID controller parameters resulting in design recommendations. A comparative study of a DC servo motor position control using various classical controllers is given in [4]. An interesting approach to DC motor servo system design is discussed in [5], where the sliding mode controller is combined with a model-free PI control system structure to compensate for the estimation errors and thus enhance system robustness and performance. Another approach to suppress DC motor servo system overshoot and/or undershoot under set-point change and load disturbance caused by state/output and input constraints is to implement robust model predictive control (MPC) techniques [6, 7].

Tube model predictive control (TMPC) has become a good option apart from robust dynamic programming [8–11] and min-max feedback model predictive control (MPC) [12–16] for formulating, analysing, and getting close to a robust control problem solution. Tube-based control techniques aim to confine all potential trajectories of an uncertain system within a tube surrounding the nominal system trajectory. The basic TMPC concept implies that the MPC controller essentially controls the nominal plant and an auxiliary controller provides robustness i.e. that perturbed plants are inside a "tube" surrounding the nominal plant. The auxiliary controller should add little additional computational requirements to the overall control, and that is why sliding mode control (SMC) [17] seems to be a good choice due to its simplicity and robust characteristics [18, 19]. Even with linear state feedback, it is difficult to calculate tube widths accurately. A set of states must be calculated within which the auxiliary controller may maintain the states of the real system. Once this set has been computed, the equivalent set within which the auxiliary control input will reside must also be calculated. To avoid confining the nominal MPC, the estimated set should be as small as possible; that is, the minimal robustly positive invariant (mRPI) set has to be calculated for the system under auxiliary control. There are various tractable convex tube MPC formulations for linear systems, including fully parameterized TMPC [20, 21], homothetic TMPC [22–24], elastic TMPC [25], and rigid TMPC [26, 27]. The best way to implement robust MPC depends on balancing conservatism and improving computational run-time performance. The parameterization of the tube is often based on polytopes, ellipsoids, or other sets, which affects the run-time and performance of the associated controller. In [28], the authors have utilized a modified reference system that operates similarly to the nominal system, with the distinction that the transition between different modes of the piecewise affine system is influenced by the real system state. A new type of configuration-constrained polytopic robust invariant tubes is presented in [29], allowing for a combined parameterization of their facets and vertices. The work described in [30] shows an innovative formulation of a resilient MPC designed to track dynamic targets through the unified optimization approach. The controller formulation contains parameters that offer additional degrees of freedom, and the new parameters enable the management of control objectives like disturbance rejection, output offset prioritization, or expansion of the domain of attraction. This paper also demonstrates how these parameters can be computed offline.

Another approach for TMPC design is not to define the exact tube but to calculate the tube widths under the auxiliary controller [31], which is based on [32] where the robustly positive invariant set is not calculated directly. In this way, one should determine the measure for the nominal MPC control constraints tightening. The authors also presented the tightening procedure along with introducing the auxiliary sliding mode controller. In [33], the authors used Laguerre orthogonal functions for the nominal MPC design. Both latter control approaches are experimentally verified on DC motor servo system. The goal of introducing orthogonal functions is to handle a large control horizon while having a small optimization problem [34]. By decreasing the degrees of freedom in optimization, one reduces the computational burden, issued by a large number of parameters for online optimization problem-solving. This allows the controller to handle both slow and fast(er) dynamics with a small number of decision variables.

This paper presents the modification of the traditional MPC design method within the TMPC framework, where the discrete-time generalized Malmquist orthogonal functions are introduced into MPC design. Their employment is justified by the assumption that the control increment behaves similarly to a stable system's impulse response, which can be expressed using the Malmquist impulse response model. Employing this class of functions has the advantage of having only a few tuning parameters that are independent of the sampling time T, which makes the closed-loop tuning process easier. To the authors' knowledge, discrete-time Malmquist functions have never been used in TMPC formulation before. A nominal TMPC is employed, as in the original TMPC, to emphasize the robustness provided by the auxiliary controller. However, the SMC-based auxiliary controller suggested in this work is also applicable to a TMPC formulation in which feedback is also used in the MPC component of the TMPC. The auxiliary controller input is very important because it affects the closed-loop system dynamics when disturbance is present. There are different procedures for its calculation. Here, two well-known SMC approaches for the auxiliary controller design are used [17, 35].

In the next sections, a brief definition of the problem is given, followed by the TMPC design with the introduction of nominal Malmquist functions based MPC and auxiliary sliding mode controller. Afterwards, the experimental results and conclusion remarks are presented. The contributions of this paper are the following:

(1) the design of the TMPC method with the nominal MPC tuned by orthogonal Malmquist functions parameters, and the auxiliary controller defined by traditional and chattering-free SMC algorithms,
(2) the robustness of the proposed TMPC, where the MPC is used to handle the nominal system (nominal model) and the deviations from the nominal is handled by sliding mode controller,
(3) lowering the computational load by employing a parameterization approach based on the orthogonal functions for the MPC design, and
(4) the online computational complexity is related only to solving a nominal MPC scheme, and is not influenced by auxiliary controller.

2. PROBLEM STATEMENT

Let us consider the state-space model of a single-input-single-output plant in discrete-time domain, described by:

$$\begin{aligned} x_{k+1} &= Ax_k + Bu_k + Ew_k \\ y_k &= Cx_k \end{aligned} \tag{1}$$

where $x \in \mathbb{R}^{n_x}$ is the system state, $u \in \mathbb{R}$ is the control input signal, $y \in \mathbb{R}$ is the output, and $w \in \mathbb{R}^{n_x}$ is a disturbance. The control input increment $\Delta u_k = u_k - u_{k-1}$ is also determined at time instant k. It is assumed that polyhedral, bounded and full dimensional sets $\mathbb{U}$, $\mathbb{U}_\Delta$, and $\mathbb{W}$ have the origin in their interior and define the constraints on the input and its increment as

$$\begin{aligned} u_k \in \mathbb{U} &= \{u_k | \Gamma u_k \leq \chi\}, \\ \Delta u_k \in \mathbb{U}_\Delta &= \{\Delta u_k | \Gamma_\Delta \Delta u_k \leq \chi_\Delta\}, \mathbb{U}_\Delta \subset \mathbb{U} \end{aligned} \tag{2}$$

and also define the constraints on the possible range of disturbances by

$$w \in \mathbb{W} = \{w | \mathrm{H} w_k \leq \lambda\}. \tag{3}$$

The main idea behind the TMPC approach is to split the system state x into two components, a nominal component $\bar{x}$ that is going to be controlled by nominal MPC without considering disturbances, and the deviation from the nominal $z = x - \bar{x}$ where the control problem is defined as a feedback control problem providing that the actual state x follows the nominal state $\bar{x}$.

The block scheme of the proposed TMPC is given in Fig. 1.

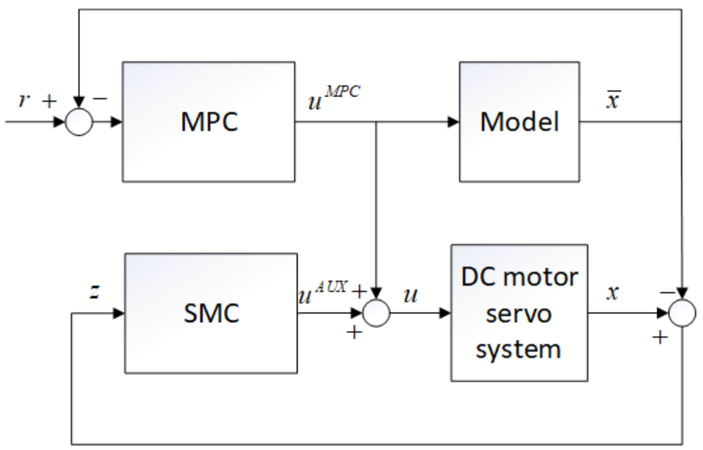

Fig. 1 – Block scheme of the proposed TMPC.

Following this concept, the nominal system is defined by

$$\bar{x}_{k+1} = A\bar{x}_k + Bu_k^{MPC}, \tag{4}$$

whereas the perturbed one is described by

$$z_{k+1} = Az_k + Bu_k^{AUX} + Ew_k. \tag{5}$$

It is obvious from (1), (4), (5) and Fig. 1 that the control input is also divided into two parts as well. One comes from the nominal MPC denoted by u^{MPC}, and the other from the auxiliary controller, denoted by u^{AUX}, yielding

$$u = u^{MPC} + u^{AUX}. \tag{6}$$

3. NOVEL TUBE MODEL PREDICTIVE CONTROL DESIGN

This section elaborates TMPC approach, wherein the conventional MPC is substituted by the control method that utilizes generalized discrete-time Malmquist orthogonal functions together with sliding mode controller as an auxiliary controller. The aim for introducing these functions in the controller design is to decrease computational complexity of proposed TMPC that is obtained using Malmquist orthogonal function parameters as a decision variable. Comparing to the traditional MPC approach, this method provides handling a large control horizon while having a small optimization problem which affects computational burden. The Malmquist functions based MPC design is presented first, and then the description of the used SMC approaches is shown in the sequel.

3.1. Malmquist functions based MPC

In [36–38], discrete-time generalized Malmquist orthogonal functions for system identification have been developed. To define the appropriate form of the Malmquist functions for the controller design, the same approach from [36] is applied. The following transfer function for the Malmquist network (depicted in Fig. 2) is obtained as

$$W_N(z) = \frac{z}{z-a_1}\prod_{k=2}^{N}\frac{z-a_{k-1}^*}{z-a_k}; a_k^* = \frac{1}{a_k}, a_k \in \mathbb{R}, a_k = [0,1], \tag{7}$$

where the symmetric transformations $a_k^* = f(a_k)$ and $a_k = f(a_k^*)$ are used for mapping the zeroes and poles as $a_k^* = \frac{1}{a_k}$ and $a_k = \frac{1}{a_k^*}$, respectively.

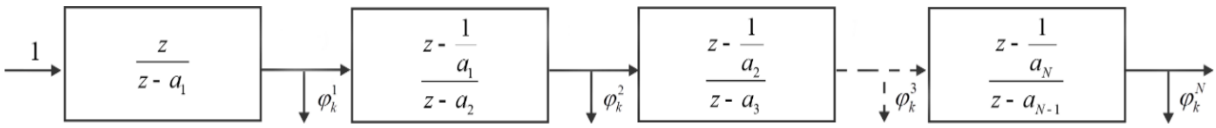

Fig. 2 – Discrete Malmquist network.

Here, these functions are used for the design of nominal MPC. Commonly, generalized discrete-time Malmquist functions can be obtained using the inverse Z-transform of the Malmquist network. However, applying the inverse Z-transform of the network is not a simple instrument for the calculation of the Malmquist functions in discrete-time domain. The practical way of deriving the Malmquist functions is deploying the state-space representation of the network as a result of their cascade form. If the Malmquist functions are represented in a vector form $\Phi_k = \left[\varphi_k^1 \;\; \varphi_k^2 \;\; \dots \;\; \varphi_k^N\right]^T$, the difference equation defining discrete-time Malmquist functions can be derived as

$$\Phi_{k+1} = A_\phi \Phi_k, \tag{8}$$

where the matrix A_ϕ, containing the parameters $a_1, a_2, \dots, a_N$ of the Malmquist functions, is given in the following form

$$A_{\Phi} = \begin{bmatrix} a_1 & 0 & 0 & \cdots & 0 \\ a_1 - \frac{1}{a_1} & a_2 & 0 & \cdots & 0 \\ a_1 - \frac{1}{a_1} & a_2 - \frac{1}{a_2} & a_3 & \cdots & \vdots \\ \vdots & \vdots & \vdots & \ddots & 0 \\ a_1 - \frac{1}{a_1} & a_2 - \frac{1}{a_2} & \cdots & a_{N-1} - \frac{1}{a_{N-1}} & a_N \end{bmatrix}, \tag{9}$$

with the initial conditions defined by $\Phi_0 = [1 \;\; 1 \;\; \cdots \;\; 1]^T$.

To design the Malmquist functions based MPC, an augmented state-space model needs to be derived. If $\Delta\bar{x}_{k+1} = \bar{x}_{k+1} - \bar{x}_k$ and $\Delta u_k = u_k - u_{k-1}$ denotation is introduced, the system with embedded integrator can be represented by:

$$\begin{gathered} \begin{bmatrix} \Delta\bar{x}_{k+1} \\ y_k \end{bmatrix} = \begin{bmatrix} A & \mathbf{0}_{n_{\bar{x}}}^T \\ CA & 1 \end{bmatrix} \begin{bmatrix} \Delta\bar{x}_k \\ y_k \end{bmatrix} + \begin{bmatrix} B \\ CB \end{bmatrix} \Delta u_k^{MPC}, \\ y_k = [\mathbf{0}_{n_{\bar{x}}} \quad 1] \begin{bmatrix} \Delta\bar{x}_k \\ y_k \end{bmatrix}, \end{gathered} \tag{10}$$

where it is assumed that the nominal system state $\bar{x}$ and output y are always available and measurable. If the following notation: $\overline{\boldsymbol{x}}_{k+1} = \begin{bmatrix} \Delta\bar{x}_{k+1} \\ y_k \end{bmatrix}$, $\boldsymbol{A} = \begin{bmatrix} A & \mathbf{0}_{n_{\bar{x}}}^T \\ CA & 1 \end{bmatrix}$, $\boldsymbol{B} = \begin{bmatrix} B \\ CB \end{bmatrix}$, $\boldsymbol{C} = [\mathbf{0}_{n_{\bar{x}}} \quad 1]$ is used, the condensed form of (10) is obtained as

$$\begin{gathered} \overline{\boldsymbol{x}}_{k+1} = \boldsymbol{A}\overline{\boldsymbol{x}}_k + \boldsymbol{B}\Delta u_k^{MPC}, \\ y_k = \boldsymbol{C}\overline{\boldsymbol{x}}_k, \end{gathered} \tag{11}$$

where $\overline{\boldsymbol{x}} \in \mathbb{R}^{n_x+1}$, $\Delta u \in \mathbb{R}$, $y \in \mathbb{R}$, and it will be used in the sequel for the nominal MPC design.

The nominal control input and its increment constraints are defined as a set of linear inequalities determined by (2) as

$$\begin{gathered} \Delta u_{\min}^{MPC} \le \Delta u_k^{MPC} \le \Delta u_{\max}^{MPC}, \\ u_{\min}^{MPC} \le u_k^{MPC} \le u_{\max}^{MPC}, \end{gathered} \tag{12}$$

where $\bullet_{\min}^{MPC}$ defines lower and $\bullet_{\max}^{MPC}$ upper limits on the appropriate control input signals at each time instant k. The control increment trajectory Δu_k^{MPC} is approximated by using a set of Malmquist functions

$$\Delta u_{k+1} = \sum_{j=1}^{N} c_k^j \varphi_i^j, \tag{13}$$

where $\left[c_k^1 \;\; c_k^2 \;\; \cdots \;\; c_k^N\right]^T = \tau_k$ is a vector of Malmquist functions coefficients. To obtain the optimal value of the control input increment Δu_k^{MPC}, the optimal Malmquist functions parameters τ_k have to be calculated and have to satisfy the new linear set of constraints defined by

$$\Psi\tau_k \le \Xi, \tag{14}$$

where

$$\Psi = \begin{bmatrix} \Phi_i^T \\ -\Phi_i^T \\ \sum_{j=0}^{i-1} \Phi_j^T \\ -\sum_{j=0}^{i-1} \Phi_j^T \end{bmatrix}, \; \Xi = \begin{bmatrix} \Delta u^{max} \\ -\Delta u^{min} \\ u^{max} - u_{k-1} \\ -u^{max} + u_{k-1} \end{bmatrix}. \tag{15}$$

The convexity of the objective function is provided by linearity in the decision variables. Many optimization routines can handle this type of optimization problem [39]. Now, the control input, calculated as

$$u_k^{MPC} = u_{k-1}^{MPC} + \Delta u_k^{MPC}, \tag{16}$$

is applied to the plant (1) and the nominal system (4). The deviation from nominal system dynamics caused by disturbance, parameter uncertainties and nonlinearities is cancelled by using the auxiliary control input u^{AUX} as it is defined in (6). To make sure that the control constraints (2) are respected by (6), the nominal constraints on the u^{MPC} input have to be narrowed. Under assumption that the disturbance is constant over the prediction horizon, the calculation method for offline constraints tightening [31] should be used. The design of the proposed auxiliary sliding mode controllers is described in the next subsection.

3.2. Auxiliary Sliding Mode Controllers

To apply the auxiliary sliding mode controller, one should consider the perturbed system dynamics (5). Two different SMC algorithms are used herein to suppress the disturbance action. The proposed auxiliary SMC laws are generally defined by

$$u^{AUX} = -(KB)^{-1}(KAz_k - g_k + \sigma(g_k)) \tag{17}$$

where $g_k = Kz_k$ is a switching function, $g_k = 0$ is a sliding surface, and K represents a vector of sliding surface parameters. In the first SMC algorithm, which is the traditional relay-based SMC, $\sigma(g_k)$ is given by

$$\sigma(g_k) = \Delta_{u^{smc}} sgn(g_k) \tag{18}$$

where $\Delta_{u^{SMC}}$ represents the relay constant.

If one substitute (17) and (18) in (5) implying that $g_k = Kz_k$, the switching function dynamics in the prediction horizon is then defined by

$$g_{k+i+1} = g_{k+i} - \Delta_{u^{smc}} sgn(g_{k+i}) + KEw_{k+i};\ i \in \{0, 1, \ldots, N\}. \tag{19}$$

The second used auxiliary controller component is the chattering-free SMC, where $\sigma(g_k)$ is described as [35, 40]

$$\sigma(g_k) = \min(|g_k|, \Delta_{u^{SMC}})\,\mathrm{sgn}(g_k), \tag{20}$$

Substituting (17) and (20) into (5), using $g_k = Kz_k$ yields

$$g_{k+i+1} = g_{k+i} - \min(I|g_{k+i}|, \Delta_{u^{SMC}})\,\mathrm{sgn}(g_{k+i}) + KEw_{k+i},\ i \in \{0, 1, \ldots, N\}. \tag{21}$$

Equations (19) and (21) defines the switching function dynamics inside the prediction horizon. The following Theorem defines the existence conditions of a sliding mode.

THEOREM [31]. *If $\Delta_{u^{SMC}}$ satisfies the inequality*

$$\Delta_{u^{SMC}} > \Omega > \max|KEw_k|, \tag{22}$$

where Ω is a positive real number, there exists a positive integer number $k_0 = k_0(g_k) < N$ for every initial state g_k, such that the system phase trajectory $g_{k+i+1}, i \in \{0,1,\ldots,N\}$, enters the domain defined by $G = \{g_{k+i}: |g_{k+i}| < \Delta_{u^{SMC}} + \Omega\}$, after k_0 timesteps and stays there for all $i > k_0$.

This condition ensures stable switching function dynamics. The Proof of the Theorem for both sliding mode auxiliary controllers can be found in [31, Appendices A and B].

The closed loop system dynamics in sliding mode is then described by

$$\begin{gathered} z_{k+1} = (A - B(KB)^{-1}K(A - I)z_k \\ Kz_k = 0. \end{gathered} \tag{23}$$

when the equivalent control $u_{eq}^{AUX} = -(KB)^{-1}(KAz_k - g_k)$ is applied in (5) [40].

4. EXPERIMENTAL RESULTS

The proposed TMPC algorithms were applied to the modular DC motor servo system [41]. This experimental setup consists of a DC motor, tachogenerator, encoder, and inertia load of 2 kg connected to the motor (see Fig. 3). It enables the rapid real-time application of different control algorithms. The system is linked to MATLAB/Simulink using an RT-DAC4/USB board. The DC motor is controlled by a pulse-width modulation

Fig. 3 – DC servo system setup.

(PWM) signal that is modulated based on the scaled input voltage $U(t) = V(t)/V_{max}$, $-1 \le U(t) \le 1$, $V_{max} = 12[V]$. The accompanying toolbox allows the identification of the DC motor parameters needed for deriving an appropriate mathematical model. The nonlinearity existing in DC servo motor angular position tasks, is unmodeled disturbance in the form of Coulomb's friction described by $w = F_c \text{sign}(x_2)$. The dead zone which also affects the control signal is between –0.15 and +0.15.

By neglecting these static and dry frictions, as well as saturation, the transfer function of this system can be represented as

$$G(s) = \frac{\theta(s)}{u(s)} = \frac{K_s}{s(T_s s + 1)} \tag{24}$$

Using the system identification tool, the DC motor gain $K_s = 184.73$ and time constant $T_s = 0.9$s are obtained. By applying the sampling period $T = 0.01s$ and the zero-order hold circuit, and by choosing the state variables as angular position, $x_1 = \theta$, measured in [rad], and angular velocity, $x_2 = \dot{\theta} = \omega$, measured in [rad/s], the following matrices of the discrete-time state-space model are calculated

$$A = \begin{bmatrix} 1 & 0.0099 \\ 0 & 0.9890 \end{bmatrix},\ B = \begin{bmatrix} 0.0102 \\ 2.0412 \end{bmatrix},\ C = [1 \quad 0]. \tag{25}$$

The parameters for the nominal Malmquist functions-based MPC are the following:

- the prediction horizon $N_p = 180$,
- the number of Malmquist network terms $N = 4$, and
- the Malmquist functions parameters $a_1 = \frac{1}{2}, a_2 = \frac{1}{3}, a_3 = \frac{1}{4}, a_4 = \frac{1}{5}$.

The SMC parameter $\Delta_{u^{SMC}}$ is chosen to cope with DC motor servo system uncertainties. A standard approach [40] to the design of the sliding surface parameters K is used that is based on the transformation of the original system model (5) into the normal form. The system dynamics in sliding mode is defined by eigenvalue $z = \exp(-\alpha T)$ with $\alpha = 1.9$. Parameters for both SMC auxiliary controllers are selected as

- $\Delta_{u^{SMC}} = 0.3$, and
- $K = [-0.9220 \quad -0.4853]$.

The control input constraints are determined from the physical system limitations which are defined by

$$-1 \le u_k \le 1 \tag{26}$$

As explained in the previous section, the control input is divided into two components and both of them have to be constrained separately to satisfy (14). The constraints on Malmquist functions-based MPC and its increments are

$$-0.7 \le u_k^{MPC} \le 0.7, -0.25 \le \Delta u_k^{MPC} \le 0.25. \tag{27}$$

The DC servo motor angular position θ response is presented in Fig. 4. In the first experiment, the DC motor servo system is controlled by the proposed TMPC with traditional relay-based SMC (18). It is shown that the output reaches the reference signal in less than 300 time steps. Figures 5 and 6 depict the nominal Malmquist functions-based MPC signal and its increment trajectories. It is shown that both signals Δu^{MPC} and u^{MPC} do not violate constraints defined by (27). The disturbance is rejected, but there is chattering in the SMC signal (Fig. 7).

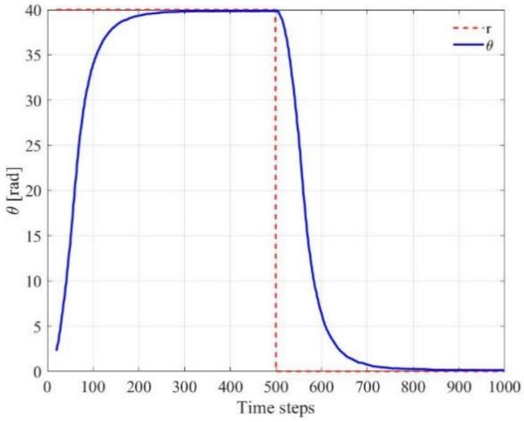

Fig. 4 – Angular position θ.

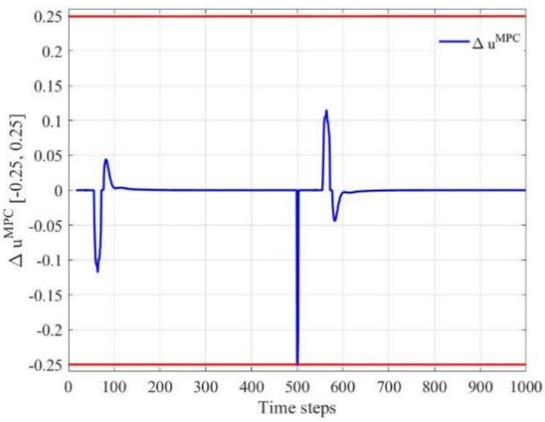

Fig. 5 – Nominal MPC input increment Δu_k^{MPC}.

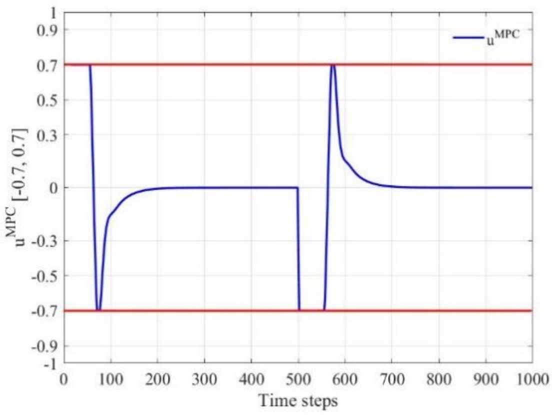

Fig. 6 – Nominal MPC input Δu_k^{MPC}.

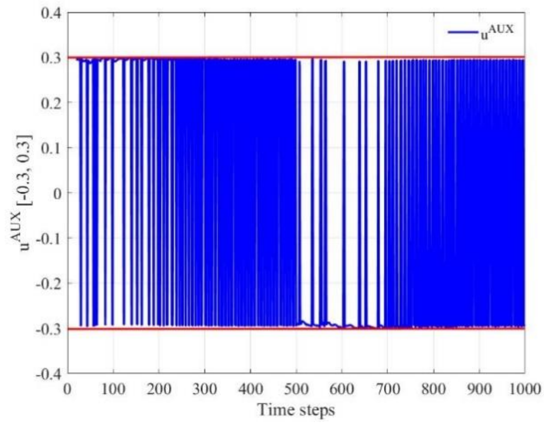

Fig. 7 – Auxiliary SMC input Δu_k^{AUX}.

The next experiment utilizes the control algorithm with the nominal MPC defined by (16) along with the auxiliary chattering-free SMC component (21). The angular position θ response is shown in Fig. 8. The nominal Malmquist functions-based MPC input and its increment are the same as in the previous experiment (Figs. 5 and 6). Figure 8 demonstrates that the used auxiliary component effectively suppresses disturbance, and also reduces the chattering phenomenon. The applied auxiliary sliding mode controller input is presented in Fig. 9.

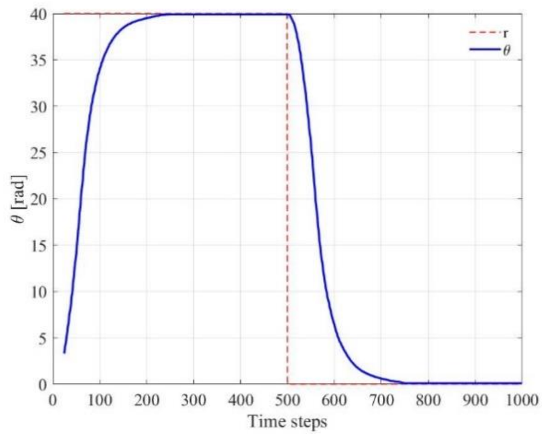

Fig. 8 – Angular position θ.

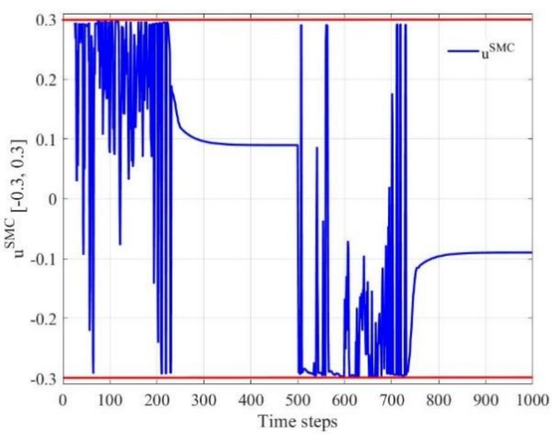

Fig. 9 – Auxiliary SMC input Δu_k^{AUX}.

In order to emphasis and justify the robustness properties of the proposed TMPC, gained by using the auxiliary SMC, an experiment is done where only nominal MPC is applied to the DC motor servo system. In this set-up, the constraints on Malmquist functions-based MPC input and its increment are

$$-1 \le u_k^{MPC} \le 1, \quad -0.25 \le \Delta u_k^{MPC} \le 0.25. \tag{28}$$

Figure 10 depicts the nominal angular position θ_{nom} and the angular position of the real-time DC motor servo system θ responses. One can see that there is discrepancy of around 12 rad between the responses of the nominal model and real system outputs. This highlights the nominal MPC's lack of robustness when applied to a real-time DC servo system with disturbance.

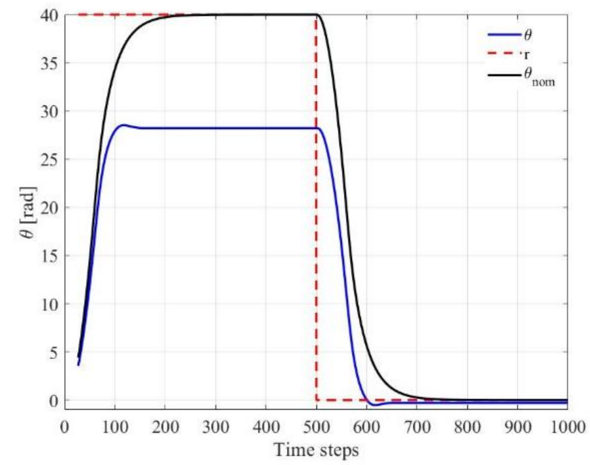

Fig. 10 – Angular positions θ and θ_{nom}.

To demonstrate the ability of Malmquist orthogonal functions to better tune the nominal MPC, the comparison of Laguerre functions based MPC (LbMPC) [34] with the proposed Malmquist functions based MPC (MbMPC) is performed with the same number of terms N and also the same value of the prediction horizon N_p of both control methods., The advantage of MbMPC is in the fact that more distinct function parameters can be used for better (fine) tuning of the nominal controller. For LbMPC, all parameters have the same value, *i.e.* $a_1 = a_2 = a_3 = a_4 = a = \frac{1}{2}$. The parameters of the MbMPC are defined as in the first experiment $a_1 = \frac{1}{2}, a_2 = \frac{1}{3}, a_3 = \frac{1}{4}, a_4 = \frac{1}{5}$. Figure 11 shows slightly faster output response of the proposed nominal MbMPC.

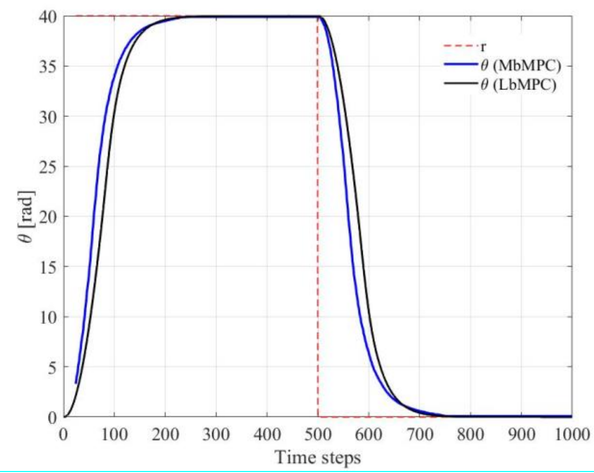

Fig. 11 – Angular positions θ for LbMPC and MbMPC.

5. CONCLUSION

This paper examines the tube model predictive control (TMPC) utilizing generalized discrete-time Malmquist orthogonal functions with a sliding mode control (SMC) as an auxiliary controller applied to enhance the robustness of DC motor servo system. Usage of Malmquist orthogonal function parameters as decision variables reduces computational complexity of the proposed TMPC, and significantly decreases the optimization problem size. In that manner, such approach enables handling a large control horizon with low computational burden. Offline tightening constraints on the control input of the nominal model predictive control (MPC) component has been performed due to the presence of SMC term. The SMC algorithms, both the traditional and the chattering-free ones, are implemented as auxiliary control components to reduce disturbances, parameter variations and effects of nonlinearities and enhance the performance of the real DC motor servo system. The proposed TMPC formulation demonstrates the efficient online solving of constrained optimization problems for Malmquist functions based MPC and the straightforward implementation of the proposed SMC methods. This approach is demonstrated by conducting several real-time experiments on the modular DC motor servo system where satisfactory system dynamical behaviour are attained.

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