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ROBUST ADAPTIVE ASYMMETRIC STATE CONSTRAINT CONTROL OF BRUSH DC MOTOR SYSTEMS DRIVING A ONE-LINK ROBOT MANIPULATOR: A FEASIBILITY-CONDITION-FREE METHOD

Yongcheng ZHOU[1], Yuan LI[2], Ao LIU[1], Liyuan YANG[3]

[1] Chongqing University, School of Automation, Chongqing 400030, China.
[2] Wuhan Polytechnic, School of Mechanical and Electrical Engineering, 430072, China.
[3] Anhui University, School of Artificial Intelligence, Hefei 230090, China.
Corresponding author: Liyuan YANG, E-mail: liyuan.yang@stu.ahu.edu.cn

Abstract. In this paper, we investigate the tracking control problem for a class of brush direct current (DC) motor systems driving a one-link robot manipulator subject to asymmetric full-state constraints. By constructing a state-dependent nonlinear transformation function (NTF), we present an adaptive robust dynamic surface control (DSC) strategy that can directly address both symmetric and asymmetric state constraints, so that there is no need to convert the problem of state constraint into the constraints on tracking errors as necessitated by the Barrier Lyapunov Function (BLF)-based existing works. Furthermore, by employing the first-order filter and constructing a new coordinate transformation, the demanding feasibility condition imposed on the BLF methods is removed, allowing the designer more freedom to select design parameters. Moreover, it is worth mentioning that under the proposed nonlinear transformation function the extra condition on the constraining function is not required. The effectiveness of the proposed control is verified via the Simulation results.

Keywords: tracking control, states constraints, brush DC motor systems, Lyapunov stability analysis.

1. INTRODUCTION

For the practical engineering systems [1–3], they face challenges in constrained operation for stability and performance, i.e., the permanent magnet brush DC motor systems [4–9]. These complexities have attracted extensive research, leading to innovative strategies for state constraint control, advancing related fields. Beginning with the control techniques, reference [4] introduced the concept of integrator backstepping control to warrant the load position tracking performance for DC systems in the presence of parameter uncertainties. Rauf et al. [10] employed a continuous non-singular terminal sliding mode control for the converter-driven DC motor systems to guarantee the system performance. Based on the state observer technique, Yao et al. [11] presented a output-feedback robust adaptive control method to cope with the structured and unstructured uncertainties. However, in the aforementioned works the problem of state/output constraint is not considered in the control design and stability analysis. If the constraint is not properly accommodated [12–14], it might result in control inaccuracy, system instability, or even accident, rendering the underlying control problem for DC systems extremely critical and challenging.

To solve the problem of state constraint and meanwhile to guarantee the closed-loop stability of the DC systems, there are fruitful results in recent years. Reference [15] provided explicit expressions for the controllability time and lower bounds based on low-dimensional system transformations so that the convex state constraints can be ensured. Recently, BLF has been employed to handle state constraints for nonlinear systems in Brunovsky form [16]. Such a function yields a value that approaches infinity whenever its arguments approach some limits. Inspired by this idea, the authors in [17] proposed a BLF-based adaptive control algorithm for DC systems to guarantee the time-varying state constraints. By developing an extended disturbance observer, Yang et al in [18] developed a BLF-based adaptive control algorithm for DC motors in the presence of uncertain disturbance, so that the time-varying output constraints can be ensured. However, it is worth noting that under the above BLF-based works, the original output/state constraints are handled indirectly by imposing transformed constraints on the errors, which imposes extra requirements on the initial states. To handle this problem, by employing the integral BLF, Liu et al. [19,20] introduced an adaptive control approach to guarantee the time-varying state constraints directly without the need of error constraint transformation. However, current BLF based state-constrained control methods for DC motor systems may involve the feasibility conditions on virtual controllers, in other words, the virtual controllers must satisfy certain pregiven constrained region, which poses significant difficulty for the design and implementation of the corresponding control schemes. In fact, as implied in the pioneering work [21], only if we are able to find a set of design parameters that satisfies the conditions, the BLF based methods involving those parameters would be feasible. Clearly, the existence of the design parameters satisfying the feasibility conditions is crucial for most existing BLF methods, which, prior to the implementation, require offline constrained optimization to verify and to obtain the optimal design parameters [22–24]. This is a highly undesirable process. It is therefore highly desirable to remove such restrictive conditions for control design. Although the nonlinear transformation function has been proposed in [22], there is no guidance on how to design a robust adaptive control method for DC motor systems.

In this paper, inspired by the previous work [23], we present a robust adaptive control method for the brush DC motor systems turning a robotic load in the presence of state constraints without involving the demanding feasibility conditions. The main contributions of this work can be summarised as follows:

- Firstly, by introducing a state-dependent nonlinear transformation function (NTF), the original state-constrained system is converted into an equivalent "unrestricted" system, so that the corresponding proposed control framework can directly address both symmetric and asymmetric state constraints, so there is no need to convert the state constraints into the constraints on tracking errors;
- Secondly, by employing the first-order filter and constructing a new coordinate transformation, the demanding feasibility condition imposed on the BLF methods is removed, allowing the designer more freedom to select design parameters. Furthermore, it is worth mentioning that under the proposed nonlinear transformation function the extra condition on the constraining function is not required.

The remainder of this paper is structured as follows: Section 2 delineates the problem formulation alongside pertinent preliminaries and assumptions. Section 3 articulates the development of a control scheme tailored to address state constraints. Section 4 validates the efficacy of the proposed algorithm through simulation results. The paper culminates with Section 5, which provides concluding remarks. Throughout this paper, $\mathbb{R}$ denotes the set of real numbers, $\mathbb{R}_+ := [0,+\infty)$ denotes the non-negative real numbers, and $\mathbb{R}^{m\times n}$ denotes the set of $m\times n$ real matrices. $\|\cdot\|$ represents the Frobenius norm for matrices and Euclidean norm for vectors, $|\cdot|$ is the absolute value of real numbers. $\mathbb{N}_+$ is a set including all positive integers.

2. PROBLEM FORMULATION AND PRELIMINARIES

In this paper, we consider the bursh DC motor systems driving a one-link robot manuipulator as the following form [4]:

$$\begin{cases} M\ddot{q}+B\dot{q}+N\sin(q)=I, \\ L\dot{I}=u_e-RI-K_B\dot{q}, \end{cases} \tag{1}$$

where

$$M=\frac{J}{K_\eta}+\frac{mL^2}{3K_\eta}+\frac{ML^2}{K_\eta}+\frac{2MR^2}{5K_\eta},\ \ B=\frac{b}{K_\eta},\ \ N=\frac{mLG}{2K_\eta}+\frac{MLG}{K_\eta},$$

where $J\in\mathbb{R}$ is the rotor inertia, $m\in\mathbb{R}$, $M\in\mathbb{R}$, and $L\in\mathbb{R}$ are the link mass, load mass and link length, respectively, $G\in\mathbb{R}$ is the gravity coefficient, $b\in\mathbb{R}$ is the coefficient of viscous friction at the joint, $q\in\mathbb{R}$ is the angular motor position, $I\in\mathbb{R}$ is the motor armature current, and $K_\eta\in\mathbb{R}$ is the coefficient which characterizes the electromechanical conversion of armature current to torque function, $R\in\mathbb{R}$ is the armature resistance, K_B is the back-emf coefficient, and $u_e\in\mathbb{R}$ is the input control voltage.

Let $x_1 = q$, $x_2 = \dot{q}$, $x_3 = I$, $u = u_e$, then the system in (1) can be transformed to the following strict-feedback form:

$$\begin{cases} \dot{x_1} = x_2, \\ \dot{x_2} = g_2 x_3 + \theta_1 \sin(x_1) + \theta_2 x_2, \\ \dot{x_3} = g_3 u + \theta_3 x_2 + \theta_4 x_3. \end{cases} \tag{2}$$

where x_1, x_2, x_3 denote the system states, $u \in \mathbb{R}$ and $y \in \mathbb{R}$ represent the control input and system output, respectively, $g_2 = \frac{1}{M}$, $g_3 = \frac{1}{L}$, $\theta_1 = \frac{-N}{M}$, $\theta_2 = \frac{-B}{M}$, $\theta_3 = \frac{-K_B}{L}$, and $\theta_4 = \frac{-R}{L}$. In practice, the considered permanent magnet brush DC motor is subject to the following full-state constraints defined by:

$$x_i \in D_i := \{(t, x_i) \in \mathbb{R}_+ \times \mathbb{R} \,|\, \kappa_{il}(t) < x_i < \kappa_{ih}, \;\; \kappa_{il}, \; \kappa_{ih} \in \mathbb{R}\}, \tag{3}$$

where $x_i, (i = 1, 2, 3)$ is the system state, $x_i(0) \in D_i$ is the initial value of state. The lower constraining function κ_{il} and upper constraining function κ_{ih} belong to:

$$\Theta := \left\{ \kappa_{il}(t) : \mathbb{R}^+ \to \mathbb{R}, \; \kappa_{ih}(t) : \mathbb{R}^+ \to \mathbb{R} \mid \kappa_{il}(t) < \kappa_{ih}(t) \right\} \tag{4}$$

which are governed/generated dynamically by:

$$\dot{\kappa}_{il} = \hat{h}_{il}(t, \kappa_{il}), \quad \kappa_{il}(0) \in \Omega_{il}, \quad i = 1, 2, 3 \tag{5}$$

$$\dot{\kappa}_{ih} = \hat{h}_{ih}(t, \kappa_{ih}), \quad \kappa_{ih}(0) \in \Omega_{ih}, \quad i = 1, 2, 3, \tag{6}$$

for all $(t, \kappa_{ij}) \in \mathbb{R}_+ \times \mathbb{R}, j = l, h$, where $\kappa_{il}(0)$ and $\kappa_{ih}(0)$ are the initial values of constraining functions, and Ω_{il}, Ω_{ih} are some known bounded compact sets. For practicality, the stability assumption is extended to hold for all (t, κ_{ij}) such that κ_{ij}, $\dot{\kappa}_{ij}$, and $\ddot{\kappa}_{ij}$ are continuous and bounded. It is important to highlight that the state constraints introduced by (3)–(6) are inherently dynamic and asymmetric, which differs from the existing studies that deal with symmetric and static state constraints or predefined time-varying constraints.

In this paper, we propose a robust adaptive control methodology for nonlinear DC motor systems as articulated in (1) so that: 1) all signals in the closed-loop systems are bounded; and 2) the time-varying yet asymmetric state constraints (3) are guaranteed without involving the demanding feasibility conditions.

To this end, we impose the following assumptions.

Assumption 1. There exist positive constants μ_i such that $\kappa_{ih}(t) - \kappa_{il}(t) \geq \mu_i > 0, \quad i = 1, 2, 3.$

Assumption 2. The reference trajectory and its derivatives up to second order are known and bounded. In addition, there exist time-varying functions $\kappa_{dl}(t)$, $\kappa_{dh}(t)$ satisfying $\kappa_{dl}(t) < \kappa_{dh}(t)$ and positive constants θ_d, θ_d such that $\kappa_{1h}(t) - \kappa_{dh}(t) \geq \theta_d > 0$, $\kappa_{dl}(t) - \kappa_{1l}(t) \geq \theta_d > 0$, and $y_d \in U_d := \{(t, y_d) \in [0, \infty) \times \mathbb{R} : \kappa_{dl}(t) \leq y_d \leq \kappa_{dh}(t)\}$.

3. MAIN RESULTS

3.1. Nonlinear transformation function

To prevent the system states from violating the constraints dynamically generated by (3)-(6), we introduce a nonlinear transformation function, defined as follows:

Definition 1 [22]. A scalar function ρ of the variable x on an open region U is a nonlinear transformation function (NTF) if it can be used to handle the constraining all cases simultaneously without the need for changing the function structure; and It exhibits the property that $\rho \to \pm\infty$ as x approaches the boundary of U and satisfies $\rho \leq B$ for all $x \in U' \subsetneq U$ under $x(0) \in U$, where B represents some bounded constant and U' is a closed interval.

Now we construct such a state-dependent NTF as follows:

$$\rho_1 = \frac{x_1 - \overline{\kappa}_{1l}}{x_1 - \kappa_{1l}(t)} + \frac{x_1 - \underline{\kappa}_{1h}}{\kappa_{1h}(t) - x_1}, \tag{7}$$

and

$$\rho_i = \frac{x_i - \overline{\kappa}_{il}}{x_i - \kappa_{il}(t)} + \frac{x_i - \underline{\kappa}_{ih}}{\kappa_{ih}(t) - x_i}, \quad i = 2,3 \tag{8}$$

with the initial states satisfying $x_i(0) \in U_i$ and the constants $\overline{\kappa}_{il}$ and $\underline{\kappa}_{ih}$ obeying the following inequalities:

$$\begin{cases} \kappa_{il}(t) < \overline{\kappa}_{il}, \\ \underline{\kappa}_{ih} < \kappa_{ih}(t) \end{cases} \tag{9}$$

It is clearly seen that the proposed NTF ρ_i as defined in (7)-(8) satisfy the property in Definition 1, i.e., for any initial states $x_i(0) \in U_i$,

$$\begin{cases} \rho_i \to -\infty \text{ if and only if } x_i \to \kappa_{il}(t), \\ \rho_i \to +\infty \text{ if and only if } x_i \to \kappa_{ih}(t). \end{cases} \tag{10}$$

Therefore, it can be deduced that for any initial conditions $x_i(0) \in U_i$, as long as the values of ρ_i are confined within the specific bounds via the appropriate control, the system states will persist within the respective regions U_i. In other words, the state constraints are guaranteed. Inspired by this insight, we now turn our attention to addressing the challenge of dynamically imposed asymmetric state constraints by preserving the boundedness of ρ_i.

Note that the expression of ρ_i as given in (7)–(8) can be rewritten as:

$$\rho_i = \rho_{i1} x_i + \rho_{i2}, \tag{11}$$

where

$$\rho_{i1} = \frac{\overline{\kappa}_{il} - \kappa_{il} + \kappa_{ih} - \underline{\kappa}_{ih}}{(x_i - \kappa_{il})(\kappa_{ih} - x_i)}, \text{ and } \rho_{i2} = \frac{\kappa_{il}\underline{\kappa}_{ih} - \overline{\kappa}_{il}\kappa_{ih}}{(x_i - \kappa_{il})(\kappa_{ih} - x_i)} \tag{12}$$

Taking the derivative of ρ_i in (11) w.r.t. time yields:

$$\dot{\rho}_i = \mu_{i1}\dot{x}_i + \mu_{i2}, \tag{13}$$

$$\mu_{i1} = \frac{x_i^2 - \kappa_{il}\kappa_{ih}}{(x_i - \kappa_{il})^2(\kappa_{ih} - x_i)^2}, \quad \mu_{i2} = \frac{(\kappa_{ih}\dot{\kappa}_{il} + \dot{\kappa}_{ih}\kappa_{il})x_i - (\dot{\kappa}_{ih} + \dot{\kappa}_{il})x_i^2}{(x_i - \kappa_{il})^2(\kappa_{ih} - x_i)^2}. \tag{14}$$

As $\rho_2 = \rho_{21}x_2 + \rho_{22}$, then one has

$$x_2 = \frac{\rho_2 - \rho_{22}}{\rho_{21}}, \tag{15}$$

we further have

$$\dot{\rho}_1 = \mu_{11}x_2 + \mu_{12} = \mu_{11}\left(\frac{1}{\rho_{21}}\rho_2 - \frac{\rho_{22}}{\rho_{21}}\right) + \mu_{12} \tag{16}$$

where the fact that $\dot{x}_1 = x_2$ is used.

3.2. Control design & stability analysis

We now focus on constructing the control framework for DC motor systems in the presence of state constraints. Since backstepping technique is the most effective method for strict-feedback/pure-feedback systems [25–28], then we can conduct the control design with backstepping technique step by step in this work. To directly handle the dynamic yet asymmetric state constraints and to remove the feasibility conditions on the virtual controllers in the existing works, we employ the following coordinate transformations:

$$\begin{cases} z_1 = \rho_1 - \rho_d, \\ z_2 = \rho_2 - \alpha_{2f}, \\ z_3 = \rho_3 - \alpha_{3f}, \end{cases} \tag{17}$$

where $\rho_d = \frac{y_d-\overline{\kappa}_{1l}}{y_d-\kappa_{1l}(t)} + \frac{y_d-\kappa_{1h}}{\kappa_{1h}(t)-y_d}$ is a computable function for control design, α_{if}, $i=2,3$, is the output of the following first-order filter:

$$\begin{cases} \xi_2\dot{\alpha}_{2f}+\alpha_{2f}=\rho_{21}\alpha_1, \\ \xi_3\dot{\alpha}_{3f}+\alpha_{3f}=\rho_{31}\alpha_2, \end{cases} \tag{18}$$

where α_1 and α_2 are the virtual controllers which will be designed later.

To facilitate the control design, we define the following filtered errors:

$$\begin{cases} y_2=\alpha_{2f}-\rho_{21}\alpha_1, \\ y_3=\alpha_{3f}-\rho_{31}\alpha_2, \end{cases} \tag{19}$$

then it is seen from (17) and (19) that

$$\rho_i = z_i+\alpha_{if} = z_i+y_i+\rho_{i1}\alpha_{i-1}, \quad i=2,3. \tag{20}$$

Step 1. According to (19), the equation of (16) can be written as:

$$\dot{\rho}_1 = \mu_{11}\left[\frac{1}{\rho_{21}}(z_2+y_2)+\alpha_1-\frac{\rho_{22}}{\rho_{21}}\right]+\mu_{12}, \tag{21}$$

then the derivative of the first virtual error $z_1=\rho_1-\rho_d$ with respect to time is

$$\dot{z}_1 = \dot{\rho}_1-\dot{\rho}_d = \mu_{11}\alpha_1+\mu_{11}\left[\frac{1}{\rho_{21}}(z_2+y_2)-\frac{\rho_{22}}{\rho_{21}}\right]+\mu_{12}-\dot{\rho}_d \tag{22}$$

where

$$\begin{aligned} \dot{\rho}_d &= \mu_{1d}\dot{y}_d+\mu_{2d}, \\ \mu_{1d} &= \frac{y_d^2-\kappa_{1l}\kappa_{1h}}{(y_d-\kappa_{1l})^2(\kappa_{1h}-y_d)^2}, \\ \mu_{2d} &= \frac{(\kappa_{1h}\dot{\kappa}_{1l}+\dot{\kappa}_{1h}\kappa_{1l})y_d-(\dot{\kappa}_{1h}+\dot{\kappa}_{1l})y_d^2}{(y_d-\kappa_{1l})^2(\kappa_{1h}-y_d)^2}. \end{aligned}$$

Then the derivative of the quadratic function $\frac{1}{2}z_1^2$ is

$$z_1\dot{z}_1 = z_1\mu_{11}\alpha_1+\Delta_1, \tag{23}$$

with

$$\Delta_1 = z_1\mu_{11}\left[\frac{1}{\rho_{21}}(z_2+y_2)-\frac{\rho_{22}}{\rho_{21}}\right]+z_1\mu_{12}-z_1\dot{\rho}_d$$

Upon employing Young's inequality, one has

$$z_1\mu_{11}\frac{1}{\rho_{21}}z_2 \le g_2\mu_{11}^2\frac{1}{\rho_{21}^2}z_1^2z_2^2+\frac{1}{4g_2}, \tag{24}$$

$$z_1\mu_{11}\frac{1}{\rho_{21}}y_2 \le \mu_{11}^2z_1^2+\frac{1}{4\rho_{21}^2}y_2^2, \tag{25}$$

$$-z_1\mu_{11}\frac{\rho_{22}}{\rho_{21}} \le z_1^2\mu_{11}^2\left(\frac{\rho_{22}}{\rho_{21}}\right)^2+\frac{1}{4}, \tag{26}$$

$$z_1\mu_{12} \le z_1^2\mu_{12}^2+\frac{1}{4}, \tag{27}$$

$$-z_1\dot{\rho}_d \le z_1^2\dot{\rho}_d^2+\frac{1}{4}. \tag{28}$$

Hence, Δ_1 can be upper bounded by

$$\Delta_1 \leqslant z_1^2\Phi_1 + g_2\mu_{11}^2\frac{1}{\rho_{21}^2}z_1^2z_2^2 + \frac{1}{4\rho_{21}^2}y_2^2 + \frac{3}{4} + \frac{1}{4g_2}, \tag{29}$$

where

$$\Phi_1 = \dot{\rho}_d^2 + \mu_{12}^2 + \mu_{11}^2\left(\frac{\rho_{22}}{\rho_{21}}\right)^2 + \mu_{11}^2 \tag{30}$$

is a computable function. Then (23) can be further expressed as

$$z_1\dot{z}_1 \leqslant z_1\mu_{11}\alpha_1 + z_1^2\Phi_1 + g_2\mu_{11}^2\frac{1}{\rho_{21}^2}z_1^2z_2^2 + \frac{1}{4\rho_{21}^2}y_2^2 + \frac{3}{4} + \frac{1}{4g_2} \tag{31}$$

The virtual controller α_1 is designed as

$$\alpha_1 = \frac{1}{\mu_{11}}(-c_1z_1 - z_1\Phi_1) \tag{32}$$

with $c_1 > 0$ being a design parameter.

Substituting the virtual controller as shown in (32) into (31), we have

$$z_1\dot{z}_1 \leq -c_1z_1^2 + g_2\mu_{11}^2\frac{1}{\rho_{21}^2}z_1^2z_2^2 + \frac{1}{4\rho_{21}^2}y_2^2 + \frac{3}{4} + \frac{1}{4g_2}. \tag{33}$$

Choosing the first Lyapunov function candidate as:

$$V_1 = \frac{1}{2}z_1^2 + \frac{1}{2}y_2^2 \tag{34}$$

then the derivative of V_1 along (33) is

$$\dot{V}_1 = z_1\dot{z}_1 + y_2\dot{y}_2 \leq -c_1z_1^2 + g_2\mu_{11}^2\frac{1}{\rho_{21}^2}z_1^2z_2^2 + \frac{1}{4\rho_{21}^2}y_2^2 + \frac{3}{4} + \frac{1}{4g_2} + y_2\dot{y}_2. \tag{35}$$

Noting that

$$\dot{y}_2 = \dot{\alpha}_{2f} - (\rho_{21}\alpha_1)' = -\frac{y_2}{\varepsilon_2} + h_1(\cdot),$$

where h_1 is a continuous function over the compact set, then one further has

$$y_2\dot{y}_2 \leqslant \left(\frac{1}{4} - \frac{1}{\varepsilon_2}\right)y_2^2 + h_1^2, \tag{36}$$

then (35) can be expressed as

$$\dot{V}_1 \leq -c_1z_1^2 + g_2\mu_{11}^2\frac{1}{\rho_{21}^2}z_1^2z_2^2 + \left(\frac{1}{4} + \frac{1}{4\rho_{21}^2} - \frac{1}{\varepsilon_2}\right)y_2^2 + \frac{3}{4} + \frac{1}{4g_2} + h_1^2 \tag{37}$$

Let

$$\frac{1}{\varepsilon_2} = \frac{1}{4} + \frac{1}{4\rho_{21}^2} + \varepsilon_2^*$$

with $\varepsilon_2^* > 0$ being an arbitrary constant, we arrive at:

$$\dot{V}_1 \leqslant -c_1z_1^2 - \varepsilon_2^*y_2^2 + g_2\mu_{11}^2\frac{1}{\rho_{21}^2}z_1^2z_2^2 + \frac{3}{4} + \frac{1}{4g_2} + h_1^2.$$

Step 2. Differentiating the second virtual error $z_2 = \rho_2 - \alpha_{2f}$, we obtain

$$\dot{z}_2 = \dot{\rho}_2 - \dot{\alpha}_{2f} = \mu_{21}\dot{x}_2 + \mu_{22} - \dot{\alpha}_{2f} = \mu_{21}\left[g_2 x_3 + \theta_1 \sin(x_1) + \theta_2 x_2\right] + \mu_{22} - \dot{\alpha}_{2f} \tag{38}$$

As

$$x_3 = \frac{1}{\rho_{31}}(z_3 + y_3) + \alpha_2 - \frac{\rho_{32}}{\rho_{31}},$$

then one has

$$\begin{aligned} \dot{z}_2 &= \mu_{21}\left[g_2 \cdot \frac{1}{\rho_{31}}(z_3 + y_3) + g_2\alpha_2 - g_2\frac{\rho_{32}}{\rho_{31}} + \theta_1 \sin(x_1) + \theta_2 x_2\right] + \mu_{22} - \dot{\alpha}_{2f} \\ &= \mu_{21} g_2 \alpha_2 + \mu_{21}\left[\frac{g_2}{\rho_{31}}(z_3 + y_3) - \frac{g_2\rho_{32}}{\rho_{31}} + \theta_1 \sin(x_1) + \theta_2 x_2\right] + \mu_{22} - \dot{\alpha}_{2f} \end{aligned} \tag{39}$$

and

$$z_2\dot{z}_2 = \mu_{21} g_2 z_2 \alpha_2 + \Delta_2 \tag{40}$$

where

$$\Delta_2 = z_2\mu_{21}\left[\frac{g_2}{\rho_{31}}(z_3 + y_3 - \rho_{32}) + \theta_1 \sin(x_1) + \theta_2 x_2\right] + z_2\mu_{22} - z_2\dot{\alpha}_{2f}.$$

Choose the second the Lyapunov function candidate as

$$V_2 = V_1 + \frac{1}{2}z_2^2 + \frac{1}{2}y_3^2 + \frac{1}{2r_2}\tilde{b}_2^2 \tag{41}$$

where $r_2 > 0$ is a design parameter, $\tilde{b}_2 = b_2 - \hat{b}_2$ is the parameter estimate error with $b_2 = \max\{g_2, \theta_0^2, 1\}$ being an unknown parameter and $\hat{b}$ being the parameter estimate. Its derivative is

$$\dot{V}_2 = \dot{V}_1 + z_2\dot{z}_2 + y_3\dot{y}_3 - \frac{1}{r_2}\tilde{b}_2\dot{b}_2 \leqslant -c_1 z_1^2 - \varepsilon_2^* y_2^2 + \frac{3}{4} + \frac{1}{4g_2} + h_1^2 + \mu_{21} g_2 z_2 \alpha_2 + \Delta_2^1 + y_3\dot{y}_3 - \frac{1}{r_2}\tilde{b}_2\dot{\hat{b}}_2 \tag{42}$$

where $\Delta_2^1 = \Delta_2 + g_2\mu_{11}^2\frac{1}{\rho_{21}^2}z_1^2 z_2^2$.

Note that

$$\begin{aligned} z_2\mu_{21}\frac{g_2}{\rho_{31}}z_3 &\leq g_3\mu_{21}^2\frac{1}{\rho_{31}^2}z_2^2 z_3^2 + \frac{g_2^2}{4g_3}, \\ z_2\mu_{21}\frac{g_2}{\rho_{31}}y_3 &\leq g_2^2\mu_{21}^2 z_2^2 + \frac{1}{4\rho_{31}^2}y_3^2, \\ -z_2\mu_{21}\frac{g_2}{\rho_{31}}\rho_{32} &\leq g_2 z_2^2\mu_{21}^2\left(\frac{\rho_{32}}{\rho_{31}}\right)^2 + \frac{g_2}{4}, \\ z_2\left(\mu_{22} - \dot{\alpha}_{2f}\right) &\leq g_2 z_2^2\left(\mu_{22} - \dot{\alpha}_{2f}\right)^2 + \frac{1}{4g_2}, \end{aligned}$$

then we have

$$\Delta_2^1 \leqslant g_2 b_2 z_2^2 \Phi_2 + g_3\mu_{21}^2\frac{1}{\rho_{31}^2}z_2^2 z_3^2 + \Gamma_2 + \frac{1}{4\rho_{31}^2}y_3^2 \tag{43}$$

where $b_2 = \max\{g_2, \theta_0^2, 1\}$, $\Gamma_2 = \frac{1}{2g_2} + \frac{1}{4} + \frac{g_2^2}{4g_3}$, and

$$\Phi_2 = \mu_{21}^2 + \mu_{21}^2\left(\frac{\rho_{32}}{\rho_{31}}\right)^2 + \left(1 + x_2^2\right)^2\mu_{21}^2 + \left(\mu_{22} - \dot{\alpha}_{2f}\right)^2 + \mu_{11}^2 \cdot \frac{1}{\rho_{21}^2}z_1^2$$

is a computable function.

Therefore (42) can be further rewritten as

$$\dot{V}_2 \leq -c_1 z_1^2 - \varepsilon_2^* y_2^2 + \frac{3}{4} + \frac{1}{4g_2} + h_1^2 + \mu_{21} g_2 z_2 \alpha_2 + g_2 b_2 z_2^2 \Phi_2 + g_3 \mu_{21}^2 \frac{1}{\rho_{31}^2} z_2^2 z_3^2 + \Gamma_2$$
$$+ \frac{1}{4\rho_{31}^2} y_3^2 + \left(\frac{1}{4} - \frac{1}{\varepsilon_3}\right) y_3^2 + h_2^2 - \frac{1}{r_2} \widetilde{b}_2 \dot{\hat{b}}_2. \tag{44}$$

Choosing the virtual controller α_2 as

$$\alpha_2 = \frac{1}{\mu_{21}} \left(-c_2 z_2 - \hat{b}_2 z_2 \Phi_2\right) \tag{45}$$

with c_2 being a positive parameter and $\hat{b}_2$ being the parameter estimate that is updated by

$$\dot{\hat{b}}_2 = r_2 z_2^2 \Phi_2 - \sigma_2 \hat{b}_2, \;\; \hat{b}_2(0) \geq 0 \tag{46}$$

where $\sigma_2 > 0$, $\hat{b}_2(0)$ is the arbitrarily chosen initial estimate $\hat{b}_2(t)$.

Let $\frac{1}{\varepsilon_3} = \frac{1}{4} + \frac{1}{4\rho_{31}^2} + \varepsilon_3^*$ with $\varepsilon_3^* > 0$ being a positive constant, then substituting the virtual controller and adaptive law as shown in (45)–(46) into (44), we have

$$\dot{V}_2 \leqslant -\sum_{k=1}^{2} c_k z_k^2 - \frac{\sigma_2}{2r_2} \tilde{b}_2^2 - \sum_{k=2}^{3} \varepsilon_k^* y_k^2 + \sum_{k=1}^{2} h_k^2 + \Gamma_{21} + g_3 \mu_{21}^2 \frac{1}{\rho_{31}^2} z_2^2 z_3^2, \tag{47}$$

with $\Gamma_{21} = \frac{3}{4} + \frac{1}{4g_2} + \tau_2 + \frac{\sigma_2}{2r_2} b_2^2$.

Step 3. The derivative of the third virtual error z_3 is

$$\dot{z}_3 = \mu_{31} \left(g_3 u + \theta_{31} x_2 + \theta_{32} x_3\right) + \mu_{32} - \dot{\alpha}_{3f}. \tag{48}$$

Choosing the Lyapunov function candidate as

$$V_3 = \frac{1}{2} z_3^2 + V_2 + \frac{1}{2r_3} \widetilde{b}_3^2, \tag{49}$$

where $\widetilde{b}_3 = b_3 - \hat{b}_3$ is the parameter estimate error with $b_3 = \max\{\theta_3^2, 1\}$ being an unknwon constant and $\hat{b}_3$ being the parameter estimate, $r_3 > 0$ is a design parameter.

Noting that the derivative of $\frac{1}{2} z_3^2$ is

$$z_3 \dot{z}_3 = g_3 \mu_{31} z_3 u + z_3 \mu_{31} \left(\theta_{31} x_2 + \theta_{32} x_3\right) + z_3 \left(\mu_{32} - \dot{\alpha}_{3f}\right),$$

then differentiating V_3, we have

$$\dot{V}_3 \leqslant -\sum_{k=1}^{2} c_k z_k^2 - \frac{\sigma_2}{2r_2} \widetilde{b}_2^2 - \sum_{k=2}^{3} \varepsilon_k^* y_k^2 + \sum_{k=1}^{2} h_k^2 + \Gamma_{21} + g_3 \mu_{31} z_3 u + \Delta_3' - \frac{1}{r_3} \widetilde{b}_3 \dot{\hat{b}}_3 \tag{50}$$

where $\Delta_3' = z_3 \mu_{31} \left(\theta_{31} x_2 + \theta_{32} x_3\right) + z_3 \left(\mu_{32} - \dot{\alpha}_{3f}\right) + g_3 \mu_{21}^2 \frac{1}{\rho_{31}^2} z_2^2 z_3^2$.

Since

$$z_3 \mu_{31} \left(\theta_{31} x_2 + \theta_{32} x_3\right) \leq g_3 z_3^2 \mu_{31}^2 \theta_3^2 \left(x_2^2 + x_3^2 + \frac{1}{2}\right) + \frac{1}{4g_3},$$
$$z_3 \left(\mu_{32} - \dot{\alpha}_{3f}\right) \leq g_3 z_3^2 \left(\mu_{32} - \dot{\alpha}_{3f}\right)^2 + \frac{1}{4g_3},$$

then one has

$$\Delta_3^1 \leq g_3 b_3 z_3^2 \Phi_3 + \frac{1}{2g_3}, \tag{51}$$

where $b_3 = \max\{\theta_3^2, 1\}$ is an unknown constant and

$$\Phi_3 = \mu_{31}^2 \left(x_2^2 + x_3^2 + 1\right) + (\mu_{32} - \dot{\alpha}_{3f})^2 + \frac{\mu_{21}^2}{\rho_{31}^2} z_2^2 z_3^2 \tag{52}$$

is a computable function, then one further has

$$\dot{V}_3 \leqslant \; -\sum_{k=1}^{2} c_k z_k^2 - \frac{\sigma_2}{2r_2}\widetilde{b}_2^2 - \sum_{k=2}^{3} \varepsilon_k^* y_k^2 + \sum_{k=1}^{2} h_k^2 + \Gamma_{21} + g_3 \mu_{31} z_3 u + g_3 b_3 {z_3}^2 \Phi_3 + \frac{1}{2g_3} - \frac{1}{r_3}\tilde{b}\dot{\hat{b}} \tag{53}$$

The actual controller is designed as

$$u = \frac{1}{\mu_{31}}(-c_3 z_3 - \hat{b}_3 z_3 \Phi_3) \tag{54}$$

where $c_3 > 0$ is a design parameter and $\hat{b}_3$ is updated by

$$\dot{\hat{b}}_3 = r_3 z_3^2 \Phi_3 - \sigma_3 \Phi_3 \hat{b}_3, \;\; \hat{b}_3(0) \geq 0, \tag{55}$$

where $\sigma_3 > 0$, $\hat{b}_3(0)$ is the arbitrarily chosen initial estimate $\hat{b}_3(t)$.

Now, we are ready to state the following result.

THEOREM 1. *Consider the permanent magnet brush direct current (DC) motor systems* (1) *subject to the asymmetric state constraints* (3)*, if the actual controller* (54)*, the virtual controllers* (32) *and* (45) *as well as the adaptive laws are applied, under the Assumptions 1–2, we can deduce that: 1) all signals in the closed-loop system are bounded; and 2) the full-state constraints are guaranteed without involving the feasibility conditions.*

Proof. Substituting the actual control law u and adaptive law $\dot{\hat{b}}_3$ into (53), one has

$$\dot{V}_3 \leqslant -\sum_{k=1}^{2} c_k z_k^2 - \sum_{k=2}^{3} \frac{\sigma_k}{2r_k}\widetilde{b}_k^2 - \sum_{k=2}^{3} \varepsilon_k^* y_k^2 + \sum_{k=1}^{2} h_k^2 + \Gamma_3, \tag{56}$$

with $\Gamma_3 = \Gamma_{21} + \frac{1}{2g_3} + \frac{\sigma_3}{2r_3} b_3^2$, then we have

$$\dot{V}_3 \leqslant -\Upsilon V_3 + \Delta_3 \tag{57}$$

where $\Upsilon = \min\{c_k, \sigma_j, 2\varepsilon_j^*\}$, $k = 1,2,3;\ j = 2,3$, and $\Delta_3 = \sum_{k=1}^{2} h_k^2 + \Gamma_3$. With the above analysis, we now can proceed to prove the following results. 1) We first prove that the boundedness of all signals is ensured. According to (56), it can be concluded that V_3 converges to the set $\Omega_1 = \{V_3 || V_3| \leq \frac{\Delta_3}{\Upsilon}\}$ as time goes by, which further implies that z_i, $\tilde{b}_2$, $\tilde{b}_3$ y_2, and y_3 are ultimately uniformly bounded. Then, it is obvious that ρ_i is L_∞, so α_{2f} and α_1 are bounded due to the fact that in (24). It follows that the state x_1 is bounded. According to (14) and (30), we can get μ_{11} and Φ_1 are bounded. Following the same procedure, we can conclude that all the internal signals are bounded. 2) Next, we prove that the full-state of the system satisfies the asymmetry constraint. Due to $\rho_i \in L_\infty$, it obtains that for any initial value $\kappa_{il}(t) < x_i(0) < \kappa_{ih}$, the system state remains in the constrained region $\kappa_{il}(t) < x_i(t) < \kappa_{ih}(t)$ for $t \geq 0$.

Remark 1. The control implementation in this work is straightforward. For example, for the actual control law u as shown in (54), we only need the variables μ_{31}, z_3, Φ_3, and the parameter estimate $\hat{b}_3$. As z_3 is defined in (17), then it is easy to obtain the corresponding value; μ_{31} can be obtained via (14); Φ_3 can be computed via (52); and the parameter estimate $\hat{b}_3$ can be guaranteed via integrating the adaptive law as shown in (55). With the similar procedure, the virtual controllers α_1, α_2, and the parameter estimate $\hat{b}_2$, can be computed easily.

4. SIMULATIONS

To verify the effectiveness of the proposed control scheme, we consider the permanent magnet brush DC motor systems (1), where the detailed values of system paarameters are given as: $K_\eta = 0.9$, $J = 1.625 \cdot 10^{-3}$, $m = 0.506$, $M = 0.1$, $L = 0.1$, $R = 0.023$, $b = 16.25 \cdot 10^{-3}$, $L = 2 \cdot 10^{-3}$, $R = 5$, $K_B = K_\eta$, $B = \frac{b}{K_\eta}$, and the gravitational acceleration $G = 9.8$. In the simulation, the system states are required to remain in the following asymmetric sets:

$$x_i \in U_i := \{(t, x_i) \in \mathbb{R}_+ \times \mathbb{R} \mid \kappa_{il}(t) < x_i < \kappa_{ih}(t)\}, \kappa_{il} \in \mathbb{R}, \kappa_{ih} \in \mathbb{R}\} \tag{58}$$

for $i = 1, 2, 3$, where the constraining functions are governed dynamically by:

$$\begin{aligned}
&\kappa_{11} = -0.4 + 0.2\sin(t), \quad \kappa_{11}(0) = -0.3 \\
&\kappa_{12} = -0.25\exp(-t) - 0.25\sin(t) + 0.25\cos(t) + 1.2, \quad \kappa_{12} = 0.6 \\
&\kappa_{21} = -0.5\sin(t) + 0.3\cos(t) - 1.8, \quad \kappa_{21} = -0.3 \\
&\kappa_{22} = -0.6\exp(-t/2) - 0.2\sin(t) + 0.4\cos(t) + 1, \quad \kappa_{22} = 0.6 \\
&\kappa_{31} = -0.5\sin(t) + 0.3\cos(t) - 2, \quad \kappa_{31} = -0.3 \\
&\kappa_{32} = -0.6\exp(-t/2) - 0.2\sin(t) + 0.4\cos(t) + 1, \quad \kappa_{32} = 0.6.
\end{aligned} \tag{59}$$

In the simulation, the reference signal is given as $y_d = 0.1 + 0.5\sin(t)$. In order to guarantee the full-state constraints, the initial values of the DC system must be within the constrained sets, therefore, they are chosen as: $x_1(0) = 0.3$, $x_2(0) = -0.2$, $x_3(0) = -0.6$. The initial values of the parameter estimates are given as $\hat{b}_2 = \hat{b}_3 = 0$. The design parameters are chosen as: $c_1 = 1.65$, $c_2 = 2.5$, $c_3 = 1.1$, $\sigma_1 = 0.2$, $\sigma_2 = 0.1$, $\sigma_3 = 0.1$, $\gamma_1 = 2$, $\gamma_2 = 0.01$, $\gamma_3 = 0.001$, $\varepsilon_2 = 10$, and $\varepsilon_3 = 0.0002$.

Under the actual controller (54), the virtual controllers (32) and (45) as well as the adpative laws, the simulation results are shown in Figs. 1–5, in which the trajectories of x_1, x_2, and x_3 under the dynamic asymmetric state constraints are plotted in Figs. 1–3, from which it is observed that the not only the system states are effectively constrained within the constrained regions, but also the systems states are bounded.

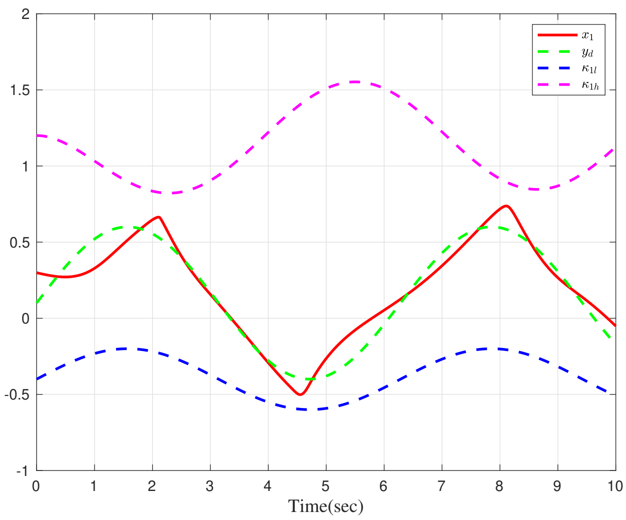

Fig. 1 – The trajectories of x_1 and y_d under constraint.

Furthermore, the evolutions of the virtual controllers are plotted in Fig. 4. It should be stressed that, if the BLF methods are applied, one has to ensure that the feasibility conditions on virtual controllers α_i, $i = 1, 2$, must be satisfied, i.e., $-\kappa_{il} < \alpha_i < \kappa_{ih}$. To this end, the offline optimization process must be implemented to obtain the optimal design parameters. However, this is a demanding condition. If the states are to be constrained in a small set, such a optimization solution may not exist (namely, the optimal design parameters do not exist). To

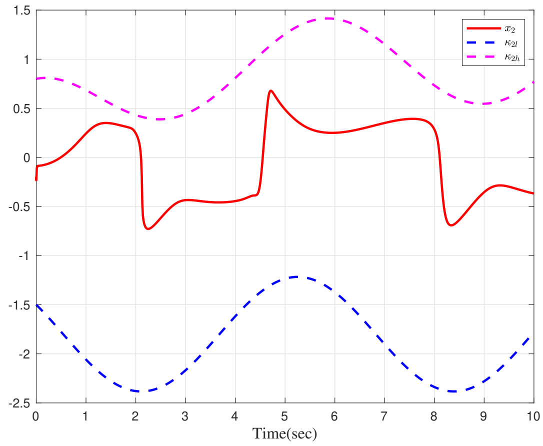

Fig. 2 – The trajectory of x_2 under constraint.

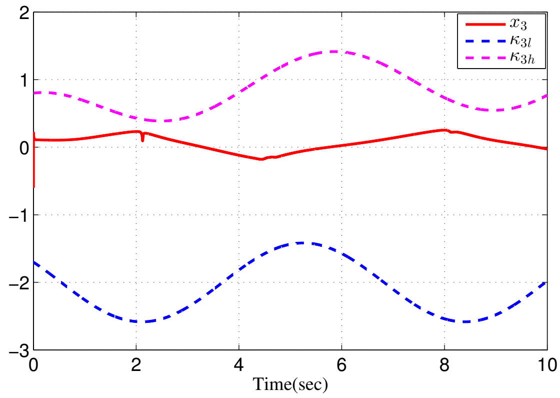

Fig. 3 – The trajectory of x_3 under constraint.

solve this issue, in this work a novel state-dependent nonlinear transformation function is developed, based on which we construct a different coordinate transformation and we only need to ensure the boundedness of the virtual controllers (rather than to guarantee that the virtual controllers must be within the constrained regions), which has been verified from Fig. 4. It is seen that, although the trajectories of the virtual controllers are not confined within the constraining regions (making traditional BLF/IBLF methods invalid/inapplicable), the proposed control can still ensure that the asymmetric full-state constraints imposed dynamically on the states are not violated as observed from Fig. 4. Moreover, the evolutions of the virtual errors are plotted in Fig. 5, which are also bounded.

5. CONCLUSION

In this paper, we introduce a robust adaptive control strategy for the permanent magnet brush DC motor systems in the presence of asymmetric full-state constraints. Firstly, by constructing a state-dependent nonlinear transformation function, we convert the problem of state constraint into the stabilization of a new variable. Secondly, by employing the first-order filter and constructing a new coordinate transformation, the demanding feasibility condition imposed on the BLF methods in the existing works is removed, allowing the designer more freedom to select design parameters. The effectiveness of the proposed control is verified via the Simulation results. It is worth noting that the virtual controllers and the actual control law are designed by using the backstepping technique, it is somewhat complicated as it contains some partial derivations, some parameter estimations, system states and other related nonlinear functions, then how to reduce the computational burden represents an interesting topic in future work.

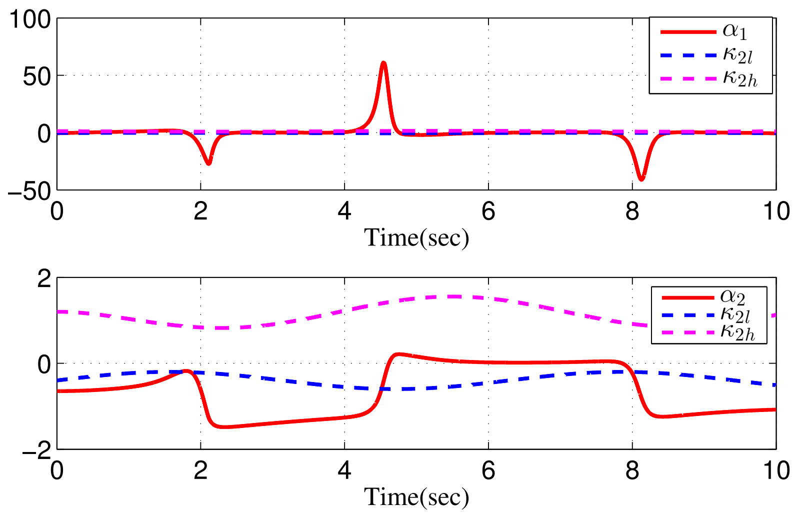

Fig. 4 – The trajectories of virtual controllers α_1 and α_2.

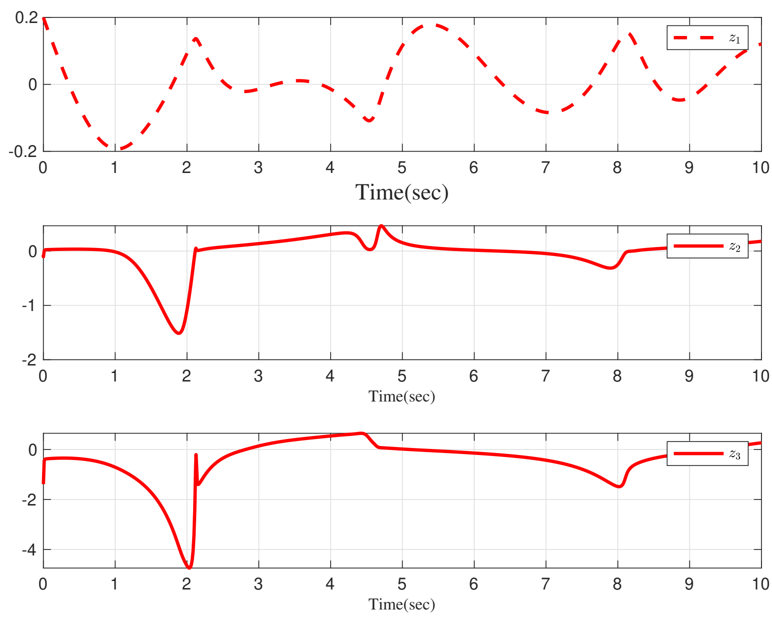

Fig. 5 – The evolutions of the virtual errors z_k, $k = 1,2,3$.

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