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A NEW CONTRIBUTION TO THE NEWTON-LEIBNITZ DIFFERENTIATION OF POLYNOMIAL APPROXIMATIONS IN THERMAL PROCESSES

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Abstract. This paper presents the determining of the critical cooling time and preheating temperature for welding. The primary equation which represents the cooling temperature is transcendent and, as such, it is not suitable for explicit solving. Hence, the author has investigated the appropriate interpolation polynomial which should align the polynomial level with the possibility of providing good accuracy of the approximation. Since the application and design of the welding included calculation of the derivatives of the polynomial, it is concluded that the accuracy of the derivative is not precise, nor does it follow the accuracy of the polynomial approximation. This research showed that the current theory for differentiation of the polynomial approximation does not provide precise results, although all the requirements of differentiability are provided. There are certain shortcomings in these actions, which imply that the currently known method of differentiation cannot be used. The new method of differentiating includes relatively small differentiating error regarding the exact value of the derivative, and the error for the polynomial approximation relating to the exact formula is not surpassed. The attempt to solve the design problem in thermal processes also makes a contribution in differentiation.

Keywords: interpolation, approximation, differentiation, welding, temperature.

1. INTRODUCTION

The research in this work was developed through analysis and study of the literature provided in [1–5]. It is important to mention also the results of famous researchers presented in [6–10], whose results have contributed to the work of other researchers in the area of thermal processes, heating and welding. The research in this work is focused on a significant number of professional and scientific works that deal with determining the critical cooling time and preheating temperature, as reported in references [10–24].

These works used theoretical models and dependencies, simulation packages, results from practice, and the determination of the mentioned dimensions according to the carbon equivalent or CE equivalent, including various other experiments.

Since the previous formulas for the calculation of the essential components of thermal processes have proved to be rough in terms of accuracy and resolution, simple analytical expressions for the preheating temperature were derived using numerous and selected studies in mathematics and numerical analysis [25–37], which will be of great use to researchers and technologists dealing with welding technology. The published works [38–49] have contributed to the results of this work with their originality and inventiveness.

The main objective of this work is to approximate an important transcendent relation from welding using a polynomial with sufficient accuracy and the possibility of being solved in an analytical way, and to make this available to many researchers and experts in practice. Approximations can be conducted in one of the following ways: 1) through the application of interpolations with various polynomials, 2) through middle-square approximation, and 3) through the approximation of the smallest square method. The resulting polynomials that have been differentiated created problems because they made large errors. This paper deals with these researches by solving the problem of differentiating polynomials using a new method.

2. THE NEW MODEL FOR CALCULATING THE PREHEATING TEMPERATURE FOR WELDING

The distribution of temperature on the welding plate can be described by the Fourier equation

$$\frac{\partial T}{\partial t} = a\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right), \tag{1}$$

where $a = \frac{\lambda}{c\gamma}$ is the coefficient of temperature conductivity, c – specific heat and γ – specific mass.

Using experiments from [6–9], the solution of the Fourier equation develops to

$$T(r,x) = \frac{q}{2\pi\lambda}\exp\left(-\frac{vx}{2a}\right)K_0\left[r\left(\frac{v^2}{4a^2}+\frac{b}{a}\right)\right]^{0.5} \tag{2}$$

where the symbol K_0 represents the modified Bessel function of type II and zero order. When the intensity of heating q is higher, the previous equation develops to equation

$$T(y_0,t) = \frac{q}{vd\sqrt{4\pi\lambda c\gamma t}}\exp\left(-\frac{y^2}{4at}-bt\right), \tag{3}$$

where it is assumed that the previous equation is the heat flux along the axis $x = 0$. In case of the highest speed of cooling in the zone of the weld, it can be concluded that $y = 0$, so that part in the exponent can be neglected. Therefore, we get the equation

$$T(t) = \frac{q}{vd\sqrt{4\pi\lambda c\gamma t}}. \tag{4}$$

During welding, the dimensions of time are important relating to cooling, after welding at temperatures of 800 °C and 500 °C. So far, in [1–10], these modes have been described and researched using the 2D and 3D Rosenthal model of equations. For the 2D model for determining the welding dimensions, there is a certain relation

$$t_{8,5} = \frac{q^2}{4\pi\lambda\gamma c d^2}\left[\frac{1}{(500-T_p)^2}-\frac{1}{(800-T_p)^2}\right], \tag{5}$$

where d – thickness of the material, q – the entering heat, T_p – preheating temperature, $t_{8,5}$ – critical time of cooling, λ – thermal conductivity [$\mathrm{Js^{-1}\,m^{-10}\,C}$], c – specific heat [$\mathrm{J\,kg^{-10}\,C^{-1}}$] and γ – specific weight [$\mathrm{kg\,m^{-3}}$]. For the 3D model for determining the welding dimensions, there is a relation

$$t_{8,5} = \frac{q}{2\pi\lambda}\left[\frac{1}{500-T_p}-\frac{1}{800-T_p}\right]. \tag{6}$$

To select the model for welding, the equation

$$d_{\mathrm{gr}} = \left[\frac{q}{\rho c}\left(\frac{1}{500-T_p}\right)+\left(\frac{1}{800-T_p}\right)\right]^{0.5}, \tag{7}$$

for the 2D type model will be used. Otherwise, the 3D model would be used. All of this is intended to improve the accuracy.

Equations (5) and (6) were used for a long time, during which their critical analysis was not conducted during practical applications, which included the entire scope of the use of variables and various types of welding. Some books mentioned that formulas adopted in advance were used. Also, some PhD theses used relations (5) and (6), which suggested the use of these relations in this research.

A smaller number of papers mentioned deviation of the calculated cooling time, but no corrections, confirmation or proofs of the accuracy of the solution were provided. Therefore, in work [14], which includes 15 examples, none of the solutions for $t_{8,5}$ was correct and the deviations were high. Due to deviations in the accuracy of the cooling time, during the development of British Standards for this type of welding [28], the correction was conducted in terms of more accurate calculations, which are provided by Eqs. (8) and (9) respectively, for the 2D and 3D models

$$t_{8,5} = (4\,300 - 4.3T_p)\frac{q^2}{d^2}\left[\frac{1}{(500 - T_p)^2} - \frac{1}{(800 - T_p)^2}\right] \tag{8}$$

$$t_{8,5} = (6\,700 - 5T_p)q\left[\frac{1}{500 - T_p} - \frac{1}{800 - T_p}\right]. \tag{9}$$

Through the applied researches using numerical calculations, it was concluded that there were deviations in relations (8) and (9) in some parts of their scope of use.

3. PRESENTATION OF THEORETICAL AND APPLIED RESULTS WITH A CONTRIBUTION TO THE NEW METHOD OF DIFFERENTIATION

For the sake of rationalization in the paper, the part that refers to various types of differentiation is given in [51]. The numbers of the tables and figures are linked in the paper to the document numbers from the site for easier monitoring. Every reference to the table numbers in the paper refers to the mentioned website document.

3.1. Approximate determination of the preheating temperature using the Lagrange interpolation method

In order to accurately determine the critical cooling time and preheating temperature in the works [38–49], in addition to exact calculation, iterative methods were also applied in solving the transcendental equations.

These solutions can be found in a graphical way, but this is not always a good approach in the case of preparing the technology for various welding processes.

Therefore, the relation (3) will be solved in such a way as to approximate the transcendent part, which includes the square root and exponential function using Lagrange's interpolation according to the experiment in [49], which used a 15 s step for interpolation that led to greater mistakes. In this paper, the use of [31, 32] provided the optimal interpolation of $h = 2$, which created the conditions for introducing an efficient new differentiation. Since, in many cases of welding, the process $T(t)$ ends ata maximum of 50 seconds, this scope will be observed for approximation in the next example.

Example 1. The task is to design the welding of steel sheets using the arc method. The thickness of the sheets is 7.4 mm and the entering heat is q_1=13 610 J/cm. The optimal temperature of preheating of the steel material should be calculated in order to obtain a good weld without disturbing the structure of the base material in the welding product. Steel with a critical cooling speed of 15 ^{o}C/s is used. In order to achieve the cooling speed, Eq. (3) will be differentiated by t, which will develop.

$$W_0 = \frac{\mathrm{d}T}{\mathrm{d}t} = \frac{q}{vd\sqrt{4\pi\lambda c\gamma t}}\exp\left(-\frac{y^2}{4at} - bt\right)\frac{\sqrt{t}}{t}\left[\left(\frac{y^2}{4at^2} - b\right)\sqrt{t} - \frac{1}{2\sqrt{t}}\right]. \tag{10}$$

Substituting (3) into (10) gives Equation (11).

$$W_0 = \frac{\mathrm{d}T}{\mathrm{d}t} = T(y,t)\left[\frac{1}{2t}\left(\frac{y^2}{2at} - 1\right) - b\right]. \tag{11}$$

The relation (11) defines the cooling speed of the material, which represents the first derivative of the cooling temperature and will be used for calculation of the preheating temperature. If $y = 0$, Equation (11) becomes

$$W_0 = \frac{\mathrm{d}T}{\mathrm{d}t} = -T(t)\left[\frac{1}{2t} + b\right]. \tag{12}$$

In relation (12), which is the original with the basic expression for temperature T, it is observed that the expression in the middle bracket represents the “differentiating operator”. In this way, it is possible to calculate the cooling speed in a shorter way and to use it for calculation of the other elements needed for the preheating temperature.

For this example, known elements will be separated from elements that need to be approximated so that Equation (3) can become

$$T(t) = k\frac{\mathrm{e}^{-bt}}{\sqrt{t}}, \tag{13}$$

and, after replacement of the known dimensions, the equation will be developed.

$$T(t) = 4015\frac{\mathrm{e}^{-bt}}{\sqrt{t}}. \tag{14}$$

Equation (14) showed that the part with the square root and exponential article are replaceable and the approximation will be conducted using Lagrange’s interpolation polynomial of the third order, which will provide easier analytical solving of the task.

In common cases, for the unknown function $f(x)$, which is added by a table containing numerical values, Lagrange’s interpolation [25–34] is used to define the solution in the form of the polynomial

$$y = \sum_{i=0}^{m} a_i x^i \tag{15}$$

and to define the unknown coefficients a_i .

For different values of the time t, the time-dependent element of Eq. (15) is calculated. For pairs selected in that way:

$$(26; 0.1417),\ (28; 0.1332),\ (30; 0.1255)\ \text{and}\ (32; 0.1185), \tag{16}$$

Lagrange’s interpolation polynomial in the form of Eq. (17) will be developed.

$$P_3(t) = -0{,}000002617\, t^3 + 0{,}00032402\, t^2 - 0{,}01603391\, t + 0{,}385538\,. \tag{17}$$

The function given by expression (14) approximates our transcendent article of Eq. (14) with a certain error that must be explained and soluble in an analytical way. Since this is a polynomial of the third order, it can be solved using known mathematical Cardan patterns. Figure 2 presents a graph of the cooling speeds using values from Table 2. Analysis of this graphic showed that the cooling speed and the first derivative by the approximate formula significantly deviate from the exact value, which leads to impreciseness of the differentiation. Because of that, this work will separately investigate this problem. In that way, the time t_8, which represents the time when the temperature on the $T(t)$ diagram reaches the value of 800°C, can be calculated along with time t_5, when the temperature of 500 °C is reached. Their difference represents the critical time of cooling in thermia, and it is known as $t_{8,5}$. In order to develop a pattern for the speed of cooling, it is important to differentiate polynomial (17) by t, to obtain the expression

$$W(t) = \frac{\mathrm{d}P_3(t)}{\mathrm{d}t} = -0{,}000007851\, t^2 + 0{,}00064804\, t - 0{,}016033391\,. \tag{18}$$

In calculating the values of expressions (17) and (18), it is important to multiply them with the constant $k = 4015$ to obtain nominal values for the real temperature and speed of cooling.

Table 1 gives the temperature values according to relations (14) and (17).

Figure 1 presents the graph of the exact value of the cooling temperature (14) and the approximate value (17) obtained using Lagrange’s interpolation formula of the third order. It can be concluded that the approximation curve follows the real curve within the given range of temperatures.

Figure 2 presents the graph of the cooling speed using the values from Table 2. Analysis of this graph shows that the cooling speed (12) and first derivative by the approximate formula (18) significantly deviate from the exact value, which leads to impreciseness of the differentiation. Because of that, this work will separately investigate this problem.

Table 1

Values of temperature for exact values and approximate values obtained using Lagrange's interpolation polynomial with deviation in percentages

t [s]	**Temperature $T^T(t)$** [°C]	**Temperature $T^A(t)$** [°C]	**Relative error** [%]
6	1520.7	1206.25	20.6
8	1284	1110.84	13.5
10	1120	1023.82	8.7
12	997.6	944.71	**5.3**
14	900.8	872.99	**3.1**
16	821.8	808.18	**1.65**
18	755.7	749.76	**0.78**
20	699.19	697.24	**0.3**
22	650.2	650.12	**0.01**
24	607	607.9	**0.15**
26	569	570.1	**0.2**
28	534.7	536.12	**0.26**
30	503.81	505.57	**0.35**
32	475.76	477.92	**0.45**
34	450	452.65	**0.6**
36	426.7	429.3	**0.6**
38	405	407.3	**0.56**
40	385	386.17	**0.3**
42	366.5	365.45	**0.28**
44	349	344.62	**1.2**
46	333	323.16	**2.95**
48	318	300.6	**5.47**
50	304	276.4	9.1
52	291	250.1	14
54	278	221.17	20.4
56	266	189.11	28.9
58	255	153.44	39.8

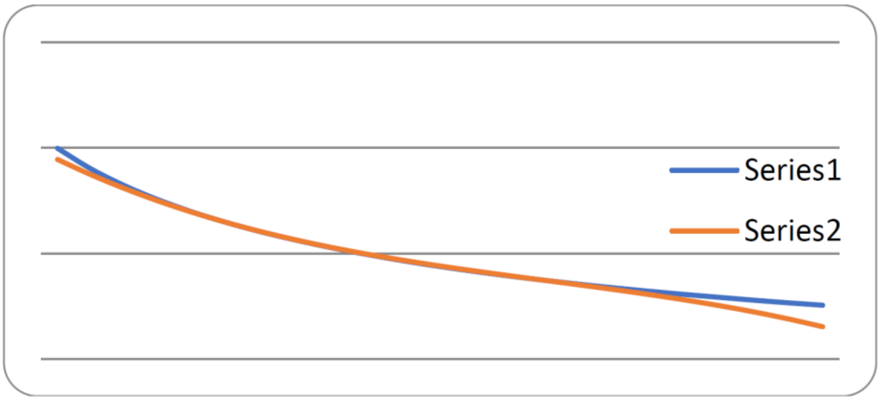

Fig. 1 – Presentation of graphic for exact (14) and approximate temperature (17) T [°C] as a function of time t [s] ($t=10+2i$, $i=1,...,24$).

Since this work suggests the introduction of the new type of differentiation for a more accurate cooling speed, Table 3 is compiled. Its first column includes the exact values for the cooling speed (12) and the second column includes values obtained according to expressions (10) and (11) for concrete values from the suggested example. This column presents approximate values of the cooling speed (18) which are very close to the exact value, which is confirmed by the values of the relative percentage of deviation.

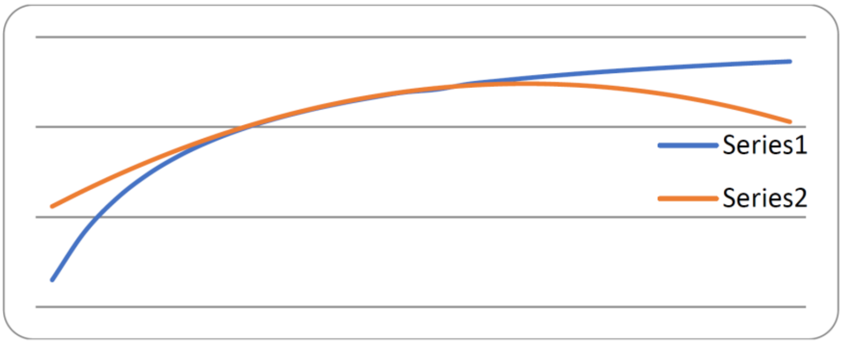

Fig. 2 – Graphics of cooling speeds [ºC/s] for the exact (curve 1) (11) and approximate formula (curve 2) (18) developed by classic and current differentiation as a function of time *t* [s] ($t=10+2i$, $i=1,\ldots,24$).

However, the mentioned error of the first deviation is the same as the errors of the interpolation approximation which are given in Table 1.

Since the differentiation errors that are given in Table 3 are significantly smaller than the values presented in Table 2, it can be concluded that the new principle of temperature differentiation $T(t)$ provides more precise differentiation results. This will make it possible to calculate more accurately the real and optimal temperatures for preheating of the welding material.

Table 2

Percentage of deviation of the first derivative (speeds of cooling) for exact and approximate formula from the expression (18) and according to existing theory

***t* [s]**	**First derivative** [ºC/s]	**Approximate first derivative (31)** [ºC/s]	**Relative error** [%]
6	–145.7	–49.89	65.7
8	–96.3	–45.56	52.7
10	–70	–41.49	40.7
12	–54.04	–37.66	30.3
14	–43.43	–34.09	21.5
16	–35.95	–30.76	14.4
18	–30.44	–27.69	9
20	–26.22	–24.87	**5.1**
22	–22.9	–22.29	**2.6**
24	–20.23	–19.97	**1.28**
26	–18.05	–17.9	**0.83**
28	–16.23	–16.08	**0.9**
30	–14.69	–14.51	**1.2**
32	–13.38	–13.19	**1.4**
34	–12.24	–12.12	**0.98**
36	–11.6	–11.3	**2.6**
38	–10.39	–10.73	**3.3**
40	–9.63	–10.41	8.1
42	–8.94	–10.34	15.6
44	–8.33	–10.53	26.4
46	–7.78	–10.96	40.9
48	–7.29	–11.65	59.8
50	–6.84	–12.58	87
52	–6.43	–13.76	114
54	–6.05	–15.2	151
56	–5.7	–16.89	196
58	–5.39	–18.82	249

Table 3

Percentage of deviation of the first derivative (speed of cooling) exact and using the new type of differentiating

t [s]	**Exact first derivative** [ºC/s]	**First derivative with new method** $-P_3(t)\left[\frac{1}{2t}+b\right]$ [ºC/s]	**Relative error** [%]
6	–145.7	–115.6	20.6
8	–96.3	–83.3	13.5
10	–70	–63.98	8.6
12	–54.04	–51.2	**5.2**
14	–43.43	–42.1	**3**
16	–35.95	–35.3	**1.8**
18	–30.44	–30.2	**0.7**
20	–26.22	–26.15	**0.2**
22	–22.9	–22.9	**0**
24	–20.23	–20.26	**0.1**
26	–18.05	–18.09	**0.2**
28	–16.23	–16.27	**0.2**
30	–14.69	–14.7	**0.06**
32	–13.38	–13.4	**0.1**
34	–12.24	–12.31	**0.5**
36	–11.6	–11.3	**2.5**
38	–10.39	–10.45	**3.3**
40	–9.63	–9.65	**0.2**
42	–8.94	–8.82	**0.2**
44	–8.33	–8.22	**1.3**
46	–7.78	–7.55	**2.9**
48	–7.29	–6.89	**5.4**
50	–6.84	–6.22	9
52	–6.43	–5.53	13.9
54	–6.05	–4.81	20
56	–5.7	–4.05	28
58	–5.39	–3.24	39

Figure 3 is the graphical representation of the cooling speeds that are numerically presented in Table 3. The two curves are nearly aligned and they are significantly favourable, unlike the curves from Fig 2, which shows a high deviation. It is concluded that the new way of differentiating the polynomial approximation and the new and original approach adopted in this work are superior to the current traditional practice of differentiating in a classical way. Details of this error will be explained later in this work.

When observing the values of the relative percentage deviation in the cooling speed – the first derivatives of the interpolation polynomial, a large error in the calculation of the first derivative is present, although the function from expression (17) is differentiable and fulfils all the requirements for differentiation according to the references in the attachment of this paper. Hence, in the point of the interpolation polynomial for the time of 50 s, the relative deviation is about 87%, while for 54 s this deviation is 151%, which is an unacceptably large deviation.

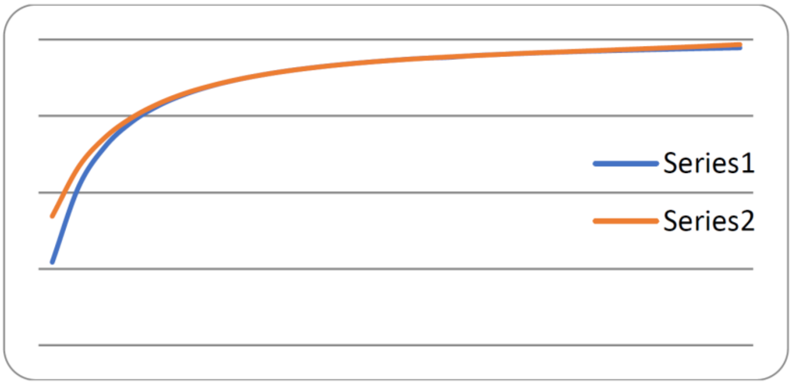

Fig. 3 – Graphic of exact speed of cooling (curve 1) (11) [°C/s] and speed obtained by new and original way of differentiating (curve 2), Eq. (22) of polynomial as a function of time t [s] (t=4+2i, i=1,...,27).

3.2. Analysis of finding higher derivatives using new type of differentiation and observing the accuracy of the method

The first derivative of the cooling temperature after welding was given by the earlier relation in the form of

$$W_0 = \frac{\mathrm{d}T}{\mathrm{d}t} = \frac{q}{vd\sqrt{4\pi\lambda c\gamma t}} \exp\left(-\frac{y^2}{4at} - bt\right)\frac{\sqrt{t}}{t}\left[\left(\frac{y^2}{4at^2} - b\right)\sqrt{\mathrm{t}} - \frac{1}{2\sqrt{\mathrm{t}}}\right], \tag{19}$$

which, after elementary transformations, turns into the form

$$W_0 = \frac{\mathrm{d}T}{\mathrm{d}t} = T(y,t)\left[\frac{1}{2t}\left(\frac{y^2}{2at} - 1\right) - b\right]. \tag{20}$$

When $y = 0$, we get the relation

$$W_0 = \frac{\mathrm{d}T}{\mathrm{d}t} = -T(t)\left[\frac{1}{2t} + b\right], \tag{21}$$

which, after the replacement of the expression for the temperature with the expression obtained using the interpolation polynomial, provides the relation

$$\frac{\mathrm{d}T}{\mathrm{d}t} = -P_3(t)\left(b + \frac{1}{2t}\right). \tag{22}$$

The part of the work that numerically processed multiple derivatives was omitted due to the scope of the work. This was also the case because higher derivatives are not needed in this application, which is why those equations and tables were omitted. Research shows that the errors of higher derivatives compared to the first derivative are significantly greater. The error according to the method of this paper is the same for all derivatives and equal to the error that occurs when obtaining the polynomial approximation.

4. ANALYSIS OF RESULTS AND DISCUSSION

The previous part of the work included the successfully approximated dependency for the temperature $T(t)$, which created the conditions for simple analytic calculations of the temperature as a function of time and the critical cooling time. The real temperature is equal to the approximate one using the introduced correction

$$T(t) = P_3(t) + R\,, \tag{23}$$

while R is a permissible approximation error. Therefore, the current expression for the speed of cooling is

$$\frac{\mathrm{d}T}{\mathrm{d}t} = -P_3(t)\left(b + \frac{1}{2t}\right). \tag{24}$$

Regarding Example 1, the replacement with W_0 =15 °C/s, and for $P_3(t)$ from the relation (17) provided the value for the time t_p as 29.5 s, which is common and critical for determining the preheating temperature. The replacement of this part for t provides the temperature

$$T(t) = 500.6\ ^\circ\mathrm{C} \tag{25}$$

and the difference with the temperature of 800 °C presents the preheating temperature as

$$T_p = 800 - 500.6 = 299.39\ ^\circ\mathrm{C}. \tag{26}$$

Welding from Example 1 should be obtained previously in order to provide preheating at the temperature of 300 °C. This would be an original way in which, beside the problem with incorrect differentiating, the original correct analytical algorithm can be provided. This algorithm would be used for solving problems related to thermia, through the introduction of the new type of differentiation. The new differentiation is applied from the expression obtained after differentiating the correct expression $T(t)$ in such a way as to use the new expression without $T(t)$ and to multiply the interpolation polynomial that is obtained in this paper using Lagrange’s interpolation.

For the case of applying expression (18), there are two values of the preheating temperature, which is a kind of problem related to choice and large deviations in the observed interval, averaging 44%. These situations are not favourable for application in practice. As a consequence, it would result in a low quality weld with a high percentage of carbon and thick structure as in [4]. The errors made by interpolations are dependent on the kind of function that is approximated and on the degree of polynomial approximation obtained. The first derivative, as a result of first differentiating the obtained polynomial, is correct but it significantly deviates from the first derivative of the main function, which needs to be approximated. This is confirmed by calculation of the first derivative of the real function on the one hand, and according to the definition of the derivative provided by calculation of tg of the angle of the tangent line at the point of derivative calculation, on the other hand. Many tables and graphics have been produced which proved that these claims are correct. Problems remain with the calculation of higher derivatives where the deviation is higher. This work provides the evidence that the problem lies in the nature of the functions obtained using interpolation polynomials.

The curve obtained by approximations is oscillatory, “twisted” around the real curve, and it thus changes the tan of the angle of the tangent line, which is the actual first derivative. This also happens with the definition of the higher derivatives. This is also clear in works [35–37], since they process the method of connecting certain functions which corrects the curves obtained by the interpolations, but without grading the correctness of the differentiating of the functions obtained in this way.

For the purpose of engineering practice, the approximation is defined in order not to exceed the error of 5%. In that way, the approximate polynomial satisfied these requirements in the scope from 10 to 50 s. All the tables in this work – in the column of errors – include in bold the numbers of the value which fulfils the mentioned requirements. Numerous tables and graphs of the dimensions confirm the validity of the claims and accuracy of the newly provided differentiation.

For the designing of filters in electronics and communications, polynomials assumed to be error-free in differentiating have been largely used. This is not favourable given the scope of shortcomings, sensitivity of filters and other relevant parameters. The results of applied research, which are described in references [41, 42], indicate that polynomial differentiation failures when designing filters for MTK devices have required additional hardware intervention on the device for adjustment.

All of this implies that this is not only a theory but confirmed in seriously practically realized devices which have been tested by accredited institutions.

Interpolation polynomials are widely applied in chemistry. The author of this paper therefore believes that this work will be very useful. Also, the mentioned approximations and interpolation polynomial are highly present in cubature formulas for the numerical calculation of integrals. Therefore, the results of the new differentiating method are useful for both theorists, according to which they will get numerous ideas for development, and practitioners who will get closer to practice.

5. CONCLUSION

This work is the result of theoretically applied researches over a long period of time. The condition of the techniques was processed and presented at a time when more formulas for calculation of the essential dimensions in thermia were being provided. Thus, it is concluded that such formulas do not provide precise results in all the segments of calculation which are needed in thermia regarding welding. These formulas are applied in numerous works, books, and papers without identifying the accuracy of the results.

For the approximation of transcendent equations, Lagrange's interpolation was selected, so a soluble polynomial of the third order is obtained analytically for applied research. Since derivatives are also needed for calculation of the cooling speed, it is concluded that they introduce significant errors regarding the real values. Hence, the work is extended and aimed at solving this problem. After identification, it is concluded that the interpolation polynomial does not provide precise results after differentiation. The reason for that is the feature of the polynomial to twist around the approximated curve, while the tangent lines which present the first derivatives vary significantly. With further differentiation for derivatives of higher degree, the errors increase. Due to such problems, the idea was to realise a new form of differentiation, which does not make any error in k's derivative, except that it transfers the error made by the polynomial in the observed approximation interval.

This problem inspired an inventive investigation to obtain the correct first derivative (which represents the cooling speed) in a way that directly multiplies the interpolation polynomial with a separate expression for differentiating, which originates from the main function. The results were correct, as was confirmed by numerous tables and graphs. The analytical expressions obtained for the cooling temperature and speed are not transcendent, but they are soluble for applied research and practice.

The new method of differentiation could be used in many applications, among which it is worth mentioning the solving of filters based on the interpolation polynomial, the solving of cubature formulas for obtaining integrals, and many other applications in technical sciences. Due to its common features, the new method of differentiation will become part of numerical analysis and mathematics as a whole.

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