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IMPROVED NEWTON-RAPHSON METHOD WITH SIMPLIFIED JACOBIAN MATRIX AND OPTIMIZED ITERATION RATE FOR POWER FLOW CALCULATION OF POWER SYSTEM

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Abstract. With high penetration of renewable energy and novel loads connected to the distribution network, the voltage fluctuation becomes more severe and frequent, which may cause over- and under-voltage. The distribution system operator should calculate the power flow and validate the state to optimize the distribution network. Power flow calculation is the solution to the multivariate nonlinear problem, and the Newton-Raphson method is an effective algorithm for solving nonlinear problems. However, calculating the Jacobian matrix is a crucial process of the Newton-Raphson method, which is time-consuming. Therefore, this paper proposed an improved Newton-Raphson method, which simplifies and decreases the iterations of the calculation process of the Jacobian matrix to improve the calculation rate. To verify the effectiveness of the proposed method, the power flow of the IEEE 33-node power distribution system is calculated by the improved Newton-Raphson method and the conventional Newton-Raphson method.

Keywords*:* Newton-Raphson method, Jacobian matrix, power flow, distribution network.

1. INTRODUCTION

Distributed generations (DGs) have the advantages of low investment cost, environmentally friendly characteristics, high energy utilization, close to the occupants, and convenient grid connection [1]. Therefore, the DGs have been promoted and developed rapidly with a series of policy incentives and support. The output power, connection mode, and connection position of the DGs affect the power flow of the distribution network and cause voltage problems of over-voltage, under-voltage, and voltage fluctuation [2]. To improve the voltage quality of the distribution network, the distribution system operator should analyze the power flow of the distribution network and formulate an optimization strategy to guarantee the power quality of the distribution network. Therefore, the power flow calculation is crucial to optimize the distribution network [3].

The power flow calculation of the power system involves the solution of nonlinear equations. The conventional methods include admittance, Newton-Raphson, and P-Q decomposition. The admittance method has a simple calculation principle, which is an iteration process based on the impedance matrix [4]. Due to the integrality of the impedance matrix, the calculation burden increases. Besides, the Newton-Raphson method based on the admittance matrix can effectively calculate the power flow of the distribution network [5]. The sparsity of the admittance matrix can improve the calculation rate and convergence. However, each iteration process needs to calculate the Jacobian matrix, which will consume a lot of time and calculation resources. To overcome the weakness of the Newton-Raphson method, the P-Q decomposition method is proposed [6]. However, the number of iterations increases, the calculation time decreases, and calculation resources are released. The P-Q decomposition method can approximately ignore the influence of voltage amplitude change on active power distribution and voltage phase change on reactive power distribution. Therefore, the P-Q decomposition method is unsuitable for calculating the power flow of the distribution network with a small X/R (X/R represents the ratio of reactance to resistance) ratio.

The Newton-Raphson algorithm is an iterative numerical method used to solve numerical approximation problems of systems of nonlinear equations. In power system power flow calculations, it is usually used to solve complex nonlinear equations of node voltages and phase angles, and is used to analyze parameters such as voltage, phase angle, active power, and reactive power of each node in the power system [7], [8], which is essential for determining the voltages and phase angles of various nodes in an electrical network. Although the Newton-Raphson method consumes an enormous amount of time, it is also widely applied to analyze the power flow of the distribution network [9], [10]. To improve the calculation efficiency of the Newton-Raphson method, an enormous number of papers on the Newton-Raphson power flow algorithm have been proposed. Authors in [11] proposed a V–Q sensitivity analysis method based on the integrated Jacobian matrix. The method improved the convergence rate of the Newton-Raphson method. Authors in [12] proposed a novel approach to solve the power flow for islanded microgrids using a Modified Newton-Raphson, which took into account the droop characteristics of DGs. Authors in [13] used stochastic gradient descent to avoid the local optima and saddle points, and the Newton-Raphson method was used to accelerate the convergence when the iterations were close to the global optimal. Authors in [14] proposed a simplified Newton-Raphson power-flow solution method, which employs nonlinear current mismatch equations instead of the commonly used power mismatches to simplify overall equation complexity. The proposed simplified Newton-Raphson method spent less execution time than the conventional Newton-Raphson method with similar convergence characteristics.

The key to using the Newton-Raphson method to calculate power system power flow is the calculation of the Jacobian matrix in iterative operations. During the iteration of the Newton-Raphson method, the complex calculation of the Jacobian matrix needs a large amount of computing resources. which will decrease the calculation efficiency. Simplifying the calculation process of the Jacobian matrix becomes particularly important. On the other hand, the number of iterations also affects the calculation efficiency. Insufficient iterations will cause the accuracy of the Newton-Raphson algorithm to decrease, while too many iterations will increase computational and time costs. Therefore, formulating a suitable optimization strategy is the key to solving power system power flow calculation problems.

Metaheuristic algorithms have global search capabilities and adaptability, can effectively find optimal solutions [15], and reduce the waste of computing resources [16], [17]. At the same time, it can be effectively applied to various complex optimization problems [18] and achieves good results in practical applications [19, 20]. This type of optimization algorithm is applied to the power flow calculation of the Newton-Raphson method, which can well determine the iteration threshold, improve the efficiency and stability of the power flow calculation, and meet the high-efficiency requirements of power system analysis. In this paper, the Particle Swarm Optimization (PSO) algorithm is used to determine the optimal iteration threshold in the Newton-Raphson method. Through the introduction of the PSO algorithm, we can improve the performance of the power flow calculation of the entire power system while maintaining calculation accuracy, and meet the demand for efficient and robust calculation methods.

The organization structure of this paper is as follows: Section 2 introduces the conventional Newton-Raphson method, including the establishment of the node voltage equation, the solution of the nonlinear equation, and the calculation process of the Jacobian matrix. Section 3 proposes an improved Newton-Raphson method, and introduces methods for simplifying the calculation process of the Jacobian matrix and optimizing the iteration rate. Section 4 shows the experimental results and verifies the effectiveness of the improved method through its application on the IEEE 33-node power distribution network, including the reduction of computing time and the improvement of computing efficiency. Section 5 summarizes the main findings and conclusions of the study.

2. CONVENTIONAL NEWTON-RAPHSON METHOD

The steps of the conventional Newton-Raphson algorithm in power system power flow calculation problems are as follows: First, establish the power balance equation of the node, including active power balance and reactive power balance. Then the system parameters are initialized, and then the Jacobian matrix is calculated and continuously iterated and updated. After iteration, it is judged whether it has converged. When the algorithm converges, the power flow calculation result of the power system under given conditions is obtained. Calculation of the power flow in the distribution network is performed from the node voltage equation [20]. According to the admittance matrix of the distribution network, the equation of voltage and power is established, which can be expressed as follows:

$$\sum_{j=1}^{n} \boldsymbol{Y}_{ij}\cdot\dot{\boldsymbol{U}}_j=\frac{\dot{\boldsymbol{S}}_i}{\dot{\boldsymbol{U}}_j} \tag{1}$$

where $\boldsymbol{Y}_{ij}$ is the admittance of nodes i and j, $\dot{\boldsymbol{U}}_j$ is the voltage of node i, $\dot{\boldsymbol{S}}_I$ is the apparent power of node i, so the equation can be expressed as follows:

$$\boldsymbol{P}_i+\mathrm{j}\boldsymbol{Q}_i=\dot{\boldsymbol{U}}_i\cdot\sum_{j=1}^{n} \boldsymbol{Y}_{ij}\cdot\dot{\boldsymbol{U}}_j \tag{2}$$

where $\boldsymbol{P}_I$ and $\boldsymbol{Q}_I$ are the active power and reactive power of node i, respectively. The left part of the equation expresses active power and reactive power injected into the node, and the right part of the equation expresses the required active power and reactive calculated by admittance matrix and node voltage. When the two parts are not equal, the node voltage will have a deviation, which will affect the power supply quality of the distribution network. Therefore, the active power and reactive power should be corrected to make the deviation of the node voltage tend to zero.

In the Newton-Raphson power flow calculation process, the equations $\boldsymbol{P}_i+\mathrm{j}\boldsymbol{Q}_I$ are nonlinear. The node voltage in (2) can be expressed in the polar coordinate system:

$$\dot{\boldsymbol{U}}_i=\boldsymbol{U}_i\cdot\mathrm{e}^{\mathrm{j}\theta_i}=\boldsymbol{U}_i\cdot(\cos\theta_\mathrm{i}+\mathrm{j}\sin\theta_i) \tag{3}$$

$$\dot{\boldsymbol{U}}_j=\boldsymbol{U}_j\cdot\mathrm{e}^{\mathrm{j}\theta_j}=\boldsymbol{U}_j\cdot(\cos\theta_j-\mathrm{j}\sin\theta_j) \tag{4}$$

$$\boldsymbol{Y}_{ij}=\boldsymbol{G}_{ij}+\mathrm{j}\boldsymbol{B}_{ij} \tag{5}$$

$$\boldsymbol{P}_i+\mathrm{j}\boldsymbol{Q}_i=\boldsymbol{U}_i\cdot(\cos\theta_i+\mathrm{j}\sin\theta_i)\cdot\sum_{j=1}^{n}(\boldsymbol{G}_{ij}+\mathrm{j}\boldsymbol{B}_{ij})\cdot\boldsymbol{U}_j\cdot(\cos\theta_j-\mathrm{j}\sin\theta_j)=0 \tag{6}$$

where $\boldsymbol{G}_{ij}$ and $\boldsymbol{B}_{ij}$ are the real and imaginary parts of admittance of the node i and j, θ_i is the phase angle of the node i. The active power and reactive of (6) can be expressed respectively:

$$\boldsymbol{P}_i=\boldsymbol{U}_i\sum_{j=1}^{n}\boldsymbol{U}_j\cdot(\boldsymbol{G}_{ij}\cdot\cos\theta_{ij}+\boldsymbol{B}_{ij}\cdot\sin\theta_{ij})=0 \tag{7}$$

$$\boldsymbol{Q}_i=\boldsymbol{U}_i\sum_{j=1}^{n}\boldsymbol{U}_j\cdot(\boldsymbol{G}_{ij}\cdot\sin\theta_{ij}-\boldsymbol{B}_{ij}\cdot\cos\theta_{ij})=0 \tag{8}$$

When the nodes of the distribution network are ‘PQ’ nodes, the active power and reactive power injected into the node are known as $\boldsymbol{P}_1'$, $\boldsymbol{Q}_1'$. The active and reactive power should be corrected to balance supply and demand. The regulation of active power and reactive power can be expressed as follows:

$$\Delta\boldsymbol{P}_i=\boldsymbol{P}_1'-\boldsymbol{U}_i\sum_{j=1}^{n}\boldsymbol{U}_j\cdot(\boldsymbol{G}_{ij}\cdot\cos\theta_{ij}+\boldsymbol{B}_{ij}\cdot\sin\theta_{ij})=0 \tag{9}$$

$$\Delta\boldsymbol{Q}_i=\boldsymbol{Q}_1'-\boldsymbol{U}_i\sum_{j=1}^{n}\boldsymbol{U}_j\cdot(\boldsymbol{G}_{ij}\cdot\sin\theta_{ij}-\boldsymbol{B}_{ij}\cdot\cos\theta_{ij})=0 \tag{10}$$

(9)and (10) expend in the Taylor series and neglect the high-order parts to obtain the modified equation:

$$\begin{bmatrix}\Delta\boldsymbol{P}\\ \Delta\boldsymbol{Q}\end{bmatrix}=\begin{bmatrix}\boldsymbol{H} & \boldsymbol{N}\\ \boldsymbol{M} & \boldsymbol{L}\end{bmatrix}\begin{bmatrix}\Delta\theta\\ \Delta\boldsymbol{U}/\boldsymbol{U}\end{bmatrix} \tag{11}$$

where $\begin{bmatrix}\boldsymbol{H} & \boldsymbol{N}\\ \boldsymbol{M} & \boldsymbol{L}\end{bmatrix}$ is the Jacobian matrix, the elements can be expressed as follows:

$$\boldsymbol{H}_{ij}=\frac{\partial\Delta\boldsymbol{P}_i}{\partial\theta_j}=-\boldsymbol{U}_i\cdot\boldsymbol{U}_j\cdot(\boldsymbol{G}_{ij}\cdot\sin\theta_{ij}-\boldsymbol{B}_{ij}\cdot\cos\theta_{ij}) \tag{12}$$

$$\boldsymbol{H}_{ii}=\frac{\partial\Delta\boldsymbol{P}_i}{\partial\theta_i}=\boldsymbol{U}_i\sum_{j=1,\,j\neq i}^{n}\boldsymbol{U}_{\mathrm{j}}\cdot(\boldsymbol{G}_{ij}\cdot\sin\theta_{ij}-\boldsymbol{B}_{ij}\cdot\cos\theta_{ij}) \tag{13}$$

$$\boldsymbol{N}_{ij}=\frac{\partial\Delta\boldsymbol{P}_i}{\partial\boldsymbol{U}_j}\boldsymbol{U}_j=-\boldsymbol{U}_{\mathrm{i}}\cdot\boldsymbol{U}_{\mathrm{j}}\cdot(\boldsymbol{G}_{ij}\cdot\cos\theta_{ij}+\boldsymbol{B}_{ij}\cdot\sin\theta_{ij}) \tag{14}$$

$$\boldsymbol{N}_{ii}=\frac{\partial\Delta\boldsymbol{P}_i}{\partial\boldsymbol{U}_i}\boldsymbol{U}_i=-\boldsymbol{U}_{\mathrm{i}}\sum_{j=1,\,j\neq i}^{n}\boldsymbol{U}_{\mathrm{j}}\cdot(\boldsymbol{G}_{ij}\cdot\cos\theta_{ij}+\boldsymbol{B}_{ij}\cdot\sin\theta_{ij}) \tag{15}$$

$$\boldsymbol{M}_{ij}=\frac{\partial\Delta\boldsymbol{Q}_i}{\partial\theta_j}=\boldsymbol{U}_i\cdot\boldsymbol{U}_j\cdot(\boldsymbol{G}_{ij}\cdot\cos\theta_{ij}+\boldsymbol{B}_{ij}\cdot\sin\theta_{ij}) \tag{16}$$

$$\boldsymbol{M}_{ii}=\frac{\partial\Delta\boldsymbol{Q}_i}{\partial\theta_i}=-\boldsymbol{U}_i\sum_{j=1,\,j\neq i}^{n}\boldsymbol{U}_j\cdot(\boldsymbol{G}_{ij}\cdot\cos\theta_{ij}+\boldsymbol{B}_{ij}\cdot\sin\theta_{ij}) \tag{17}$$

$$\boldsymbol{L}_{ij}=\frac{\partial\Delta\boldsymbol{Q}_i}{\partial\boldsymbol{U}_j}\boldsymbol{U}_j=-\boldsymbol{U}_i\cdot\boldsymbol{U}_j\cdot(\boldsymbol{G}_{ij}\cdot\sin\theta_{ij}-\boldsymbol{B}_{ij}\cdot\cos\theta_{ij}) \tag{18}$$

$$\boldsymbol{L}_{ii}=\frac{\partial\Delta\boldsymbol{Q}_i}{\partial\boldsymbol{U}_i}\boldsymbol{U}_i=-\boldsymbol{U}_i\sum_{j=1,\,j\neq i}^{n}\boldsymbol{U}_j\cdot(\boldsymbol{G}_{ij}\cdot\sin\theta_{ij}-\boldsymbol{B}_{ij}\cdot\cos\theta_{ij}) \tag{19}$$

The inversion of the Jacobian matrix can be expressed as the *S* matrix, which can describe the relationship between node voltage/phase angle correction and active/reactive power correction.

3. IMPROVED NEWTON-RAPHSON METHOD

In conventional Newton-Raphson power flow calculation, each iteration should calculate the Jacobian matrix. Therefore, simplifying the calculation process of the Jacobian matrix can effectively reduce the calculation time and release computation resources. Besides, the convergence rate plays a significant role in the Newton-Raphson method. Keeping the convergence rate also can improve the calculation efficiency.

3.1. Simplified Jacobian Matrix

In the low-voltage distribution network, the distribution line segments will not be for long. The power flow of each branch and the voltage difference between nearby nodes are usually small [21]. Therefore, the voltage amplitude and phase angle of nearby nodes can be expressed as follow:

$$\begin{cases}\theta_i=\theta_j\\ \theta_{ij}=\theta_i-\theta_j\approx 0\\ |\boldsymbol{U}_i|=|\boldsymbol{U}_j|\end{cases} \tag{20}$$

According to (20), each element of the Jacobian matrix can be expressed as:

$$\boldsymbol{H}_{ij}=\frac{\partial\Delta\boldsymbol{P}_i}{\partial\theta_j}=\boldsymbol{U}_i\cdot\boldsymbol{U}_j\cdot\boldsymbol{B}_{ij}\cdot\cos\theta_{ij} \tag{21}$$

$$\boldsymbol{H}_{ii}=\frac{\partial\Delta\boldsymbol{P}_i}{\partial\theta_i}=-\boldsymbol{U}_i\sum_{j=1,\,\mathrm{j}\neq\mathrm{i}}^{n}\boldsymbol{U}_j\cdot\boldsymbol{B}_{ij}\cdot\cos\theta_{ij} \tag{22}$$

$$\boldsymbol{N}_{ij}=\frac{\partial\Delta\boldsymbol{P}_i}{\partial\boldsymbol{U}_j}\boldsymbol{U}_j=-\boldsymbol{U}_i\cdot\boldsymbol{U}_j\cdot\boldsymbol{G}_{ij}\cdot\cos\theta_{ij} \tag{23}$$

$$\boldsymbol{N}_{ii}=\frac{\partial\Delta\boldsymbol{P}_{\mathrm{i}}}{\partial\boldsymbol{U}_{\mathrm{i}}}\boldsymbol{U}_i=-\boldsymbol{U}_{\mathrm{i}}\sum_{j=1,\,\mathrm{j}\neq\mathrm{i}}^{n}\boldsymbol{U}_j\cdot\boldsymbol{G}_{ij}\cdot\cos\theta_{ij} \tag{24}$$

$$\boldsymbol{M}_{ij}=\frac{\partial\Delta\boldsymbol{Q}_i}{\partial\theta_j}=\boldsymbol{U}_i\cdot\boldsymbol{U}_j\cdot\boldsymbol{G}_{ij}\cdot\cos\theta_{ij} \tag{25}$$

$$\boldsymbol{M}_{ii}=\frac{\partial\Delta\boldsymbol{Q}_i}{\partial\theta_i}=-\boldsymbol{U}_i\sum_{j=1,\,\mathrm{j}\neq\mathrm{i}}^{n}\boldsymbol{U}_j\cdot\boldsymbol{G}_{ij}\cdot\cos\theta_{ij} \tag{26}$$

$$\boldsymbol{L}_{ij}=\frac{\partial\Delta\boldsymbol{Q}_i}{\partial\boldsymbol{U}_j}\boldsymbol{U}_j=\boldsymbol{U}_i\cdot\boldsymbol{U}_j\cdot\boldsymbol{B}_{ij}\cdot\cos\theta_{ij} \tag{27}$$

$$\boldsymbol{L}_{ii}=\frac{\partial\Delta\boldsymbol{Q}_i}{\partial\boldsymbol{U}_i}\boldsymbol{U}_i=\boldsymbol{U}_i\sum_{j=1,\, j\neq i}^{n}\boldsymbol{U}_j\cdot\boldsymbol{B}_{ij}\cdot\cos\theta_{ij} \tag{28}$$

where $\boldsymbol{H}_{ij}=\boldsymbol{L}_{ij}$, $\boldsymbol{N}_{ij}=-\boldsymbol{M}_{ij}$, by simplifying and replacing the element of the Jacobian matrix, the time used to calculate the Jacobian matrix can be significantly reduced, and the rate of power flow calculation is accelerated.

3.2. Optimized Iteration Rate

The Newton-Raphson method needs to calculate the Jacobian matrix repeatedly at each iteration. The calculation in each iteration is huge, which greatly reduces the calculation rate. When the iterations tend to converge, the changes in the elements of the Jacobian matrix obtained are small. Since the changes can be negligible, the same Jacobian matrix can be used in several iteration processes. To evaluate the change in the Jacobian matrix, an iteration threshold is set. When the changes of the Jacobian matrix in the iterations are less than the threshold, the matrix is considered stable enough, which will not be calculated and called a frozen Jacobian matrix. Based on the frozen Jacobian matrix, the calculation time of the power flow calculation can be significantly reduced, and the calculation rate will increase.

The setting of the iteration threshold requires comprehensive consideration of the dynamic characteristics of the system and the required calculation accuracy. If the threshold is set too low, the algorithm may not be able to fully utilize the update of the Jacobian matrix to improve the convergence speed, thereby increasing the number of unnecessary iterations. On the contrary, if the threshold is set too high, the algorithm may start using the frozen Jacobian matrix before the system has truly converged, which may cause the convergence to slow down or even diverge. To more effectively optimize the iteration rate of the Newton-Raphson method, this paper uses the Particle Swarm Optimization (PSO) algorithm to explore the optimal iteration threshold. The way particles search in the PSO algorithm is very similar to the way a flock of birds searches for food. Each particle in the particle swarm represents a solution, and each particle will obtain a corresponding fitness value based on the objective function.

The PSO algorithm is an evolutionary computation to obtain the optimal solution In the PSO algorithm, each particle in the particle swarm represents a solution, and each particle will obtain a corresponding fitness value based on the objective function. Combined with the iteration threshold, we set the fitness function as $t_{pf}=f(\xi)$, where t_{pf} is the time spent in power flow calculation, ξ is the iteration threshold, and $f(\xi)$ represents the time of power flow calculation. The lower fitness value of a single particle indicates that t_{pf} will be shorter and the iteration threshold is better.

The speed update of the particle swarm can be expressed as:

$$v_i(t+1)=w\cdot v_i(t)+c_1\cdot r_1\cdot(p_i-x_i(t))+c_2\cdot r_2\cdot(g-x_i(t)) \tag{29}$$

where $v_i(t+1)$ and $v_i(t)$ are the updated velocity of the particle and the current velocity of the particle, w is the inertia factor, p_i is the individual best value, g is the global best value. C_1 and c_2 are acceleration constants, r_1 and r_2 are random numbers between [0,1], $x_i(t)$ is the position of the particle at time t. Updating the velocity of the particle swarm represents the numerical change of the particles during each iteration. Therefore, for the optimization of the iteration threshold ξ, we replace $x_i(t)$ with $\xi_i(t)$, replace $v_i(t)$ with $\Delta\xi_i(t)$, formula (29) can be modified as:

$$\Delta\xi_i(t+1)=w\cdot\Delta\xi_i(t)+c_1\cdot r_1\cdot(p_i-\xi_i(t))+c_2\cdot r_2\cdot(\xi_{best}-\xi_i(t)) \tag{30}$$

The position update formula of the particle swarm can be expressed as:

$$x_i(t+1)=x_i(t)+v_i(t+1) \tag{31}$$

According to formula (30), formula (31) can be modified as:

$$\xi_i(t+1)=\xi_{\mathrm{i}}(t)+\Delta\xi_i(t+1) \tag{32}$$

where $\xi_{\mathrm{i}}(t)$ is the current iteration threshold, $\xi_i(t+1)$ is the updated iteration threshold, and ξ_{best} is the best iteration threshold found from initialization to the current search. The time of Newton-Raphson power flow calculation is $f(\xi_{\mathrm{i}}(t))$, if $f(\xi_i(t))<f(p_i)$, then $p_i=\xi_i(t)$; if $\min f(\xi_i(t))<f(\xi_{best})$, then $\xi_{best}=\xi_i(t)$. Iteration will

be continued until the set number of iterations is reached. Then, the iteration threshold will finally select $\xi = \xi_{best}$. Through the PSO algorithm, we can obtain the most suitable threshold for the Newton-Raphson power flow calculation to improve computational efficiency.

The flowchart for optimizing the Newton-Raphson method using the PSO algorithm is given in Fig. 1.

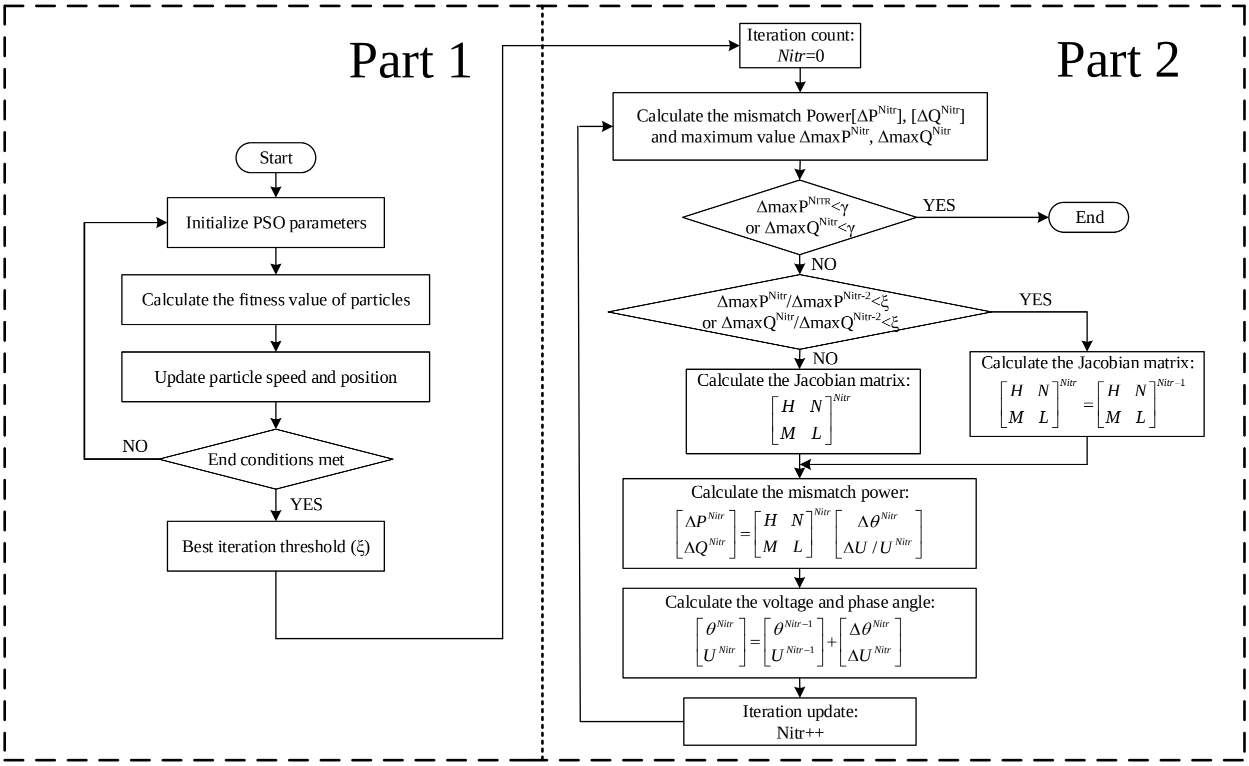

Fig. 1 – The flowchart of the optimization process.

Figure 1 is divided into two parts. The first part is the process of using the particle swarm optimization algorithm to derive the optimal iteration threshold, which can reduce the iteration number of power flow calculation. The second part is used to simplify the calculation of the Jacobian matrix elements in the Newton-Raphson method, which can improve the efficiency of power flow calculations. To optimize the iteration rate, the PSO algorithm calculates the optimal iteration threshold which will be used for the improved Newton-Raphson method. The notations γ and ξ indicate the error tolerance and convergence rate threshold. The ξ is the maximum error, the difference between the approximation and the true value. During the iterative process of the Newton-Raphson method, when the calculated maximum value of the unbalanced power is lower than the error tolerance, the iterative process will stop and the approximation will be determined to replace the true value to obtain the final result. Otherwise, iteration will be performed. When the unbalanced power difference is greater than the iteration threshold, the Jacobian matrix should be recalculated to improve the convergence speed. Otherwise, this iteration will use the Jacobian matrix of the previous iteration to avoid repeated calculations. Therefore, using the improved Newton-Raphson method after particle swarm optimization can reduce the number of calculations of the Jacobian matrix and make power flow calculations more efficient.

4. RESULTS

The conventional Newton-Raphson method and improved Newton-Raphson method are implemented by MATLAB R2018a on a Win10-based i7-7820HK PC. The IEEE 33-node distribution network is used to demonstrate the validity of the improved Newton-Raphson power-flow algorithm and compare it with the conventional Newton-Raphson method. Node 1 is the reference node, and the voltage value of the reference node is 1 p.u. The other nodes are all ‘PQ’ nodes in the distribution network. The base power is 100 MVA.

The structure and the admittance of the IEEE 33-node distribution network are shown in Fig. 2 and Table 1. The PSO Parameter is given in Table 2.

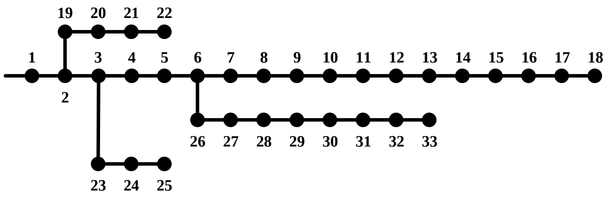

Fig. 2 – IEEE 33-node power distribution system.

Table 1

The admittance of the IEEE 33-node distribution network [22]

Node *i*	Node *j*	Admittance	Node *i*	Node *j*	Admittance
1	2	0.0922+j0.047	17	18	0.3720+j0.5740
2	3	0.4930+j0.2511	2	19	0.1640+j0.1565
3	4	0.3660+j0.1864	19	20	1.5042+j1.3554
4	5	0.3811+j0.1941	20	21	0.4095+j0.4784
5	6	0.8190+j0.7070	21	22	0.7089+j0.9373
6	7	0.1872+j0.6188	3	23	0.4512+j0.3083
7	8	0.7114+j0.2351	23	24	0.8980+j0.7091
8	9	1.0300+j0.7400	24	25	0.8960+j0.7011
9	10	1.0440+j0.7400	6	26	0.2030+j0.1034
10	11	0.1966+j0.0650	26	27	0.2842+j0.1447
11	12	0.3744+j0.1238	27	28	1.0590+j0.9337
12	13	1.4680+j1.1550	28	29	0.8042+j0.7006
13	14	0.5416+j0.7129	29	30	0.5075+ j0.2585
14	15	0.5910+j0.5260	30	31	0.9744+j0.9630
15	16	0.7463+j0.5450	31	32	0.3105+j0.3619
16	17	1.2890+j1.7210	32	33	0.3410+j0.5362
8	21	2+j2	9	15	2+j2
12	22	2+j2	18	32	0.5+j0.5

Table 2

Optimal Parameter of PSO

Number of iterations	500
Learning Factor (C_1)	2.0
Learning Factor (C_2)	1.5
Inertia weight (w)	0.1

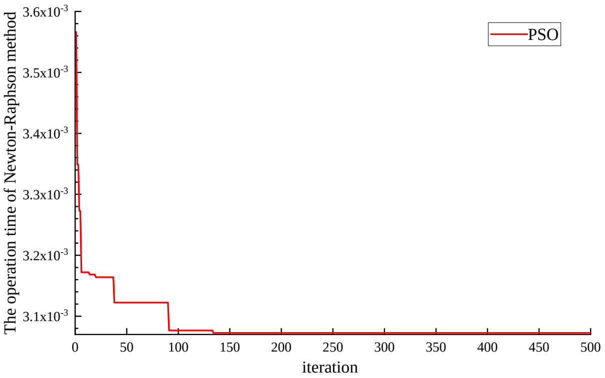

Fig. 3 – Iterative convergence of particle swarm optimization algorithm.

The results of particle swarm optimization iteration are shown in Fig. 3. The particle swarm algorithm is set to iterate 500 times, and we found that when the iteration threshold $\xi_i(t)$ is 1.2999, $f(\xi_i(t))$ converges and has a minimum value. The Jacobian matrix will have the best iteration effect and the power flow calculation efficiency is the highest.

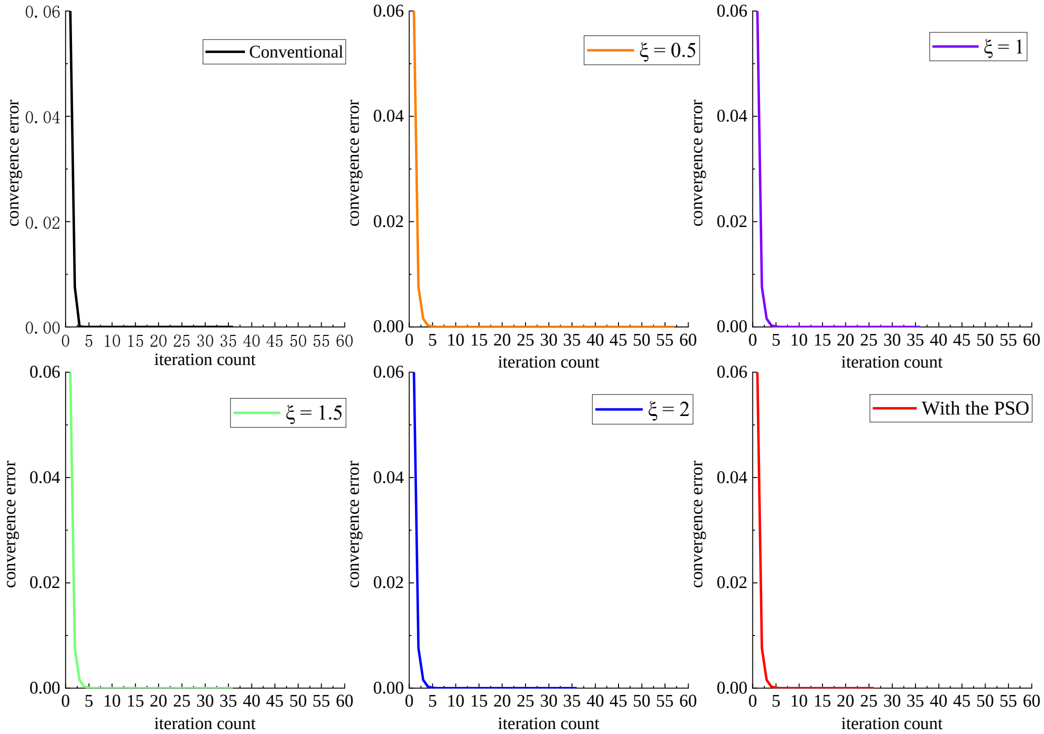

Fig. 4 – Iterative convergence of different methods.

The conventional Newton-Raphson method and the improved Newton-Raphson method are used to calculate the power flow of the IEEE-33 bus power distribution system. The convergence tolerance γ is set to 10^{-14}. The convergence processes of the two methods are shown in Fig. 4. The conventional Newton-Raphson algorithm converges after 37 iterations. In the improved Newton-Raphson algorithm without particle swarm optimization, the iteration threshold ξ is set to 0.5, 1, 1.5, 2 to compare with the improved Newton-Raphson method optimized by the particle swarm optimization. The results show that when the improved Newton-Raphson method is not optimized using particle swarm optimization, the Newton-Raphson algorithm requires 37 iterations to converge when $\xi = 1$, $\xi = 1.5$, and $\xi = 2$, and 55 iterations when $\xi = 0.5$. When using the Newton-Raphson algorithm optimized by particle swarm optimization, the number of iterations is reduced to 27, which greatly improves the efficiency of power flow calculation.

The characteristics (number of iterations, calculation time, and final convergence tolerance) of the conventional Newton-Raphson method and the improved Newton-Raphson method are shown in Table 3. While ensuring that the power fluctuation error is within the convergence tolerance, the conventional Newton-Raphson method has the longest calculation time, reaching 17.09 ms. Due to the different choices of iteration thresholds, there may be variations in the calculation time for the improved Newton-Raphson method. However, the calculation time after improvement is shorter than that of the conventional Newton-Raphson method. When $\xi = 0.5$, the operation time is 16.4 ms. When $\xi = 1.5$, the operation time is 5.62 ms. After using the particle swarm algorithm to optimize the iteration threshold, it can be seen that the power flow calculation time of the improved Newton-Raphson method is 3.04 ms, which is 17.05 ms faster than the calculation time of the conventional Newton-Raphson method, and the calculation efficiency has increased by 660.53%.

The voltage value of the two method results is shown in Table 4, the value of the node voltage calculated by the two methods is consistent, and the maximum error is 4×10^{-15}. Therefore, on the premise of keeping the accuracy of power flow calculation, the improved Newton-Raphson method can effectively improve the calculation rate of power flow.

Table 3

The result of two Newton-Raphson methods

	Conventional Newton-Raphson method	Improved Newton-Raphson method (Without PSO)				Improved Newton-Raphson method (With the PSO)
		ξ = 0.5	ξ = 1.0	ξ = 1.5	ξ = 2	
Calculation time(ms)	20.09	16.4	14.6	5.62	5.74	3.04
Iterations amount	37	58	58	37	37	27
Convergence tolerance	8.65×10^{-15}	8.54×10^{-15}	8.54×10^{-15}	9.38×10^{-15}	9.37×10^{-15}	9.32×10^{-15}

Table 4

The voltage of two Newton-Raphson methods

Node	Conventional Newton-Raphson method	Improved Newton-Raphson method	Node	Conventional Newton-Raphson method	Improved Newton-Raphson method
1	1	1	18	0.913090479361055	0.913090479361059
2	0.997032259729201	0.997032259729201	19	0.996503895654682	0.996503895654681
3	0.982937983395567	0.982937983395567	20	0.992926299531404	0.992926299531404
4	0.975456413220923	0.975456413220922	21	0.992221795820546	0.992221795820547
5	0.968059232356033	0.968059232356032	22	0.991584376857737	0.991584376857737
6	0.949658177395617	0.949658177395617	23	0.979352257335941	0.979352257335941
7	0.946172613505425	0.946172613505425	24	0.972681100969174	0.972681100969174
8	0.941328437217892	0.941328437217892	25	0.969356112454364	0.969356112454364
9	0.935059372180165	0.935059372180165	26	0.947728910132004	0.947728910132004
10	0.929244422592416	0.929244422592415	27	0.945165164232633	0.945165164232633
11	0.928384417163371	0.928384417163371	28	0.933725580913507	0.933725580913507
12	0.926884836746467	0.926884836746467	29	0.925507478359276	0.925507478359277
13	0.920771747551763	0.920771747551764	30	0.921950057873221	0.921950057873221
14	0.918504992768571	0.918504992768573	31	0.917788887087669	0.917788887087669
15	0.917092680117403	0.917092680117406	32	0.916873465734143	0.916873465734143
16	0.915724760079141	0.915724760079144	33	0.916589822133527	0.916589822133527
17	0.913697546157041	0.913697546157045			

5. CONCLUSIONS

The Newton-Raphson algorithm is an effective method for solving nonlinear problems and can be used for power flow calculations in distribution networks. To improve the computing efficiency of the conventional Newton-Raphson method, this paper proposes an improved Newton-Raphson method to increase the computing speed and release computing resources. First, considering the characteristics of the low-voltage distribution network, the calculation of the Jacobian matrix is simplified, saving the calculation time of each iteration. Secondly, the concept of iteration threshold is proposed. When the power fluctuation of power flow calculation is greater than the iteration threshold, the Jacobian matrix is recalculated to improve the accuracy of the calculation. This paper uses the particle swarm optimization algorithm to optimize the threshold and obtains the optimal threshold for calculation by the Newton-Raphson method. In addition, an effective experiment to improve computational efficiency was conducted in an IEEE-33 node distribution network, and the number of iterations, calculation time, and convergence tolerance of the improved Newton-Raphson method were compared with the conventional Newton-Raphson algorithm. The results show that on the premise of maintaining the accuracy of power flow calculation, the improved Newton-Raphson method can reduce 10-31 iterations and increase the calculation efficiency by 660.53%. The overall computing rate is improved and computing resources can be significantly released.

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