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NUMERICAL INVESTIGATION OF HYBRID NANOFLUID FLOW IN A LID DRIVEN CAVITY WITH A HEATED OBSTACLE

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Abstract. This study aims to examine heat transfer and fluid flow around a heated solid obstacle in a lid-driven cavity filled with a hybrid TiO_2-Cu/water nanofluid. The geometry being analyzed is a two-dimensional cavity with an aspect ratio of 5. The upper wall moves at a constant velocity of U_{lid}. The solid obstacle connected to the bottom wall of the cavity is kept at a higher temperature than the top and lower walls, while the remaining walls are insulated. The hybrid nanofluid flow is considered to be Newtonian, laminar, and incompressible. The Richardson number's impact is analyzed by keeping the Reynolds number constant at 100 and adjusting the Grashof number from 10^2 to 10^4. The volume fractions of each nanoparticles range from 0% to 8%. The results are presented in terms of streamlines, isotherms, and profiles of the average Nusselt number. Numerical data indicates that cells rotating in opposite directions are formed inside the rectangular container as a result of the combined influence of natural and forced convection. Increasing the Richardson number from 0.01 to 1, due to heightened buoyancy effect, results in a 4.5% increase in the Nusselt number. An increase of 8% in the volume percentage of nanoparticles for each Richardson number results in a heat transfer rate enhancement of around 9.8%.

Keywords: Richardson number, rectangular cavity, Nusselt number, nanoparticles.

Nomenclature

C_p – heat capacity [J/kg·K\
g – gravitational acceleration [m/s^2\
Gr – Grashof number
H – height of the cavity [m\
h – height of the heated block [m\
k – thermal conductivity [W/m·K\
ℓ – height of the heated block [m\
L – kength of the cavity [m\
Nu – Nusselt number
P – dimensionless pressure
p – pressure [Pa]
Pr – Prandtl number
Re – Reynolds number
Ri – Richardson number
T – dimensionless temperature
u, v – Velocity components [m/s]
U, V – Dimensionless velocity components
x, y – Cartesian coordinates [m]
X, Y – Dimensionless cartesian coordinates.

Greek symbols
α – thermal diffusivity [m^2/s]
β – coefficient of volumetric expansion [1/K]
θ – temperature [K]
μ – dynamic viscosity [kg/m·s]
ρ – density [kg/m^3]
φ – volume concentration [%]

Subscripts
ave – average
hnf – hybrid nanofluid
f – base fluid
c – cold
h – hot
1 – Copper nanoparticles
2 – TiO_2 nanoparticles

1. INTRODUCTION

Over the past three decades, much research has been aimed at developing efficient cooling of electric, electronic and nuclear devices and controlling the fluid flow and heat exchange of the solar thermal operations and thermal storage. Early studies demonstrated analytically and experimentally the potential of nanofluids for cooling of high power density devices [1–3].

Twenty years ago, researchers began to explore convection inside lid-driven cavities. This type of convection involves both natural and forced convection currents in the same environment, and is a useful tool in many different fields of engineering, particularly in heat transfer applications. These studies typically focused on mixed convection and utilized various numerical methods, such as the finite difference method [4], the finite element method [5], and other methods [6–7] to simulate the fluid flow and heat transfer in the cavity.

In recent years, researchers have continued to investigate the effects of using a nanofluid in a lid-driven cavity with a focus on more complex phenomena such as MHD [8], Joule heating [9], and hybrid nanofluid approach [10]. The studies found that the use of a nanofluid in the cavity can enhance the heat transfer and fluid flow in the system. These studies have also made use of more advanced numerical methods such as Lattice Boltzmann Method LBM, Artificial Neural Network ANN [11] and High-Order Compact Scheme [12].

The LBM is used for the first time by Karimipour *et al* [13] to investigate the nanofluid mixed convection in an inclined lid driven cavity. This method is developed to predict the fluid flow and heat transfer of air through the inclined lid driven 2-D cavity. The shallow condition is selected to present the possible practical applications. The Boltzmann collision term and hydrodynamic boundary conditions should be modified to include both buoyancy forces and inclination angle effects together with the cavity lid motion. Results show that Nusselt numbers increases for high values of the inclination angle and nanoparticles volume fraction at free convection domination. The same method is used by Goodarzi *et al* [14] to simulate the heat transfer induced by a heated obstacle within a lid driven cavity. Results are presented for Richardson number that ranged from 0.1 to 10 and for various cavity inclination angles. In addition to the ability of LBM model to simulate the supposed domain, results revealed that the effects of inclination angle are more important at higher values of Richardson numbers.

The studies have also explored the influence of different parameters such as temperature distribution, curvature of the cavity [15, 16], and magnetic field on the fluid flow and heat transfer in the cavity. The heat transfer and laminar flow of Water/Al_2O_3 nanofluid in a T-shaped enclosure with lid-driven under the influence of applying magnetic field have been numerically investigated [17]. The numerical solving domain has been simulated two-dimensionally, and the Richardson number takes the following values: 0.1, 1 and 10. Nanoparticles volume fraction varied between 0 and 6%. The effect of applying homogeneous magnetic field on the natural and forced heat transfer parameters of nanofluid has been analyzed for various Hartman numbers. The results of this research revealed that, applying magnetic field has significant effect on the temperature domain and fluid flow and considerably reduces the circulation mechanisms of fluid. By increasing Richardson and Hartmann numbers, the transfer momentum of cap in the bottom layers of fluid penetrates lesser and the amount of heat transfer reduces. The enhancement of volume fraction of nanoparticles and the reduction of Richardson and Hartmann numbers, significantly enhance the heat transfer of enclosure with cold fluid. Similar results are obtained by Bakar et al [18]. Also, the changes of volume fraction of nanoparticles have less influence on the variations of velocity and the changes of magnetic field intensity greatly affect the velocity field.

Overall, the study of lid-driven cavities with a heated block inside and subjected to nanofluid flow has evolved over the years, with a focus on the use of more advanced numerical methods and the investigation of more complex phenomena.

The mixed convection within a lid-driven square cavity subjected to variable properties of nanofluid having a hot obstacle have been analyzed numerically by Esfe *et al* [19]. Finite volume method with

SIMPLER algorithm is employed for solving the Navier-Stokes and energy equations. A wide range of parameters such as Richardson number ($0.01 < \mathrm{Ri} < 100$), solid volume fraction ($0 < \varphi < 0.05$), temperature of fluid ($300 < T < 340$) and height of heated obstacle ($0.1 < h < 0.3$) have been examined. Numerical results show that the average Nusselt number decreases with increasing temperature of nanofluid for the whole range of the Richardson numbers.

Recently, the lid-driven top walls influence combined with the side walls waviness map induce the mixed convection heat transfer, flow behavior, and entropy generation of a hybrid nanofluid is studied by Maneengam *et al.* [20]. The working fluid occupies a permeable cubic chamber and is subjected to a magnetic field. The governing equations are solved by employing the GFEM method. The results show that the magnetic force significantly affects the working fluid thermal and flow behavior, where the magnetic force perpendicular direction remarkably improves the thermal distribution at $\mathrm{Re} = 500$. Also, increasing Ha and decreasing Re drops both the irreversibility and the heat transfer rate. In addition, the highest undulation number on the wavy-sided walls gives the best heat transfer rate and the highest irreversibility.

The homogenous models were predominant in hybrid nanofluid’s studies. However, a serious deficiency in observed when estimating the heat transfer coefficients compared with the experimental results. A recent study [21] focused on the development of a novel hybrid nanofluid model based on the major slip mechanisms of nanofluids caused by Brownian diffusion and thermophoresis. The problem of flow and heat transfer of a hybrid nanofluid in a lid driven square cavity filled with porous medium is studied as an example to show the correctness and validity of this new model. The average Nusselt number evaluated with the help of the quadratic multiple regression analysis justifies that the thermophoretic effect is more dominant than the Brownian motion on Nusselt numbers at both walls.

Mixed convection in a rectangular double lid-driven cavity filled with hybrid nanofluid (Al_2O_3-Cu-water) subjected to insulated sidewalls and sinusoidal temperature on horizontal walls is numerically investigated [22]. The effects of amplitude ratio, phase deviation, and Reynolds numbers on the flow and heat transfer characteristics are discussed. It is found that the rate of heat transfer is improved as the volume fraction of the hybrid nanoparticles and the amplitude ratio are increased. The non-uniform heating at cavity walls tend to provide higher heat transfer rate and the heat transfer rate increases with respect to Reynolds number. Recently, Engineers and scientists from different fields are harnessing the power of machine learning to identify the underlying relationships between data from complex systems or models of different degrees of complexity. Simulation data is used to develop a novel artificial neural network model for efficient predictions of heat and mass transfer within cavities [23–24]. M.H. Borbora *et al.* [25] provides a comprehensive review of numerical simulations focusing on lid-driven cavity flow with nanofluids. The study covers a wide range of topics related to the use of nanofluids in lid-driven cavity flows. It discusses various numerical simulation techniques employed in the study of nanofluid behavior, including their thermal and flow characteristics. The review offers insights into the advancements, challenges, and future directions in this field, providing valuable information for researchers and engineers working on nanofluid applications in fluid dynamics.

This study presents the findings of mixed convection in a lid-driven cavity containing a hybrid nanofluid with a heated obstacle. The cavity has an aspect ratio of 5, and the hybrid nanofluid consists of Copper and TiO_2 nanoparticles in water. The study examined the impact of different Grashof numbers and the volume proportion of nanoparticles.

2. GEOMETRICAL CONFIGURATION

The mixed convection of hybrid nanofluid of Titania (TiO_2) and Copper (Cu) with water as base fluid is numerically investigated in a lid driven cavity. A solid rectangular block is attached to the bottom wall of the cavity and maintained at high temperature than the upper wall, which translates with uniform velocity U_{lid}.

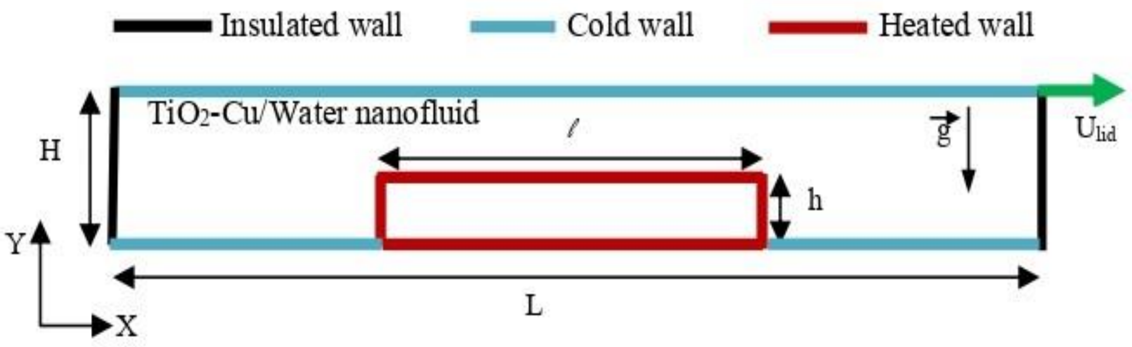

Fig. 1 – Configuration of the lid driven cavity with a hot block inside.

The side walls of the cavity are kept insulated. The aspect ratio of the cavity is fixed to L/H=5, where L is the length and H is the height of the cavity. Length and height of the solid rectangular block are respectively: h=$H/2$ and ℓ=$L/2$. A schematic view of the studied cavity is sketched in Figure 1. The thermophysical properties of water and nanoparticles are listed in Table 1.

Table 1

Thermo-physical properties of water and nanoparticles at T = 300 K ([26])

	H_2O	Cu	TiO_2
C_p (J.kg^{-1}·K^{-1})	4179	385	686.2
ρ (kg/m^3)	997.1	8933	4250
k (W.m^{-1}·K^{-1})	0.613	401	8.95
β (K^{-1})	21×10^{-5}	1.67×10^{-5}	0.9×10^{-5}
μ (kg.m^{-1}·s^{-1})	0.855×10^{-3}	–	–

3. GOVERNING EQUATIONS AND BOUNDARY CONDITIONS

The governing equations for the laminar, Newtonian hybrid nanofluid flow inside the lid driven cavity are mathematically described by the Navier-Stokes equations, and by energy equation (1–4). It is assumed that the flow is single phase and the water is in thermal balance with nanoparticles. In addition, the flow is assumed to be incompressible and the Boussinesq approximation is considered.

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\rho_{hnf}(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y})=-\frac{\partial p}{\partial x}+\mu_{hnf}\left[\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right] \tag{2}$$

$$\rho_{hnf}(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y})=-\frac{\partial p}{\partial x}+\mu_{hnf}\left[\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right] \tag{3}$$

$$\rho_{hnf}(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y})=-\frac{\partial p}{\partial y}+\mu_{hnf}\left[\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right]+(\rho\beta)_{hnf}g(\theta-\theta_c) \tag{4}$$

The following equation is used to calculate the density of the hybrid nanofluid:

$$\rho_{hnf}=(1-\varphi)\rho_f+\varphi_1\rho_1+\varphi_2\rho_2 \tag{5}$$

ρ_f, ρ_1 and ρ_2 are respectively the density of water, Copper and TiO_2 nanoparticles respectively. φ_1 and φ_2 denote respectively the volume fraction of the Cu nanoparticles and TiO_2 nanoparticles. The effective thermal conductivity and the specific heat capacity of the hybrid nanofluid respectively:

$$\alpha_{hnf} = \frac{k_{hnf}}{(\rho C_p)_{hnf}} \tag{6}$$

$$(\rho C_p)_{hnf} = (1-\varphi)(\rho C_p)_f + \varphi_1(\rho C_p)_1 + \varphi_2(\rho C_p)_2 \tag{7}$$

$$(\rho\beta)_{hnf} = (1-\varphi)(\rho\beta)_f + \varphi_1(\rho\beta)_1 + \varphi_2(\rho\beta)_2 \tag{8}$$

The hybrid nanofluid dynamic viscosity is estimated using the model of Brinkman [27]:

$$\mu_{hnf} = \mu_f(1-\varphi)^{-2.5} \tag{9}$$

The thermal conductivity of the hybrid nanofluid k_{hnf} was given by Maxwell-Garnet model [28]:

$$\frac{k_{hnf}}{k_f} = \frac{\left(\frac{\varphi_1 k_1 + \varphi_2 k_2}{\varphi}\right) + 2k_f + 2(\varphi_1 k_1 + \varphi_2 k_2) - 2\varphi k_f}{\left(\frac{\varphi_1 k_1 + \varphi_2 k_2}{\varphi}\right) + 2k_f - (\varphi_1 k_1 + \varphi_2 k_2) + \varphi k_f} \tag{10}$$

In the previous equations: $\varphi = \varphi_1 + \varphi_2$. The hybrid nanofluid consisting equal solid volume fraction of Cu and TiO_2 nanoparticles.

The boundary conditions for the considered problem can be expressed as:

- A slip velocity is considered for the upper wall: $u = U_{lid}$.
- No slip boundary conditions are considered over all the other walls.
- Temperature of the upper wall is $\theta = \theta_c$ and the temperature of the heated block is $\theta = \theta_h$.
- The side walls of the cavity are kept insulated.

The dimensionless variables for the considered problem are:

$$X = \frac{x}{H},\ Y = \frac{y}{H},\ P = \frac{p}{\rho_{nf} U_{lid}^2},\ U = \frac{u}{U_{lid}},\ V = \frac{v}{U_{lid}},\ T = \frac{\theta - \theta_c}{\theta_h - \theta_c}.$$

By using the dimensionless parameters, the governing equations are written in the following form:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \tag{11}$$

$$U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + \frac{1}{\mathrm{Re}}\frac{\mu_{hnf}}{\rho_{hnf}}\frac{\rho_f}{\mu_f}\left[\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2}\right] \tag{12}$$

$$U\frac{\partial V}{\partial X} + V\frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + \frac{1}{\mathrm{Re}}\frac{\mu_{hnf}}{\rho_{hnf}}\frac{\rho_f}{\mu_f}\left[\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2}\right] + \frac{\mathrm{Gr}}{\mathrm{Re}^2}\frac{(\rho\beta)_{hnf}}{\rho_{hnf}\beta_f}T \tag{13}$$

$$U\frac{\partial T}{\partial X} + V\frac{\partial T}{\partial Y} = \frac{1}{\mathrm{Re}\cdot\mathrm{Pr}}\frac{\alpha_{hnf}}{\alpha_f}\left[\frac{\partial^2 T}{\partial X^2} + \frac{\partial^2 T}{\partial Y^2}\right] \tag{14}$$

4. NUMERICAL METHODS

The finite volumes method is utilized to discretize the governing equations. The software used is the commercial computational fluid dynamics code FLUENT 16.0. The SIMPLE technique is utilized for linking mass and momentum equations [29]. The convergence criterion was set at 10^{-6} for the normalized residual of each equation. We applied relaxation factors of 0.7 for velocities and energy equation, and 0.3 for the pressure.

4.1. Code validation

The mathematical model and numerical method were validated by comparing them to the results of Hirpho and Ibrahim [30] for steady mixed convection in a partially heated trapezoidal enclosure. The cavity is filled with Al_2O_3-Cu/water hybrid nanofluid, presumed to be in a laminar and incompressible state. The top wall of the enclosure is insulated and moves at a constant speed U_{lid}. The side walls of the enclosure are kept at a uniforme temperature T_c. The middle portion of the bottom wall is heated and maintained at uniforme temperature T_h. The remaining parts of the bottom wall are maintained insulated.

For a constant Reynolds number of 100, various Richardson numbers of 0.1, 1 and 10 were examined, together with varying volume fractions ($0 \leq \varphi \leq 0.02$). The mean Nusselt number obtained from our numerical analysis closely matched the values reported by Hirpho and Ibrahim as demonstrated in Table 2.

Table 2

The validation of the mean Nusselt numbers for Re = 100 and $\varphi = 0.01$

Ri	Nu_{ave} (Hirpho and Ibrahim)	Nu_{ave} (This study)
0.1	4.3960	4.3325
1	9.7698	9.8891
10	17.569	17.4886

4.2. Grid independence

A grid independent technique is utilized to identify the appropriate number of nodes by employing three meshes of varying grid sizes. The Reynolds number, calculated using the inflow velocity and the microchannel height, was determined to be Re = 100. When the angle was 0%, the average difference between the first two meshes was less than 0.40%, and less than 0.70% between the last two meshes for the mean Nusselt number of the heated surface, as shown in Table 3. This is the reason why the final mesh was implemented.

Table 3

Effect of the mesh size on the mean Nusselt number

Mesh size	Mean Nusselt number	Deviation %
500×100	7.15	-
550×110	7.24	1.24
600×120	7.29	0.68

5. RESULTS AND DISCUSSIONS

The current simulation presents numerical results for the laminar mixed convection of a hybrid TiO_2-Cu/Water nanofluid in a lid-driven cavity. The Richardson numbers considered in this study range from Ri = 0.01 to Ri = 1, while maintaining a fixed Reynolds number of Re = 100. The variation in Richardson number results in the Grashof number ranging from 10^2 to 10^4. The impact of nanoparticles on heat transfer rate is examined by changing the nanoparticles fraction from 0% to 8%. Numerous studies have extensively discussed the effect of the percentage of additional nanoparticles, and almost all works concluded that a high concentration of nanoparticles boosts the thermal conductivity of the host fluid.

The Nusselt number for each horizontal section of the bottom wall can be defined as:

$$\mathrm{Nu}(x) = -\frac{k_{hnf}}{k_f}\left(\frac{\partial\theta}{\partial y}\right) \tag{15}$$

The local Nusselt number of each vertical part of the lower wall can be expressed as:

$$\mathrm{Nu}(y) = -\frac{k_{hnf}}{k_f}\left(\frac{\partial\theta}{\partial x}\right) \tag{16}$$

The average Nusselt number of the heated surface is defined by:

$$\mathrm{Nu}_{ave} = \mathrm{Nu}_m|_x + \mathrm{Nu}_m|_y \tag{17}$$

where $\mathrm{Nu}_m|_x$ and $\mathrm{Nu}_m|_y$ denote the integration of the formula (15) and (16) over the length of each horizontal and vertical part of the hot block respectively.

5.1. Effect of Richardson number

Figure 2 shows how the average Nusselt number changes as the volume percentage of nanoparticles increases. The Nusselt number grows approximately linearly with the volume percentage of nanoparticles for all Richardson values under consideration. The presence of Copper and TiO_2 hybrid nanoparticles in the receiving fluid enhances heat transfer by increasing the total thermal conductivity of the hybrid nanofluids. Higher Richardson values correspond to more significant changes in heat transmission and Nusselt number. For a wide range of Richardson numbers, similar outcomes are reported by Esfe *et al*. [19] for a lid-driven cavity with an inside obstacle located at the bottom wall of the cavity, and subjected to Al_2O_3-water nanofluid. It is also reported by Munawar *et al*. [31] that adding 2% of Ag-MgO hybrid nanoparticles causes an uprise of the average Nusselt number by about 18.3%, for the mixed convection regime in a square enclosure heated with a circular center heater.

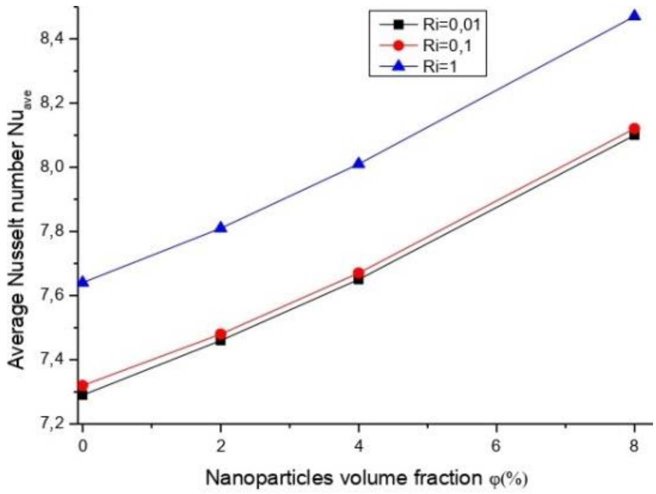

Fig. 2 – Average Nusselt number for different volume fractions.

Figure 3 displays the streamlines and isotherms for various Richardson numbers with a constant nanoparticle volume fraction of $\varphi = 0.08$. At this stage, the flow pattern shows two clockwise cells that have nearly filled the whole cavity. The first one is positioned in the upper left corner of the cavity and stretched towards the right side due to the shear force generated by the velocity of the lid wall. The second cell is situated on the right side of the heated obstacle. An growing buoyancy effect generates a small counterclockwise cell in the lower left corner of the hollow. In the most hybrid nanofluid studies, the nanoparticles fraction was limited to avoid decreasing the fluidity of the liquid, this is why in this study, the volume fraction of nanoparticles has been set at 0.08 to avoid any flow stagnation that might be affected by the high nano-concentration, and to benefit from the desirable role of these particles in heat transfer enhancement.

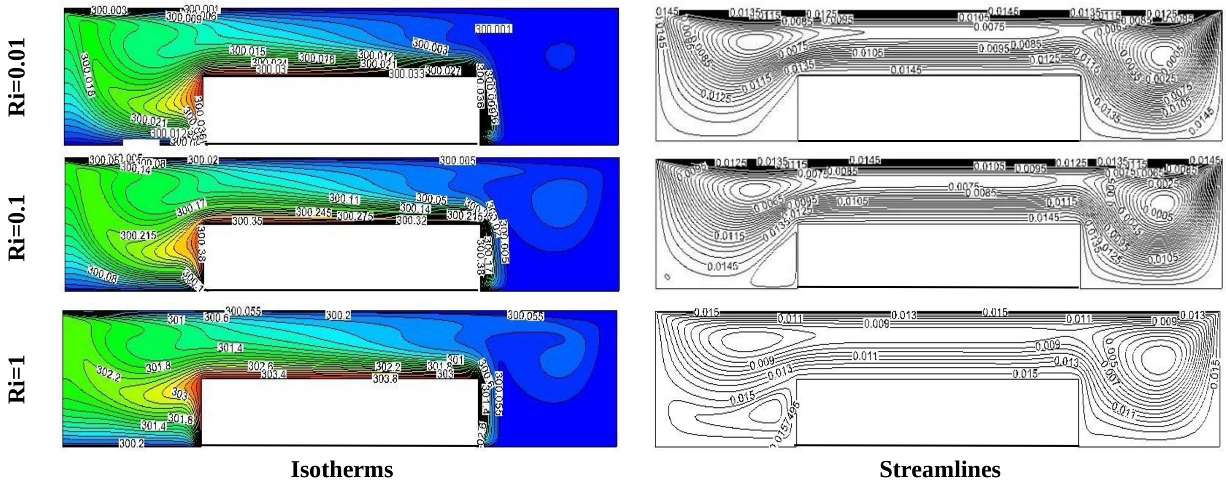

Fig. 3 – Streamlines and isotherms for φ = 0.08.

Increasing the Richardson number caused this cell to expand. Temperature lines are very prominent around the hot block, indicating a significant temperature gradient. Isotherms diverge as they move away from the hot obstruction, resulting in a decrease in temperature gradient.

5.2. Effect of nanoparticles fraction variation on heat and fluid flow

Fig. 4 displays streamlines and isotherms for Ri = 1. Two prominent cells occupy the whole cavity without notable changes in volume percentage, whereas a small cell forms on the left side of the heated block. Additionally, isotherms are closely packed and condensed near the heated block because of the temperature differential that occurs.

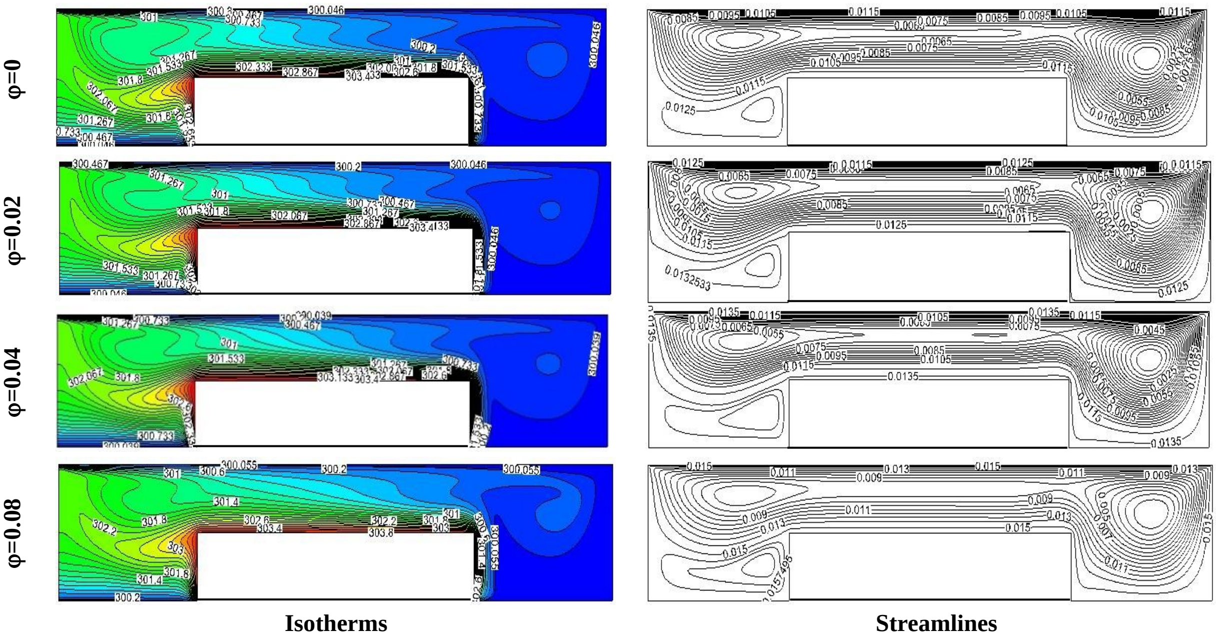

Fig. 4 – Streamlines and isotherms for Ri = 1.

Figure 5 shows how the average Nusselt number changes with the Richardson number for various nanoparticle fractions. At a specific volume fraction, hybrid nanofluids improve heat transmission in the cavity more than Cu-based and TiO_2-based nanofluids. The heat rate increases steadily as the Ri number rises because of the heightened buoyancy effect. Similar numerical results are obtained by Alsabery *et al*. [32] for a lid driven cavity with wavy walls, filled by Cu-Al_2O_3 hybrid nanofluid and having a localized solid block.

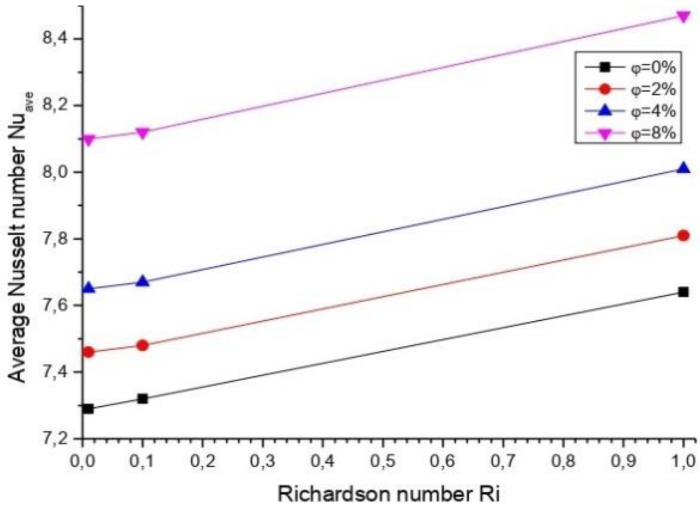

Fig. 5 – Average Nusselt number for different Richardson numbers.

6. CONCLUSIONS

The current study quantitatively examines the mixed convection of a hybrid nanofluid flow within a rectangular lid-driven cavity. A heated block is connected to the bottom wall of the cavity, while the side walls are insulated. The hybrid nanofluid consists of Copper and TiO_2 nanoparticles in thermal equilibrium with water as the base fluid. The flow is single-phase and is controlled by the Navier-Stokes equations and the energy equation in the two-dimensional scenario. The Reynolds number in the simulations is set at $Re = 100$, while the Grashof number ranges from $Gr = 10^2$ to $Gr = 10^4$. The impact of nanoparticles is evaluated by changing the volume fraction of nanoparticles from 0% to 8%. Numerical findings showed that the interaction of natural and forced convection generates clockwise and counterclockwise cells within the rectangular enclosure. Increasing the Richardson number from $Ri = 0.01$ to $Ri = 1$ results in an amplified buoyancy effect, leading to a 4.5% augmentation in the Nusselt number across all volume fractions. Moreover, the heat transfer rate increases by approximately 9.8% for each Richardson number as the volume percentage of nanoparticles is increased.

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