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A COMPACT EXPRESSION FOR THE SUPERSYMMETRIC LAGRANGIAN

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Abstract. In this work we present a short review and an alternative, more compact expression of the action of an abelian gauge supersymmetric theory with a chiral superfield. This method can be generalized to any supersymmetric Lagrangian and it is stemming from a comprehensive framework introduced in previous works that might include both gravity and quantum field theories.

Keywords: supersymmetry, alternative Lagrangian, unified theories.

1. INTRODUCTION

Since the dawn of supersymmetric theories (for a review see [1]) with their supergravity extensions and superstring generalizations these have been standing at the forefront of many theoretical endeavours and experimental verification (see for instance [2–5]).

Unfortunately up to today no hint of experimental signatures has been detected at least for the most straightforward implementations of supersymmetry in the standard model of elementary particles.

In this work we show, based on our previous works [6, 7] that there is a framework in which the presence or absence of supersymmetry is natural and can be regarded as multiple facets of the same theory.

Section II contains a short review of basic supersymmetry bricks. In section III we describe our perspective and an alternative way to implement the supersymmetric action. Section IV is dedicated to the Conclusions.

2. BASIC OF CONSTRUCTING THE SUPERSYMMETRIC LAGRANGIAN

We start with superspace which consists in the four space time coordinate x^μ and four fermionic coordinates θ^α, $\theta^\dagger_{\dot\alpha}$. Here the fermionic coordinates are constant complex anti-commuting spinors with mass dimension $m^{-\frac{1}{2}}$.

One can define a generalized coordinate,

$$y^\mu = x^\mu + i\theta^\dagger\bar\sigma^\mu\theta. \tag{1}$$

In terms of this, the chiral superfield has the explicit expression:

$$\begin{aligned}\Phi &= \phi(y) + \sqrt{2}\theta\Psi(y) + \theta\theta F(y)\\ \Phi^* &= \phi^*(y^*) + \sqrt{2}\theta^\dagger\Psi^\dagger(y^*) + \theta^\dagger\theta^\dagger F^*(y^*).\end{aligned} \tag{2}$$

In Eq. (2) the fields $\phi(y)$ ($\phi^*(y^*)$), $\Psi(y)$ ($\Psi^\dagger(y^*)$) and $F(y)$ ($F^*(y^*)$) are respectively the scalar field, the fermion field and the auxiliary field part of a supermultiplet. They are all related through supersymmetry transformations.

One can define the chiral covariant derivatives in terms of $(y,\theta,\theta^\dagger)$,

$$\begin{aligned}
\mathrm{D}_\alpha &= \frac{\partial}{\partial\theta^\alpha} - 2i(\sigma^\mu\theta^\dagger)_\alpha\frac{\partial}{\partial y^\mu}\\
\mathrm{D}^\alpha &= -\frac{\partial}{\partial\theta_\alpha} + 2i(\theta^\dagger\bar\sigma^\mu)^\alpha\frac{\partial}{\partial y^\mu}\\
\bar{\mathrm{D}}^{\dot\alpha} &= \frac{\partial}{\partial\theta^\dagger_{\dot\alpha}}\\
\bar{\mathrm{D}}_{\dot\alpha} &= -\frac{\partial}{\partial\theta^{\dagger\dot\alpha}}.
\end{aligned} \tag{3}$$

and $(y^*,\theta,\theta^\dagger)$:

$$\begin{aligned}
\mathrm{D}_\alpha &= \frac{\partial}{\partial\theta^\alpha}\\
\mathrm{D}^\alpha &= \frac{\partial}{\partial\theta_\alpha}\\
\bar{\mathrm{D}}^{\dot\alpha} &= \frac{\partial}{\partial\theta^\dagger_{\dot\alpha}} - 2i(\bar\sigma^\mu\theta)^{\dot\alpha}\frac{\partial}{\partial y^{\mu *}}\\
\bar{\mathrm{D}}_{\dot\alpha} &= -\frac{\partial}{\partial\theta^{\dagger\dot\alpha}} + 2i(\theta\sigma^\mu)_{\dot\alpha}\frac{\partial}{\partial y^{\mu *}}.
\end{aligned} \tag{4}$$

For the gauge abelian theory one needs the gauge superfield in terms of $y^\mu,\theta,\theta^\dagger$:

$$\begin{aligned}
&V(y) = \theta^\dagger\bar\sigma^\mu\theta A_\mu(y) + \theta^\dagger\theta^\dagger\theta\lambda(y) + \theta\theta\theta^\dagger\lambda^\dagger(y) +\\
&\frac{1}{2}\theta\theta\theta^\dagger\theta^\dagger[D(y) + i\partial_\mu A^\mu(y)].
\end{aligned} \tag{5}$$

Here $A_\mu(y)$ is the gauge field, $\lambda(y)$, $\lambda^\dagger(y)$ are the gauginos and $D(y)$ is a real bosonic auxiliary field.

We leave out here any details which may be found in any textbook (see for example [8]).

From Eqs. (2), (4) and (5) one may construct the supersymmetric abelian gauge invariant Lagrangian:

$$\begin{aligned}
&\mathscr{L} = \int \mathrm{d}^2\theta\int \mathrm{d}^2\bar\theta\,\Phi_i^*\exp[2gqV]\Phi_i +\\
&\int \mathrm{d}^2\theta W(\Phi) +\\
&\int \frac{1}{4}\mathrm{d}^2\theta W^\alpha W_\alpha + c.c. +\\
&\int \mathrm{d}^2\theta\int \mathrm{d}^2\bar\theta(-2kV).
\end{aligned} \tag{6}$$

Here we used:

$$\begin{aligned}
W_\alpha &= -\frac{1}{4}\bar{\mathrm{D}}\bar{\mathrm{D}}\mathrm{D}_\alpha V\\
W^\dagger_{\dot\alpha} &= -\frac{1}{4}\mathrm{D}\mathrm{D}\bar{\mathrm{D}}_\alpha V.
\end{aligned} \tag{7}$$

Moreover q is the electric charge, g is the coupling constant and k is a constant and the superpotential W is considered of the form:

$$W = \frac{1}{2}M^{ij}\Phi_i\Phi_j + \frac{1}{6}y^{ijk}\Phi_i\Phi_j\Phi_k. \tag{8}$$

In the equation above Φ_i are the chiral superfields, M^{ij} is a symmetric mass matrix and y^{ijk} are Yukawa couplings.

3. THE ALTERNATIVE ACTION

We shall first display the Lagrangian for the abelian gauge kinetic terms for the gauge field and one chiral superfield then we will show how it can be easily extended to include the rest of the terms in Eq. (6). We consider four more fermionic coordinates θ_1^{α}, $\theta_{1\dot{\alpha}}^{\dagger}$.

We introduce the Lagrangian:

$$\mathscr{L}_1 = \int d^2\theta\, d^2\bar{\theta}\, d^2\theta_1\, d^2\bar{\theta}_1 \exp[X]\exp[Y]\exp[qgV(y)]\exp[X^{\dagger}]\exp[Y^{\dagger}]\exp[gqV(y)], \tag{9}$$

where

$$\begin{aligned} X &= \theta_1\theta_1\Phi(y) \\ Y &= k_2\frac{i}{qg}\theta_1\bar{\theta}_1 D\bar{D} - k\theta_1\bar{\theta}_1\theta_1\bar{\theta}_1 V, \end{aligned} \tag{10}$$

where k_2 and k are constant chosen such as to lead to the correct coefficients of the kinetic and the Fayet-Iliopoulos terms in the Lagrangian.

If one consider the first term in the expansion of $\exp[Y] = 1 + ...$ one notices the apparition of the gauge invariant kinetic term for the superfield:

$$\begin{aligned} &\int d^2\theta\, d^2\bar{\theta}\, d^2\theta_1\, d^2\bar{\theta}_1\, \theta_1\theta_1\Phi\exp[2gqV]\bar{\theta}_1\bar{\theta}_1\Phi^* = \\ &\int d^2\theta\, d^2\bar{\theta}\, \Phi\exp[2qgV]\Phi^*, \end{aligned} \tag{11}$$

as it can be seen form the first term of Eq. (6).

We expand $\exp[Y]$ and $\exp[Y^{\dagger}]$ to get:

$$\begin{aligned} &\int d^2\theta\, d^2\bar{\theta}\, d^2\theta_1\, d^2\bar{\theta}_1 \exp[X]\exp[Y]\exp[qgV(y)]\exp[X^{\dagger}]\exp[Y^{\dagger}]\exp[gqV(y)]^* \propto \\ &\int d^2\theta\, d^2\bar{\theta}\, d^2\theta_1\, d^2\bar{\theta}_1 \exp[-2k\bar{\theta}_1\theta_1\bar{\theta}_1\theta_1 V]\times \\ &\left[1 + \theta_1\bar{\theta}_1\theta_1\bar{\theta}_1[\frac{k_2}{2}[D\bar{D}D\bar{D}V(1+gqV+...) - k_2\bar{D}D\bar{D}DV(1+qgV+...)] + D\bar{D}V\bar{D}DV + ...] + ...\right] \propto \\ &\int d^2\theta\, d^2\bar{\theta} - \frac{k_2}{2}\left[(D\bar{D}V)(D\bar{D}V) + (\bar{D}DV)(\bar{D}DV)\right] - 2kV = \\ &-\frac{k_2}{2}\left[\int d^2\theta(\bar{D}^2DV)(\bar{D}^2DV) + \int d^2\bar{\theta}(D^2\bar{D}V)(D^2\bar{D}V)\right] - kD. \end{aligned} \tag{12}$$

The first two terms in the square bracket of the third line of the above equation lead to total derivatives (see the arguments in what follows) and we can drop them.

Here we used,

$$\mathrm{D}\bar{\mathrm{D}} = -\bar{\mathrm{D}}\mathrm{D} + A, \tag{13}$$

where A is the operator resulting from the corresponding anticommutation relations, and,

$$\begin{aligned} &\mathrm{D}\bar{\mathrm{D}}V\bar{\mathrm{D}}\mathrm{D}V = \\ &-\frac{1}{2}\mathrm{D}\bar{\mathrm{D}}V(\mathrm{D}\bar{\mathrm{D}}V) - \frac{1}{2}\bar{\mathrm{D}}\mathrm{D}V(\bar{\mathrm{D}}\mathrm{D}V) + AVAV, \end{aligned} \tag{14}$$

Since A contains the momentum operator with respect to Y this will lead to a type of gauge fixing term for the vector field A_μ. We also used the chiral covariance condition for the gauge fields strengths $W_\alpha, W_{\dot{\alpha}}$.

In order to introduce the superpotential we just make the replacement $X' = X + k_3\bar{\theta}_1\bar{\theta}_1\theta_1\theta_1\bar{\theta}\theta W$ where W is the standard superpotential in terms of powers of Φ and k_3 is a dimensionless constant.

4. CONCLUSIONS

Based on previous works [6, 7] we showed that a basic gauge abelian supersymmetric theory with a chiral superfield can be implemented naturally in a comprehensive action in a framework which may include both gravitation and quantum field theory. The results in this paper can also be regarded as an alternative, more compact approach to writing the action of a supersymmetric theory which may provide more insights into its rapport with experimental consequences.

REFERENCES

[1] Martin SP. A supersymmetry primer. arXiv 1997: hep-ph/9709356.

[2] The Atlas Collaboration. Search for direct production of electroweakinos in final states with one lepton, missing transverse momentum and a Higgs boson decaying into two b-jets in pp collisions at sqrt(s) = 13 TeV with the ATLAS detector. Eur. Phys. J. 2020;80:691. arXiv: 1909.09226.

[3] The Atlas Collaboration. Search for direct production of electroweakinos in final states with one lepton, jets and missing transverse momentum in pp collisions at sqrt(s) = 13 TeV with the ATLAS detector. JHEP (Journal of High Energy Physics) 2023;12:167. arXiv 2023: 2310.08171.

[4] The CMS Collaboration. Search for supersymmetry in final states with disappearing tracks in proton-proton collisions at sqrt(s) = 13 TeV. Phys. Rev. D 2024;109:072007 .

[5] The CMS Collaboration. Search for supersymmetry in final states with a single electron or muon using angular correlations and heavy-object identification in proton-proton collisions at sqrt(s) = 13 TeV. JHEP (Journal of High Energy Physics) 2023;9:149.

[6] Jora R. Gravity and matter action in a unifying framework. Rom. J. Phys. 2022;67(7-8):112.

[7] Jora R. Gravity and matter reinterpreted. Rom. J. Phys. 2023;68(7-8):110.

[8] Aitchinson I. Supersymmetry in particle physics: an elementary introduction. Cambridge University Press; 2007.

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