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CURRENTS OF QUANTUM ORIGIN IN UNIFORM ACCELERATED FRAMES

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Abstract. In the present paper we are using the solutions to the $SO(3,1)-$gauge invariant Dirac equation written on a Rindler spacetime to work out the conserved current density components. Within the so-called semi-relativistic approximation, the *up* and *down* four-spinors are describing either spatially decaying modes or a propagating field leading to a non-zero current along Oz. A direct connection between the critical value of the acceleration and the spinor's phase is established.

Key words: Rindler spacetime, Dirac equation, conserved current.

1. INTRODUCTION

The phenomena of creation of scalars and fermions in a system of coordinates associated with an observer moving with asymptotically uniform acceleration has a long history [1].

As it is known, an accelerated charged particle is seen by a comoving observer as being at rest in a gravitational field while, for an accelerating observer, the particle in a inertial motion is freely falling in the gravitational field.

Despite of being the simplest model, the Rindler background has received considerable attention, in the context of a better understanding of the deep relation between the Larmor radiation and the equivalence principle. Following the seminal paper of Unruh and Wald [2], a lot of works have tried to find an answer to the fundamental question: *What happens when an accelerating observer detects a Rindler particle ?*

In order to understand how quantum systems are affected by the structure of spacetime, the equations describing relativistic particles moving on curved manifolds have been intensively worked out, [3–5]. As for the Rindler spacetime, recently, in a (1+1)-dimensional toy model, the Dirac equation has been discussed [6], pointing out that the Zitterbewegun of a free relativistic electron is an observable phenomenon in non-inertial frames [7].

Even though most of the authors are using a two-dimensional setting, the four-dimensional theory is necessary to be developed in view of a complete quantization procedure [8]. As a first step in studying quantum effects of charged particles in accelerated frames, it is important to derive exact solutions to the field equations. In this respect, the free of coordinates formalism based on Cartan's equations, which is employed in the present paper, is particularly useful.

Our main motivation for this work is to provide more physical results coming from the analysis developed in our previous paper [9], devoted to the $SO(3,1)-$gauge invariant Dirac equation on Rindler spacetime, in both Dirac and Weyl representations. Similar results have been recently obtained for a free scalar field satisfying the Klein-Gordon equation, where the Rindler modes have been derived in terms of the modified Bessel function of the second kind [10]. The authors are studying the deep connection between acceleration, radiation, and the Unruh effect.

In the next section, after briefly recapitulating the mathematical formalism, we are investigating the conserved current density components, with a particular emphasis on the semi-relativistic case.

Our results are different from the ones derived in the non-relativistic picture developed in [11], where the authors have started with the classical Lagrangian and they have shown that the Schrödinger equation for the accelerated particle can be reduced to the one dimensional hydrogen atom problem. As the quantized energy levels has been found identical with those of the quantum harmonic oscillator, the existence of a new kind of quanta can be predicted.

2. DIRAC EQUATION AND CURRENT COMPONENTS

Using the following transformation formulas between the inertial and hyperbolic coordinates

$$X = x\,, \quad Y = y\,, \quad Z = z\cosh(\alpha t)\,, \quad T = z\sinh(\alpha t)\,, \;\; \alpha \in \mathbf{R} \tag{1}$$

the Minkowski line element $\mathrm{d}s^2 = (\mathrm{d}X)^2 + (\mathrm{d}Y)^2 + (\mathrm{d}Z)^2 - (\mathrm{d}T)^2$ turns into the Rindler metric with

$$\mathrm{d}s^2 = (\mathrm{d}x)^2 + (\mathrm{d}y)^2 + (\mathrm{d}z)^2 - (\alpha z)^2(\mathrm{d}t)^2 \tag{2}$$

In order to develop a $SO(3,1)-$gauge invariant approach, we have introduced the ortho-normal bases [9]

$$\omega^{\mu} = \mathrm{d}x^{\mu}\,, \quad \omega^4 = (\alpha z)\mathrm{d}t\,; \quad e_{\mu} = \partial_{\mu}\,, \quad e_4 = \frac{1}{\alpha z}\,\partial_t \tag{3}$$

where $\mu = \overline{1,3}$ and derived, using the first Cartan's equation

$$\mathrm{d}\omega^a = \Gamma^a_{[bc]}\,\omega^b \wedge \omega^c\,, \quad 1 \le b < c \le 4$$

the one-form connection

$$\Gamma_{34} = \frac{1}{z}\,\omega^4 \tag{4}$$

By employing the second Cartan's equation, one may notice that the essential curvature 2-form component $\mathbf{R}_{34} = d\Gamma_{34}$ is vanishing, pointing out the existence of a flat spacetime. We only deal with an uniformly-accelerated frame with $\alpha = \mathrm{a}/c^2$, where a is the proper acceleration.

The relativistic fermion is described by the Dirac equation

$$\gamma^a \left[\psi_{|a} + \frac{1}{4}\Gamma_{bca}\gamma^b\gamma^c\,\psi \right] + m_0\psi = 0 \tag{5}$$

with $\psi_{|a} = e_a\psi$, which has the explicit form

$$\gamma^a\psi_{|a} + \frac{1}{2z}\gamma^3\psi + m_0\psi = 0$$

In terms of the defined tetrads (3), the above equation looks like

$$\gamma^1\psi_{,1} + \gamma^2\psi_{,2} + \gamma^3\left[\psi_{,3} + \frac{1}{2z}\,\psi\right] + \frac{1}{\alpha z}\gamma^4\psi_{,4} + m_0\psi \,=\, 0 \tag{6}$$

and we are using the Dirac representation, $\gamma^{\mu} = -i\beta\,\alpha^{\mu}$, $\gamma^4 = -i\beta$, where

$$\beta = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}\,, \quad \alpha^{\mu} = \begin{pmatrix} 0 & \sigma^{\mu} \\ \sigma^{\mu} & 0 \end{pmatrix}$$

in signature (+2)-convention.

As it has been shown in our previous paper [9], for positive energy stationary states, the equation (6) is satisfied by the following bi-spinor

$$\psi(\vec{x},t) = \frac{e^{-iEt}}{\sqrt{\alpha z}}\frac{\mathcal{N}}{2}\begin{bmatrix} F_+(z)+i\,e^{-i\gamma}F_-(z) \\ F_-(z)-i\,e^{i\gamma}F_+(z) \\ F_+(z)-i\,e^{-i\gamma}F_-(z) \\ F_-(z)+i\,e^{i\gamma}F_+(z)\end{bmatrix} e^{ip_A x^A} \tag{7}$$

where $A=1,2$ and $F_\pm$ are expressed in terms of $K-$Bessel functions [12]

$$F_\pm(z) = \sqrt{\alpha z}\,K_{\frac{1}{2}\mp i\omega}(\varepsilon_\perp z), \tag{8}$$

with

$$\varepsilon_\perp = \left[p_\perp^2+m_0^2\right]^{1/2}, \quad \omega = \frac{E}{\alpha}$$

One should impose the normalization condition

$$L_x L_y \int_0^\infty \psi^\dagger \psi \,\mathrm{d}z = 1 \tag{9}$$

i.e.

$$2L_x L_y \frac{|\mathcal{N}|^2}{\varepsilon_\perp}\int_0^\infty K_{\frac{1}{2}+i\omega}(\varepsilon_\perp z)\,K_{\frac{1}{2}-i\omega}(\varepsilon_\perp z)\,\mathrm{d}(\varepsilon_\perp z) = 1\,,$$

which leads, by using the relation [12]

$$\int_0^\infty K_\nu(t)K_{\nu'}(t)\,\mathrm{d}t = \frac{\pi}{4}\Gamma(1+i\omega)\Gamma(1-i\omega) = \frac{\pi}{4}\,|\Gamma(1+i\omega)|^2 = \frac{\pi^2\omega}{4\sinh(\pi\omega)}$$

to the normalization constant

$$|\mathcal{N}|^2 = \frac{2}{L_x L_y}\frac{\varepsilon_\perp}{\pi^2\omega}\sinh(\pi\omega) \tag{10}$$

At this stage, one is able to compute the conserved current density components defined by

$$j^\mu = iq\bar{\psi}\gamma^\mu\psi \tag{11}$$

and these have the following real expressions

$$\begin{aligned} j_x &= q|\mathcal{N}|^2\cos\gamma\left\{e^{i\gamma}K_{\frac{1}{2}+i\omega}(\varepsilon_\perp z)K^*_{\frac{1}{2}-i\omega}(\varepsilon_\perp z)+e^{-i\gamma}K^*_{\frac{1}{2}+i\omega}(\varepsilon_\perp z)K_{\frac{1}{2}-i\omega}(\varepsilon_\perp z)\right\} \\ j_y &= q|\mathcal{N}|^2\sin\gamma\left\{e^{i\gamma}K_{\frac{1}{2}+i\omega}(\varepsilon_\perp z)K^*_{\frac{1}{2}-i\omega}(\varepsilon_\perp z)+e^{-i\gamma}K^*_{\frac{1}{2}+i\omega}(\varepsilon_\perp z)K_{\frac{1}{2}-i\omega}(\varepsilon_\perp z)\right\} \end{aligned} \tag{12}$$

By integrating with respect to the orthogonal surface, one obtains the corresponding currents

$$J_x = \int j_x\,dydz = \frac{qL_y}{\varepsilon_\perp}|\mathcal{N}|^2\cos\gamma\left\{e^{i\gamma}\mathcal{I}_1+e^{-i\gamma}\mathcal{I}_2\right\}$$

with

$$\mathcal{I}_{1,2} \;=\; \frac{1}{2}\left(\frac{1}{2}\pm i\omega\right)\frac{\pi}{\cosh(\pi\omega)}$$

and similarly for J_y, where we have applied the relation (10). Thus, we finally get the real expressions

$$J_x = \frac{q}{L_x}\frac{c^2\alpha}{\pi E}\tanh\left(\frac{\pi E}{c\alpha}\right)\cos^2\gamma\left\{1-2\frac{E}{c\alpha}\tan\gamma\right\} \tag{13}$$

and

$$J_y = \frac{q}{L_y}\frac{c^2\alpha}{\pi E}\tanh\left(\frac{\pi E}{c\alpha}\right)\sin\gamma\cos\gamma\left\{1-2\frac{E}{c\alpha}\tan\gamma\right\} \tag{14}$$

and one may notice that, for a given $\gamma \in (0, \pi/2)$, there is a critical value of the acceleration, $\alpha_{cr} = 2E\tan\gamma$, above which both currents are turning from negative to positive values. Also, for $\gamma = 0$, only J_x survives, while for $\gamma \to \pi/2$, one deals with a finite negative current flowing along Oy,

$$J_y = -\frac{2cq}{\pi L_y}\tanh\left(\frac{\pi E}{c\alpha}\right)$$

In what it concerns the vanishing component j_z, let us write the bi-spinor (7) as the following superposition

$$\psi(\vec{x},t) \;=\; \psi_1(\vec{x},t) + \psi_2(\vec{x},t)$$

with

$$\psi_1(\vec{x},t) = \mathscr{N}\, u_1\, K_{\frac{1}{2}-i\omega}(\varepsilon_\perp z)e^{i(p_A x^A - Et)}$$
$$\psi_2(\vec{x},t) = \mathscr{N}\, u_2\, K_{\frac{1}{2}+i\omega}(\varepsilon_\perp z)e^{i(p_A x^A - Et)}$$

where the two bi-spinors

$$u_1 = \frac{1}{2}\begin{bmatrix} 1 \\ -i\,e^{i\gamma} \\ 1 \\ i\,e^{i\gamma}\end{bmatrix}, \quad u_2 = \frac{1}{2}\begin{bmatrix} i\,e^{-i\gamma} \\ 1 \\ -i\,e^{-i\gamma} \\ 1\end{bmatrix}$$

behave, with respect to the Dirac conjugation $\bar{\psi} = i\,\psi^\dagger\gamma^4$, like a null complex diad, namely $\bar{u}_1\,u_1 \;=\; 0 \;=\; \bar{u}_2\,u_2$, $\bar{u}_1\,u_2 \;=\; i\,e^{-i\gamma}$, $\bar{u}_2\,u_1 = -\,i\,e^{i\gamma}$. The two base oriented equal currents are given by

$$j_z = q|\mathscr{N}|^2 K_{\frac{1}{2}-i\omega}(\varepsilon_\perp z)K_{\frac{1}{2}+i\omega}(\varepsilon_\perp z)$$

3. THE WEAK FIELD APPROXIMATION

Let us turn now to the physically important weak field-situation, i.e. $|\alpha s| \ll 1$, where $z = \frac{1}{\alpha} + s$. With the 4-spinor given in (7), written in terms of the 2-spinors as

$$\psi(x) = \begin{bmatrix} \varphi(\vec{x}) \\ \chi(\vec{x})\end{bmatrix} e^{-iEt} \tag{15}$$

the equation (6) splits into the coupled equations

$$\begin{aligned} \sigma^\mu\,\varphi_{,\mu} + \frac{1}{2z}\sigma^3\varphi \;&=\; i\left[\frac{E}{\alpha z} + m_0\right]\chi \\ \sigma^\mu\,\chi_{,\mu} + \frac{1}{2z}\sigma^3\chi \;&=\; i\left[\frac{E}{\alpha z} - m_0\right]\varphi \end{aligned} \tag{16}$$

In the non-relativistic regime $\varepsilon \ll m_0$, with $E = m_0 + \varepsilon$, $\varepsilon \in \mathbf{R}_+$, the system (16) casts into the form

$$\begin{aligned} \sigma^\mu\,\varphi_{,\mu} + \frac{\alpha}{2}\,\sigma^3\varphi \;&=\; 2im_0\,\chi \\ \sigma^\mu\,\chi_{,\mu} + \frac{\alpha}{2}\,\sigma^3\chi \;&=\; i\,[\varepsilon - \alpha m_0 s]\,\varphi \end{aligned} \tag{17}$$

With the relation

$$\chi = -\frac{i}{2m_0}\left[\sigma^\mu\,\varphi_{,\mu} + \frac{\alpha}{2}\,\sigma^3\varphi\right], \tag{18}$$

the second equation in (17) becomes

$$\left[\partial_x^2 + \partial_y^2 + \partial_s^2\right]\varphi + \alpha\,\frac{\partial\varphi}{\partial s} + \left[2m_0(\varepsilon - \alpha m_0 s) + \frac{\alpha^2}{4}\right]\varphi = 0 \tag{19}$$

and, with the function substitution

$$\varphi(\vec{x}) = e^{-\frac{\alpha s}{2}}\mathscr{C}(\vec{x}) \tag{20}$$

this turns into the Schrödinger equation

$$\Delta\mathscr{C} + \left[2m_0\left(\varepsilon + \frac{\alpha^2}{4m_0} - \alpha m_0 s\right)\right]\mathscr{C} = 0 \tag{21}$$

where $\mathscr{C}(\vec{x}) = e^{ip_A x^A}T(s)$. We introduce the new dimensionless variable

$$\eta = \left(2\alpha m_0^2\right)^{1/3}\left[s - \frac{1}{\alpha}\left(\frac{\varepsilon}{m_0} - \frac{p_\perp^2}{2m_0^2} + \frac{\alpha^2}{4m_0^2}\right)\right] \tag{22}$$

which might be positive or negative.

In the first case ($\eta > 0$), corresponding to the condition

$$\frac{\varepsilon_\parallel}{m_0} + \frac{\alpha^2}{4m_0^2} < \alpha s \ll 1 \tag{23}$$

where $\varepsilon_\parallel = \varepsilon - p_\perp^2/(2m_0)$, the relation (21) leads to the Airy equation [12]

$$\frac{\mathrm{d}^2T}{\mathrm{d}\eta^2} - \eta T = 0 \tag{24}$$

whose solution is expressed in terms of the Bessel modified function of the first kind as

$$T(\eta) = \frac{1}{\sqrt{3\pi}}\sqrt{\eta}\,K_{1/3}\left(\frac{2}{3}\eta^{3/2}\right) \tag{25}$$

One may easily check that the above expression is decreasing from $T(0) \approx 0.63$ to zero, for large η's, pointing out the existence of decaying modes along Oz.

For the *up* 2-spinor given by

$$\varphi^{up}(\vec{x}) = \frac{1}{\sqrt{3\pi}}\,e^{ip_A x^A}\,e^{-\frac{\alpha s}{2}}\sqrt{\eta}K_{1/3}\left(\frac{2}{3}\eta^{3/2}\right)\begin{bmatrix}1\\0\end{bmatrix} \tag{26}$$

the 2-spinor χ reads

$$\chi^{up}(\vec{x}) = -\frac{i}{2m_0}\frac{1}{\sqrt{3\pi}}\,e^{ip_A x^A}\,e^{-\frac{\alpha s}{2}}\begin{bmatrix} -\left(2\alpha m_0^2\right)^{1/3}\eta K_{2/3}\left(\frac{2}{3}\eta^{3/2}\right)\\ i(p_x + ip_y)\sqrt{\eta}K_{1/3}\left(\frac{2}{3}\eta^{3/2}\right)\end{bmatrix} \tag{27}$$

so that the *up* 4-spinor $\psi(x)$, defined in (7) is

$$\psi^{up}(x) = \frac{\mathcal{N}}{\sqrt{3\pi}}\, e^{i(p_A x^A - Et)}\, e^{-\frac{\alpha s}{2}} \left[\begin{array}{c} \sqrt{\eta} K_{1/3}\left(\frac{2}{3}\eta^{3/2}\right) \\ 0 \\ i\left(\frac{\alpha}{4m_0}\right)^{1/3} \eta K_{2/3}\left(\frac{2}{3}\eta^{3/2}\right) \\ \frac{p_x + i p_y}{2m_0}\sqrt{\eta} K_{1/3}\left(\frac{2}{3}\eta^{3/2}\right) \end{array} \right] \tag{28}$$

As for the *down* 4-spinor, this can be computed with the relation

$$\varphi^{down}(\vec{x}) = \frac{1}{\sqrt{3\pi}}\, e^{i p_A x^A}\, e^{-\frac{\alpha s}{2}} \sqrt{\eta} K_{1/3}\left(\frac{2}{3}\eta^{3/2}\right) \left[\begin{array}{c} 0 \\ 1 \end{array} \right] \tag{29}$$

and has the expression

$$\psi^{down}(x) = \frac{\mathcal{N}}{\sqrt{3\pi}}\, e^{i(p_A x^A - Et)}\, e^{-\frac{\alpha s}{2}} \left[\begin{array}{c} 0 \\ \sqrt{\eta} K_{1/3}\left(\frac{2}{3}\eta^{3/2}\right) \\ \frac{p_x - i p_y}{2m_0}\sqrt{\eta} K_{1/3}\left(\frac{2}{3}\eta^{3/2}\right) \\ -i\left(\frac{\alpha}{4m_0}\right)^{1/3} \eta K_{2/3}\left(\frac{2}{3}\eta^{3/2}\right) \end{array} \right] \tag{30}$$

One may notice that, in the particular case of $p_x = p_y = 0$, the relations (28) and (30) get the simpler form:

$$\psi_0^{up} = \frac{\mathcal{N}}{\sqrt{3\pi}}\, e^{-iEt}\, e^{-\frac{\alpha s}{2}} \left[\begin{array}{c} \sqrt{\eta} K_{1/3}\left(\frac{2}{3}\eta^{3/2}\right) \\ 0 \\ i\left(\frac{\alpha}{4m_0}\right)^{1/3} \eta K_{2/3}\left(\frac{2}{3}\eta^{3/2}\right) \\ 0 \end{array} \right]$$

and

$$\psi_0^{down} = \frac{\mathcal{N}}{\sqrt{3\pi}}\, e^{-iEt}\, e^{-\frac{\alpha s}{2}} \left[\begin{array}{c} 0 \\ \sqrt{\eta} K_{1/3}\left(\frac{2}{3}\eta^{3/2}\right) \\ 0 \\ -i\left(\frac{\alpha}{4m_0}\right)^{1/3} \eta K_{2/3}\left(\frac{2}{3}\eta^{3/2}\right) \end{array} \right]$$

being eigen-vectors of the spin operator, corresponding to the eigen-values $\lambda = \pm 1/2$,

$$\Sigma_3 = -\frac{i}{4}\left(\gamma^1\gamma^2 - \gamma^2\gamma^1\right) = \frac{1}{2}\left[\begin{array}{cc} \sigma^3 & 0 \\ 0 & \sigma^3 \end{array} \right]$$

For $e^{-\alpha s} \approx 1$, the normalization constant $\mathcal{N}$ coming from the condition $\int_0^\infty \psi^\dagger \psi\, \mathrm{d}s = 1$, is given by the following relation

$$\frac{|\mathcal{N}|^2}{3\pi} \frac{1}{\left(2\alpha m_0^2\right)^{1/3}} \left\{ \left[1 + \frac{p_\perp^2}{(2m_0)^2}\right] \mathscr{I}_1 + \left(\frac{\alpha}{4m_0}\right)^{2/3} \mathscr{I}_2 \right\} = 1$$

where the two integrals, namely

$$\mathscr{I}_1 = \int_0^\infty \eta \left[K_{1/3}\left(\frac{2}{3}\eta^{3/2}\right)\right]^2 \mathrm{d}\eta \approx 2\,, \quad \mathscr{I}_2 = \int_0^\infty \eta^2 \left[K_{2/3}\left(\frac{2}{3}\eta^{3/2}\right)\right]^2 \mathrm{d}\eta = \frac{\pi}{\sqrt{3}}$$

have been computed using the relation [12]

$$\int_0^\infty t^{a-1} \left[K_\nu(t)\right]^2 dt = \frac{\sqrt{\pi}}{4\Gamma\left(\frac{a+1}{2}\right)} \Gamma\left(\frac{a}{2}\right) \Gamma\left(\frac{a}{2} - \nu\right) \Gamma\left(\frac{a}{2} + \nu\right)$$

valid for $a > 2\nu$. In explicit calculations, one may use the approximation

$$|\mathcal{N}|^2 \approx \frac{3\pi}{2}\left(2\alpha m_0^2\right)^{1/3}\left[1-\frac{p_\perp^2}{(2m_0)^2}-\frac{\pi}{2\sqrt{3}}\left(\frac{\alpha}{4m_0}\right)^{2/3}\right] \approx \frac{3\pi}{2}\left(2\alpha m_0^2\right)^{1/3} \tag{31}$$

In the other case corresponding to $\eta < 0$, i.e.

$$\alpha s < \frac{\varepsilon_\parallel}{m_0}+\frac{\alpha^2}{4m_0^2} \tag{32}$$

which is a natural condition for the weak-field situation, we can change the sign in the equation (24) and $\eta \to |\eta|$. The Bessel modified functions $K_{1/3}$ and $K_{2/3}$ will be replaced by the oscillating Hankel functions $H^{(1)}_{1/3}$ and $H^{(1)}_{2/3}$ and expression (28) of the *up* spinor becomes

$$\psi^{up}(x) = \frac{\mathcal{N}}{\sqrt{3\pi}}\, e^{i(p_A x^A - Et)}\, e^{-\frac{\alpha s}{2}} \begin{bmatrix} \sqrt{\eta} H^{(1)}_{1/3}\left(\frac{2}{3}\eta^{3/2}\right) \\ 0 \\ \frac{1}{2}(\sqrt{3}+i)\left(\frac{\alpha}{4m_0}\right)^{1/3} \eta H^{(1)}_{2/3}\left(\frac{2}{3}\eta^{3/2}\right) \\ \frac{p_x+ip_y}{2m_0}\sqrt{\eta} H^{(1)}_{1/3}\left(\frac{2}{3}\eta^{3/2}\right) \end{bmatrix} \tag{33}$$

and similarly for ψ^{down}. Unlike the previous case, these are leading to a non-vanishing real positive current component along Oz

$$j_z = q|\mathcal{N}|^2 \frac{1}{\sqrt{3\pi}}\, e^{-\alpha s}\left(\frac{\alpha}{4m_0}\right)^{2/3} \eta^{3/2}\left[H^{(2)}_{1/3}H^{(1)}_{2/3}+H^{(1)}_{1/3}H^{(2)}_{2/3}\right] \tag{34}$$

which, close to the turning point corresponding to $z \to z_*$ ($\eta \to 0$), gets the value $j_z \sim q\alpha$.

4. CONCLUDING REMARKS

The Rindler coordinates are of a major importance for describing how the Minkowski spacetime appears to an uniformly accelerated observer. In this respect, they open the way for a generalization of the Larmor formula in gravitational fields. Using a coordinate-independent method, based on Cartan's formalism, the theory developed in the present paper can be seen as a model example of solving the Dirac equation on a curved spacetime. With the mode solutions to the Dirac equation, one can constract the Rindler quantum field and the current and charge density operators.

An intriguing result is the fact that the acceleration alters the mass dimensionality of the field from 3/2 to one. Also, by inspecting the expressions of the bi-spinor, one may notice that the signs of the components and their phases are suggesting an Elko-type behavior [15]. Since the mismatch of the mass dimensionality of the new fields forbids them to enter the SM doublets, these fermions have been seen as first-principle candidates for dark matter. However, this Elko and mass dimension one fermion needs a new formalism to compute transition probabilities and observables [13, 14].

Another important result is the direct connection between the critical value of the acceleration and the proper phase of the Elko spinor $\alpha_{cr} = 2E\tan\gamma$. The currents (13) and (14) get an asymptotic behavior to constant positive values for large accelerations, i.e. $\alpha \gg 2E\tan\gamma$. Putting it backwards, it turns out that the phase of the Elko behavior is established by an effective acceleration, according to the relation $\gamma = \arctan\alpha_{cr}/(2E)$.

A special attention has been given to the semi-relativistic approximation. The equation (24) is similar to the one derived by Fowler and Nordheim in their original work on field emission of electrons [16] and agrees with the one obtained in an (1+1)-dimensional approach in [17].

As long as the natural condition (32) is satisfied, i.e. $z < z_* \approx 1/\alpha$, one deals with a propagating field with the wave function (33) and a non-zero current along Oz given in (34). Once $z > z_*$, one ends up with spatially decaying modes along Oz and the current j_z is vanishing.

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