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STUDY ON DATA-DRIVEN CONTROL OF MAGLEV TRAIN LEVITATION SYSTEM BASED ON KOOPMAN LINEAR RECONSTRUCTION

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Abstract. When the maglev train levitation system runs for a long time, the components of the system will age and the control performance of the system will decline. Moreover, the suspension system of maglev train is a complex nonlinear system, and the system has unmodeled dynamics, so it is impossible to ensure the performance of the control system through the accurate identification of the suspension system model. In this paper, a data-driven control design framework of the maglev train suspension system based on Koopman operator is proposed. According to the measured data of the suspension system, the nonlinear maglev train suspension system is reconstructed in the linear framework by using Koopman characteristic function, and the optimal control of the suspension system is realized. Simulation and experiments verify the effectiveness of the method.

***Key words**:* data driven, Koopman theory, linear reconstruction, magnetic suspension.

1. INTRODUCTION

As a new type of rail transit, maglev train has been applied on a large scale. For example, Maglev Operation lines have been built in Beijing, Changsha, Qingyuan, Fenghuang. However, after the long-term operation of maglev train, there will inevitably be problems such as line track settlement, wear of structural parts and aging of electronic components, which will change the model parameters of maglev train levitation system, and then lead to the degradation of the performance of the levitation control system designed at the factory. In order to make the maglev train always maintain the set gap value during operation, the suspension system is required to maintain good control performance.

Aiming at the control problem of maglev train suspension system, a complex nonlinear system, a variety of nonlinear control methods have been developed, including sliding mode control [1], model predictive control [2], adaptive control [3], robust control [4], reinforcement learning based control [5], etc. Although these nonlinear control algorithms have made some progress in the control of the suspension system [6], these methods usually require a lot of computational resources, or are not easy to be extended to new application scenarios of model degradation. And these nonlinear control methods do not have an overall framework. With the rise of new technologies such as big data and machine learning, these challenges are expected to be solved. Recently, Koopman operator theory has become a popular method to obtain the linear representation of nonlinear systems from data. Koopman proved in 1931 that nonlinear dynamical systems can be represented by infinite dimensional linear operators acting on Hilbert space of system state measurement function [7]. The spectral decomposition of this linear Koopman operator can fully characterize the nonlinear system.

Koopman theory has been widely used in system identification [8], estimation [9] and control [10] of nonlinear systems. However, the space of all possible measurement functions describing the state needs to be infinite dimensional, and the control law is usually based on finite dimensional approximation, so dynamic mode decomposition (DMD) [11] is often used to approximate the Koopman operator. However, DMD is based on linear measurement. For most nonlinear systems, DMD is no longer applicable [12]. Therefore, extended dynamic mode decomposition (EDMD) is needed to approximate Koopman operator [13]. Linear systems are rich in optimal estimation theory and control algorithms, while Koopman operator can provide a way to express nonlinear dynamics in a linear framework. The latest research challenge is to obtain the coordinate transformation of approximate Koopman characteristic function in nonlinear system based on data-driven method. In this paper, EDMD technology is used to re describe the suspension system in the coordinate system based on Koopman characteristic function, and then study the suspension control problem.

Therefore, the main work of this paper is to propose a data-driven Koopman characteristic function identification method for suspension system, and realize the research on the data-driven control method of suspension system based on Koopman characteristic function.

2. MODEL RECONSTRUCTION MAGLEV TRAIN LEVITATION SYSTEM

2.1. Maglev system model analysis

The research object of this paper is the suspension system of normal medium- and low- speed maglev train, which takes Fenghuang maglev train as an example. The levitation force of single section train body of a maglev train is usually provided by 20 suspension systems. These suspension systems are mechanically decoupled by means of 5 bogies. Therefore, the model analysis of maglev train levitation system can be implemented into the analysis of single suspension system model. In this section, two previous works on suspension control of maglev train are cited to establish and analyze the single suspension system model [3–4]. The simplified structure diagram of the single suspension system is shown in Fig. 1.

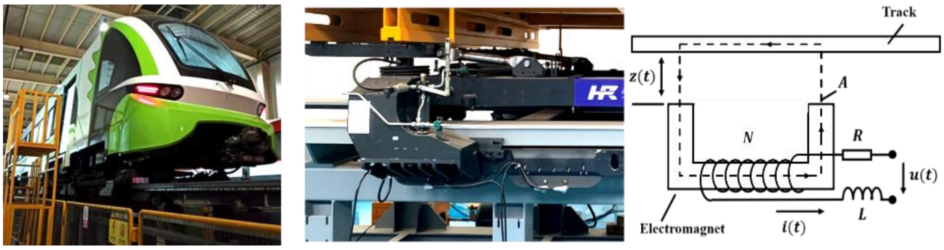

Fig. 1 – The structural diagram of the single suspension system of the normal medium- and low- speed maglev train.

Let *m* represent the equivalent mass of the suspension magnet and the vehicle body in the single suspension system, $z(t)$ is the suspension gap, *N* is the number of turns of the electromagnet coil, *A* is the cross-sectional area of the electromagnet, $u(t)$ and $i(t)$ are the voltage and current at both ends of the electromagnet coil respectively, *R* is the resistance of the electromagnet coil, *L* is the electrical inductance of the electromagnet coil, μ is the vacuum permeability, f_d is the external interference force. The single suspension system model can be obtained by quoting the previous suspension control research [3]:

$$\begin{cases} m\ddot{z} = mg - \mu A N^2 i^2 / (2z)^2 + f_d \\ u = Ri + \mu A N^2 \dot{i} / (2z) - \mu A N^2 i \dot{z} / (2z^2). \end{cases} \tag{1}$$

Taking the state variable as $\begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix} = \begin{bmatrix} z & \dot{z} & i \end{bmatrix}^T$, $H = \mu A N^2 / 4$, the state space model of the single suspension system is obtained as follows:

$$\begin{bmatrix} \dot{x}_1 & \dot{x}_2 & \dot{x}_3 \end{bmatrix}^T = \begin{bmatrix} x_2 & -Hx_3^2/(mx_1^2) + g & x_2 x_3 / x_1 - R x_1 x_3 / (2H) \end{bmatrix}^T + \begin{bmatrix} 0 & 0 & x_1/(2H) \end{bmatrix}^T u = f(x) + g(x) u . \tag{2}$$

The model is applicable to the case that the initial state of the system is near the equilibrium position. When the initial suspension position is far away from the suspension equilibrium position, the controller designed based on this model may not guarantee the stability of the suspension system.

2.2. Koopman analysis

According to formula (2), the suspension system is a third-order complex nonlinear system. In addition, the suspension system of the on-orbit maglev train still has unmodeled dynamics, which makes the nonlinear model of the suspension system unable to be established accurately. Therefore, this section uses

Koopman operator theory to obtain the data-driven mathematical model of the suspension system. Koopman operator theory studies the system according to the evolution of the state measurement function. Koopman's research shows that the infinite dimensional linear operator on the Hilbert space acting on the state measurement function of the system can be used to represent the nonlinear dynamic system. Consider the state measurement function $g()$ in infinite dimensional Hilbert space. The infinite dimensional linear Koopman operator is defined as K_t, and the discrete-time dynamic system is considered:

$$x_{k+1} = F_{\Delta t}(x_k),\ x \in M . \tag{3}$$

Then we can get that the effect of K_t on the measurement function is:

$$K_{\Delta t} g(x_k) = g(F_{\Delta t}(x_k)) . \tag{4}$$

Therefore, the state observation value $g(x_{k+1})$ at the next time can be obtained according to the Koopman operator:

$$g(x_{k+1}) = K_{\Delta t} g(x_k),\ g(x) \in \mathbb{R}. \tag{5}$$

Assuming that the Koopman eigenfunction is $\varphi(x)$ and the corresponding eigenvalue is λ, then:

$$\varphi(x_{k+1}) = K_{\Delta t}\varphi(x_k) = \lambda\varphi(x_k) . \tag{6}$$

The Koopman operator is infinite dimensional, so it is complex to calculate Koopman operator directly. Koopman analysis is not to obtain the evolution process of all measurement functions, but only to identify the evolution process of key measurement functions of dynamic system in Hilbert space. With the help of the characteristic function of Koopman operator, such a set of linear measurement functions can be obtained. Furthermore, the linear representation of the suspension system in the coordinates composed of this set of measurement functions can be obtained, which is global.

According to the chain rule, the derivative of Koopman characteristic function $\varphi(x,u)$ with control input u with respect to time is:

$$\mathrm{d}\varphi(x,u)/\mathrm{d}t = \nabla_x\varphi(x,u)\dot{x} + \nabla_u\varphi(x,u)\dot{u} = \lambda\varphi(x,u) + \nabla_u\varphi(x,u)\dot{u} . \tag{7}$$

where $\dot{u}$ can be regarded as the input of linear system based on Koopman characteristic function.

2.3. Linear reconstruction analysis of Maglev nonlinear system

The suspension system can realize the global linear representation in the coordinates composed of Koopman characteristic function. Therefore, this section applies Koopman analysis to realize the linear reconstruction of suspension system. The linear reconstruction does not need to obtain the evolution of all the measurement functions that can be used to represent the suspension system in Hilbert space, but to obtain the approximate evolution of the suspension system on a set of invariant subspaces expanded by finite measurement functions. Namely, we want to get the finite dimensional matrix representation of Koopman operator. Finally, the finite dimensional linear reconstruction system of the suspension system is obtained. This solves the local linearity limitation caused by the design of suspension controller based on the linearization for a long time, and realizes the linearization representation of the whole working range of suspension system. According to Koopman mode decomposition introduced by Mezic in 2005 [14], the measurement vector g can be expressed as:

$$g(x) = \left[g_1(x) \quad g_2(x) \quad \cdots \quad g_p(x) \right]^T = \sum_{j=1}^{\infty} \varphi_j(x) v_j,\ \ g_j(x) = \sum_{j=1}^{\infty} v_{ij}\varphi_j(x) ,$$

where $\varphi(x)$ is the characteristic function, v_j is the j-th Koopman mode associated with φ_j, and the measurement function $g_i(x)$ is represented by a basis in Hilbert Space.

The finite dimensional matrix representation of Koopman operator can be obtained from the Koopman invariant subspace expanded by the measurement function $\left\{ g_1, g_2, \cdots, g_p \right\}$, which is expanded by the finite characteristic function set of Koopman operator. The calculation method of Koopman operator based on finite dimensional approximation can realize the finite dimensional linear reconstruction of suspension system to a certain extent. Suppose that the snapshot pair of system state collected is:

$$\left\{\left(x(t_k), x(t_k')\right)\right\}_{k=1}^{m}, \quad x \in \mathbb{R}^{n\times 1}, \tag{8}$$

where, $t_k' = t_k + \Delta t$, assuming uniform sampling in time, $t_k' = t_{k+1}$, two data matrices and the driving input data matrix are constructed as follows:

$$X = \begin{bmatrix} | & | & & | \\ x(t_1) & x(t_2) & \cdots & x(t_m) \\ | & | & & | \end{bmatrix}, \; X' = \begin{bmatrix} | & | & & | \\ x(t_2) & x(t_3) & \cdots & x(t_{m+1}) \\ | & | & & | \end{bmatrix}, \; R = \begin{bmatrix} | & | & & | \\ u(t_1) & u(t_2) & \cdots & u(t_m) \\ | & | & & | \end{bmatrix} \tag{9}$$

Then, the best fitting linear operator A and driving matrix B can be obtained by DMD algorithm. And the dynamics is:

$$X' \approx AX + BR, \; A \in \mathbb{R}^{n\times n}, \; B \in \mathbb{R}^{n\times 1}, \; X \in \mathbb{R}^{n\times m}, \; U \in \mathbb{R}^{1\times m}. \tag{10}$$

Rewrite the above formula as:

$$X' \approx \begin{bmatrix} A & B \end{bmatrix} \begin{bmatrix} X & R \end{bmatrix}^T = GP, \; G \approx X'P^{\dagger}. \tag{11}$$

Do SVD for X and P respectively [15]:

$$\tilde{G} = U_2^* G \begin{bmatrix} U_2 & I \end{bmatrix}^T, \; \tilde{A} = U_2^* X' V_1 \Sigma_1^{-1} U_{11}^* U_2, \; \tilde{B} = U_2^* X' V_1 \Sigma_1^{-1} U_{12}^*. \tag{12}$$

However, DMD based on linear measurement is not enough for linear model reconstruction of suspension system. Therefore, EDMD is applied to realize linear model reconstruction of suspension system in this paper. EDMD includes not only the regression of the direct measurement value of the state, but also the regression of the nonlinear measurement value of the state. In EDMD, the state of structural augmentation [16]:

$$y = \Theta^T(x) = \begin{bmatrix} \theta_1(x) & \theta_2(x) & \cdots & \theta_p(x) \end{bmatrix}^T, \; p >> n. \tag{13}$$

Similarly, two data matrices are constructed as follows:

$$Y = \begin{bmatrix} | & | & & | \\ y_1 & y_2 & \cdots & y_m \\ | & | & & | \end{bmatrix}, \; Y' = \begin{bmatrix} | & | & & | \\ y_2 & y_3 & \cdots & y_{m+1} \\ | & | & & | \end{bmatrix}. \tag{14}$$

Finally, the data matrix of the suspension system can be expressed as:

$$Y' \approx AY + BR = \begin{bmatrix} A & B \end{bmatrix} \begin{bmatrix} Y & R \end{bmatrix}^T = GP. \tag{15}$$

The best fitting linear operator A and driving matrix B are obtained by dimensionality reduction and regression according to kernel method, respectively:

$$A = U_2^* Y' V_1 \Sigma_1^{-1} U_{11}^* U_2, \; B = U_2^* Y' V_1 \Sigma_1^{-1} U_{12}^*. \tag{16}$$

The candidate library Θ of nonlinear functions provides a rich basis $\theta_j(x)$ to approximate the Koopman operator. The Koopman characteristic function can be approximated as:

$$\varphi(x) \approx \sum_{k=1}^{p} \theta_k(x) \xi_k = \Theta(x)\xi. \tag{17}$$

As shown in Figure 2, the suspension gap of the suspension system is designed to transition from the initial 15 mm to 8 mm in normal operation. Based on the measured data of the suspension system, the best fitting linear operator A and driving matrix B are obtained by EDMD. And then the linear model reconstruction of the suspension system is realized, and the response curve with driving input is obtained according to the linear reconstruction model. As shown in the right figure of Fig. 2, the linear reconstruction model can better restore the response characteristics of the original suspension system.

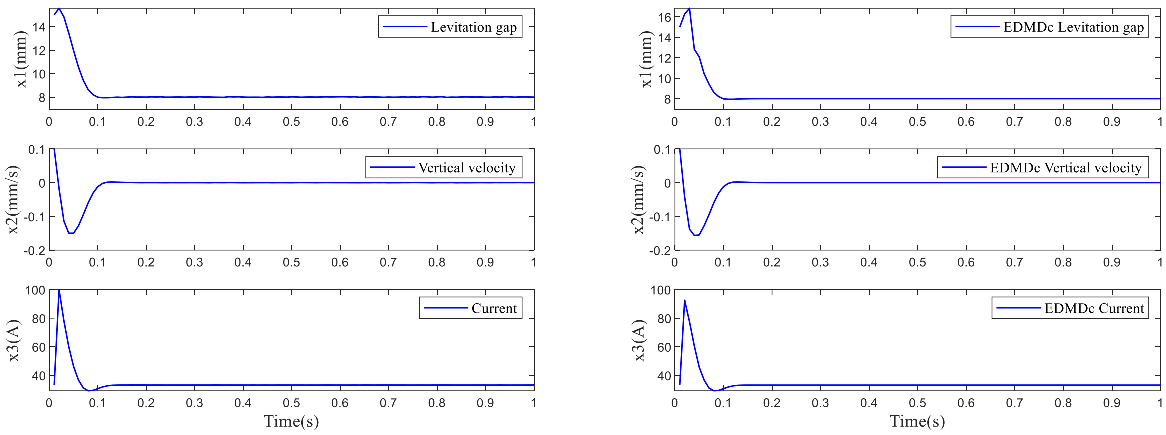

Fig. 2 – The closed loop response curve of the suspension system (left), the response curve with drive input of linear reconstruction model of the suspension system (right).

3. DESIGN OF DATA DRIVEN CONTROL SYSTEM BASED ON KOOPMAN CHARACTERISTIC FUNCTION

3.1. Sparse identification of Koopman characteristic function

In this section, the sparse identification is used to identify the dominant Koopman characteristic function, so as to construct the low dimensional approximate model of the suspension system. This method can avoid complex model structure search and calculation.

Firstly, the suspension system with driving input is constructed as follows:

$$\mathrm{d}x/\mathrm{d}t = f(x) + Bu\,. \tag{18}$$

The dynamics of Koopman characteristic function with driving input is still linear. Then, according to the chain rule, the derivative of Koopman characteristic function $\varphi(x)$ with respect to time is [16]:

$$\mathrm{d}\varphi(x)/\mathrm{d}t = \nabla\varphi(x)\cdot\left(f(x) + Bu\right) = \lambda\varphi(x) + \nabla\varphi(x)\cdot Bu\,. \tag{19}$$

Then, the approximate characteristic function can be obtained by solving equation (19) using linear control theory based on Riccati. Next, the dominant Koopman characteristic function is obtained by sparse identification. The system data matrix and its derivative matrix are constructed as follows:

$$X = \left[x(t_1)\quad x(t_2)\quad \cdots\quad x(t_m)\right]^T,\ \dot{X} = \left[\dot{x}(t_1)\quad \dot{x}(t_2)\quad \cdots\quad \dot{x}(t_m)\right]^T,\ R = \left[u(t_1)\quad u(t_2)\quad \cdots\quad u(t_m)\right]^T. \tag{20}$$

The candidate library Θ of the designed nonlinear function is the polynomial of all possible time series of state x and driving input u.

$$\Theta\left(\left[X\quad R\right]\right) = \left[1\quad X\quad R\quad X^2\quad X\otimes R\quad R^2\quad \cdots\right]. \tag{21}$$

Let the model of the suspension system be:

$$\dot{X} = \Theta\left(\left[X\quad R\right]\right)\Xi\,, \tag{22}$$

where each column ξ_k of Ξ is a coefficient vector. Then, the ξ_k of the above model is identified by the sparse regression method [16]

$$\xi_k = \arg\min_{\xi'_k} \left\| \dot{X}_k - \Theta\left(\begin{bmatrix} X & R \end{bmatrix}\right)\xi'_k \right\|_2 + \lambda \left\| \xi'_k \right\|_1 . \tag{23}$$

$\dot{X}_k$ is the k-th column of $\dot{X}$, and λ is used to adjust the sparsity.

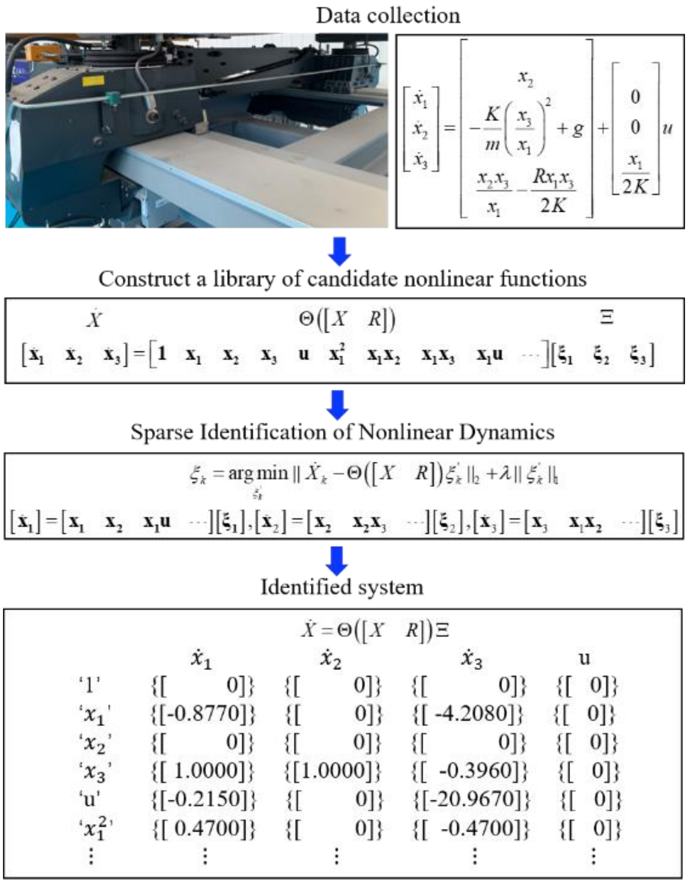

Fig. 3 – Characteristic function identification steps of the magnetic levitation system based on sparse identification.

Algorithm 1: Characteristic function identification based on sparse identification

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Input: data matrix X and its derivative matrix $\dot{X}$, candidate library $\Theta\left(\begin{bmatrix} X & R \end{bmatrix}\right)$ of nonlinear function, threshold ε

Output: coefficient vector ξ_k

$\Xi \leftarrow \dot{X} = \Theta\left(\begin{bmatrix} X & R \end{bmatrix}\right)\Xi$ %Initialize

while Non convergence do

  $\text{abs}(\Xi) \le \varepsilon$ %Find small coefficients

  for each state dimension in system do

    $\xi_k = \arg\min_{\xi'_k} \left\| \dot{X}_k - \Theta\left(\begin{bmatrix} X & R \end{bmatrix}\right)\xi'_k \right\|_2 + \lambda \left\| \xi'_k \right\|_1$ %Regress dynamics to find sparse

  end for

end while

3.2. Optimal control design based on Koopman characteristic function

Combined with the Koopman operator theory, the optimal control loop structure based on Koopman characteristic function is designed in this section. As shown in Fig. 4. The optimal suspension control is realized based on the linear reconstruction model of the suspension system. Firstly, the optimal control problem is established for a set of simplified Koopman characteristic functions.

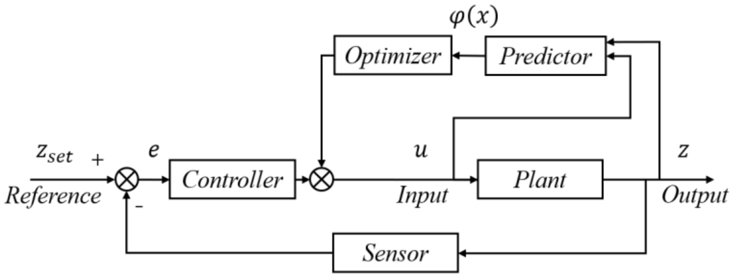

Fig. 4 – The optimal control loop structure based on Koopman characteristic function.

$$J(\varphi,u)=\frac{1}{2}\int_0^{\infty}\left(\boldsymbol{\varphi}^T(x(t))Q_\varphi\boldsymbol{\varphi}(x(t))+\boldsymbol{u}^T(t)R\boldsymbol{u}(t)\right)\mathrm{d}t\,, \tag{24}$$

where $\boldsymbol{\varphi}=\begin{bmatrix}\varphi_1 & \varphi_2 & \cdots & \varphi_r\end{bmatrix}^T$ contains r characteristic functions. Selecting a specific set of characteristic functions to build the model and formulate the cost function. Then, the relevant target value can be determined by evaluating the cost function of the system state.

Use the control input to increase the state and take the derivative of the control input as the new input.

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix}\varphi\\ u\end{bmatrix}=\begin{bmatrix}\Lambda & B_\varphi\\ 0 & 0\end{bmatrix}\begin{bmatrix}\varphi\\ u\end{bmatrix}+\begin{bmatrix}0\\ I_q\end{bmatrix}\dot{u}\,. \tag{25}$$

By transforming the nonlinearity in the control term into state dynamics, the formula (24) changes to:

$$J=\frac{1}{2}\int_0^{\infty}\left(\begin{bmatrix}\varphi^T & u^T\end{bmatrix}\begin{bmatrix}Q_\varphi & 0\\ 0 & R\end{bmatrix}\begin{bmatrix}\varphi\\ u\end{bmatrix}+\dot{u}^T\hat{R}\dot{u}\right)\mathrm{d}t\,. \tag{26}$$

Therefore, the dynamic system in the coordinate system based on Koopman characteristic function is:

$$\frac{\mathrm{d}}{\mathrm{d}t}\varphi(x)=\boldsymbol{\Lambda}\varphi(x)+\nabla_x\varphi(x)\cdot Bu\,, \tag{27}$$

where $\boldsymbol{\Lambda}=\mathrm{diag}(\lambda_1,\lambda_2,\cdots,\lambda_r)$. Then the feedback controller based on Koopman operator is:

$$u=-C_\varphi(x)\left[\varphi-\varphi^{ref}\right]. \tag{28}$$

The feedback control gain C_φ determined by solving the algebraic Riccati equation to minimize J.

4. EXPERIMENTS

This paper aims to verify the effectiveness of the linear reconstruction model of the suspension system based on Koopman eigenfunction coordinate system. The error E based on the characteristic function is used to evaluate the suspension gap of the experimental test track. Then, according to the evaluation results, a set of characteristic functions for linear reconstruction of suspension system model are selected.

$$E=\left\|\varphi\left(x(t)\right)-e^{\lambda t}\varphi\left(x(0)\right)\right\|^2, \tag{29}$$

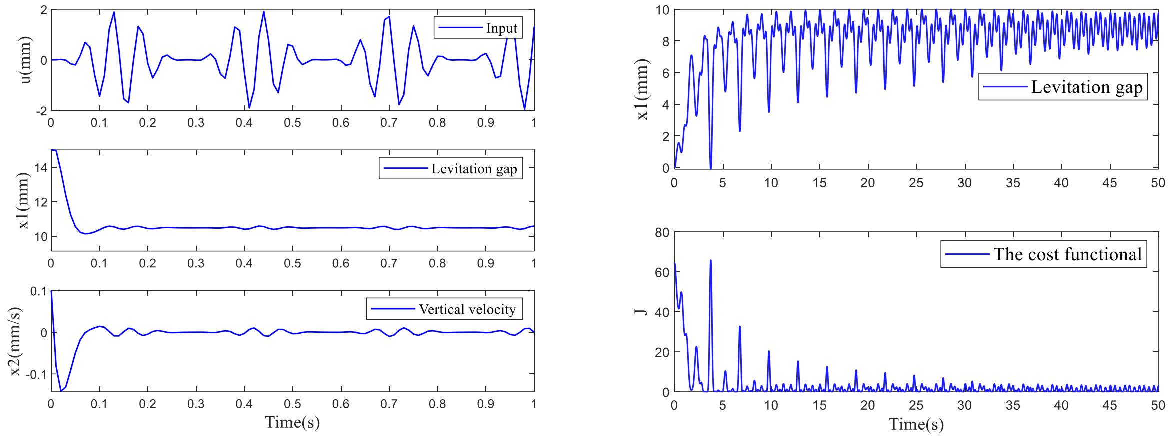

Fig. 5 – The response data of simulated excitation of the single suspension closed-loop feedback control system (left), the simulation response curve of the single suspension system (right).

where $\mu = 4\pi\times10^{-7}$ H/m, the coil current and suspension gap during stable suspension are $i_0 = 22.0$ A and $Z_0 = 8.0$ mm respectively, N = 360 turns, A = 0.038 m^2, R = 0.92 Ω, and m = 535 kg. Therefore, the system matrix described in the state space of the suspension system is $A = \begin{bmatrix} 0 & 1 \\ 5467.52 & 0 \end{bmatrix}$, control matrix $B = \begin{bmatrix} 0 & -1.99 \end{bmatrix}^T$ and output matrix $C = \begin{bmatrix} 1 & 0 \end{bmatrix}$. A state feedback controller is designed to stabilize the closed-loop single suspension model. The purpose is to obtain the response data of the closed-loop feedback control of the single suspension system by designing the simulated excitation. As shown in the Fig. 5.

Then, according to the collected response data of the suspension system, a set of Koopman characteristic functions are obtained by using EDMD and sparse identification introduced in the paper. Finally, the fourth-order Runge Kutta method is used to solve the differential equation of the system, and the linear reconstruction model of the single suspension system based on Koopman characteristic function can be obtained. Then, combined with the LQR controller, the simulation response curve of the single point suspension system can be obtained, as shown in the right figure of Fig. 5. The system can reach a stable state after a limited time.

Then, the single electromagnet suspension experiment is carried out on the suspension control experimental platform of the single electromagnet suspension system. The experimental platform is shown in the Fig. 6. Where m = 6.9 kg, Z_0 =4.0 mm, i_0 =9.5 A, A = 13.5 cm^2, N = 300 turns, and R = 4.8 Ω.

Fig. 6 – The suspension control experimental platform of the single electromagnet suspension system.

Collect the experimental data of suspension control response of single iron suspension system. And then, according to the method introduced in the paper, a data-driven linear reconstruction model of single electromagnet suspension system based on Koopman characteristic function is obtained, as shown in the left of Fig. 7. According to the linear reconstruction model of the single electromagnet suspension system based on the data drive, the single electromagnet suspension system controller based on the model is designed and applied to the suspension control experimental platform of the single electromagnet suspension system, so as to obtain the suspension experimental response curve as shown in the right figure of Fig. 7.

	$\dot{x}_1$	$\dot{x}_3$	$\dot{u}$
'1'	-103.6176	-5.5496e+05	0
'x_1'	2.7712e+04	1.4824e+08	0
'x_3'	0.5777	-2.3701e+03	0
'u'	0	0	0
'x_1x_1'	-1.8529e+06	-9.8998e+09	0
'x_1x_3'	-77.0036	3.1728e+05	0
'x_1 u'	0	0	0
'x_3x_3'	-0.0043	14.2953	0
'x_3 u'	0	0	0
'uu'	0	0	0

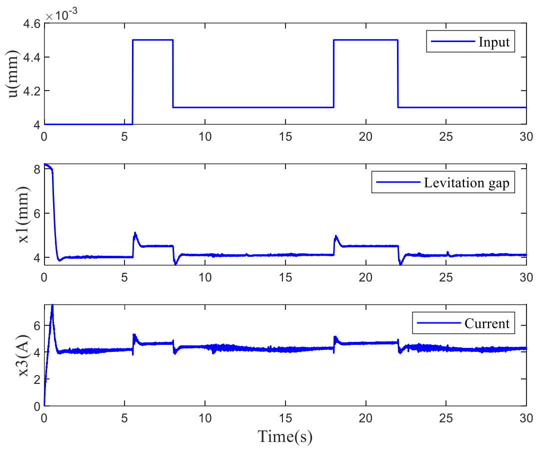

Fig. 7 – The data-driven linear reconstruction model based on Koopman characteristic function (left), the response curve of suspension control experiment of the single electromagnet suspension system (right).

It can be seen from the right figure of Fig. 7 that the linear reconstruction model of single electromagnet suspension system based on data-driven identification can well reflect the characteristics of the original system. The experimental results prove the effectiveness of the data-driven control design framework of maglev train suspension system based on Koopman characteristic function.

5. CONCLUSION

Combined with the Koopman characteristic function obtained through sparse identification, the paper designs the data-driven control design framework of maglev train suspension system based on Koopman characteristic function, and verifies the effectiveness of the linear reconstruction system of suspension control system based on the Koopman characteristic function. At the same time, according to the experimental measurement data of the single electromagnet suspension system, the Koopman characteristic function of the linear reconstruction system which can describe the suspension control system is identified by using the methods of EDMD and sparse identification. The nonlinear suspension system is re expressed in the linear framework, and the effectiveness of the linear reconstruction model applied to the actual suspension control is verified.

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