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HYPERSINGULARITIES OF 3-RRR PLANAR PARALLEL ROBOTS

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Abstract: The use of the entire workspace of a parallel robot calls for passing through type II singularities, which can be achieved by removing them from the inverse dynamics solution. Hypersingularities are type II singularities whose removal from the inverse dynamics requires satisfaction of additional higher order derivative conditions other than the acceleration-level ones for maintaining the consistency of the equations of motion. For this reason, the identification of hypersingularities constitutes an important and yet a new subject in the field of parallel robots. This paper contributes to the literature by deriving a condition for their occurrence in one of the mostly used parallel robots in applications, namely the 3-RRR planar parallel robot. It is shown that any type II singular configuration of the considered robot becomes a hypersingularity if the end-effector velocities at that configuration lie on a certain plane in the end-effector velocity space. The orientation of this hypersingularity plane changes according to the singular configuration. However, the said plane always passes through the origin of the end-effector velocity space. These findings are believed to bring new insights into the optimization of the singularity removal process of parallel robots through the minimization of the number of conditions to be satisfied in this regard.

Key words: 3-RRR planar parallel robot, kinematics, dynamics, singularity, hypersingularity.

1. INTRODUCTION

Parallel robots offer distinct advantages for many sophisticated applications such as high-performance machining [1, 2], medical robotics [3–5], microrobotics [6, 7], and reconfigurable robotics [8, 9]. “Type II singularities” [10] give rise to the main disadvantage of them, namely, the restricted usability of their workspace. Accordingly, a great deal of research has been conducted on the method of trajectory planning that will make it possible to smoothly pass through these singular configurations. In the early literature, this method had been based on keeping the dynamic equations of the robot consistent at the singularity (see, e.g., Refs. [11–13]). In a recent paper, however, Özdemir [14] proved that depending on the time derivatives of the vanishing determinant, realization of a singular path may require additional higher order derivative conditions other than the acceleration-level ones for consistency. Singularities that require such additional conditions for their removability are called in the rest of the present paper as hypersingularities. In another recent paper focusing on the two-degree-of-freedom (2-DOF) five-bar planar parallel robot with revolute joints (5R), Özdemir [15] showed that every type II singular configuration of the said robot becomes a hypersingularity (or a “high-order singularity”, as called in that reference) when the path of its end point has a certain slope at that configuration.

The present paper aims to expand the knowledge on hypersingularities by deriving a condition for their occurrence in a 3-DOF planar parallel robot, namely the 3-RRR planar parallel robot (where the underlined R represents that the base revolute joint is the actuated joint in each of the three legs). This robot is one of the mostly used parallel robots in applications [16] and has been extensively studied in the literature for its optimum design [17–21], kinematics [22], dynamics [23–25], balancing [26–28], control [29–31], energy efficiency [32], workspace [33, 34], trajectory planning [35, 36] and singularity loci [37–41]. However, hypersingularity analysis of parallel robots is quite a new subject, and its key role for removing type II singularities has been only recently explored in the literature [14, 15]. The present paper provides new insights into the optimization of the said singularity removal process through the minimization of the number of conditions to be satisfied in this regard.

2. PRELIMINARIES

The 3-RRR planar parallel robot is illustrated in Fig. 1. The base joints R_1, R_4 and R_7 are driven by motor torques T_1, T_2 and T_3, respectively. The fixed Cartesian coordinate system xy has its origin at the center of joint R_1. The coordinates of the centers of joints R_4 and R_7 in this system are (c_1, c_2) and (c'_1, c'_2), respectively. The gravitational acceleration g is assumed to be acting along the negative y-direction. The i^{th} moving link ($i = 1, 2, \ldots, 7$) has a mass m_i and a mass moment of inertia I_i about the axis which passes through its mass center G_i and is normal to its plane of motion. For each $j = 1, 2, 3$, let $L_{2j-1} = |R_{3j-2}R_{3j-1}|$, $L_{2j} = |R_{3j-1}R_{3j}|$, $d_{2j-1} = |R_{3j-2}G_{2j-1}|$ and $d_{2j} = |R_{3j-1}G_{2j}|$. Furthermore, let $L_7 = |R_3R_6|$, $L'_7 = |R_3R_9|$, $d_7 = |R_3G_7|$ and $d'_7 = |R_3E|$ where point E is the end point of the robot. The joint variables $\theta_1, \theta_2, \ldots, \theta_7$ and the constant angular parameters $\gamma_1, \gamma_2, \ldots, \gamma_7, \alpha, \beta$ are defined in Fig. 1.

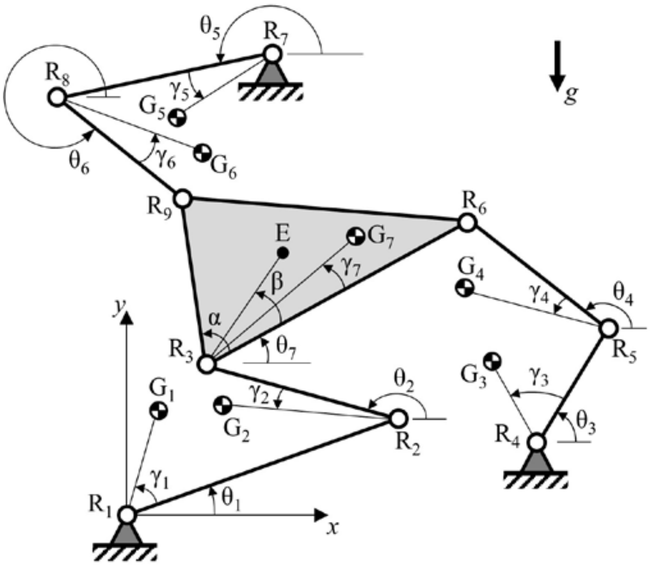

Fig. 1 – The 3-RRR planar parallel robot.

Consider a prescribed motion of the end-effector where no inverse kinematic singularity is encountered. Using the Lagrangian method, the torques required to realize this motion can be expressed as follows:

$$T_1 = M_{11}\ddot{\theta}_1 + M_{12}\ddot{\theta}_2 + M_{17}\ddot{\theta}_7 + C_1 + g\Gamma_1 - L_1\left(-\lambda_1 \sin(\theta_1) + \lambda_2 \cos(\theta_1) - \lambda_3 \sin(\theta_1) + \lambda_4 \cos(\theta_1)\right) \quad (1)$$

$$T_2 = M_{33}\ddot{\theta}_3 + M_{34}\ddot{\theta}_4 + C_3 + g\Gamma_3 - L_3\left(\lambda_1 \sin(\theta_3) - \lambda_2 \cos(\theta_3)\right) \quad (2)$$

$$T_3 = M_{55}\ddot{\theta}_5 + M_{56}\ddot{\theta}_6 + C_5 + g\Gamma_5 - L_5\left(\lambda_3 \sin(\theta_5) - \lambda_4 \cos(\theta_5)\right) \quad (3)$$

where

$$M_{11} = m_1 d_1^{\,2} + I_1 + m_2 L_1^{\,2} + m_7 L_1^{\,2} \quad (4)$$

$$M_{12} = L_1\left(m_2 d_2 \cos(\theta_2 + \gamma_2 - \theta_1) + m_7 L_2 \cos(\theta_2 - \theta_1)\right) \quad (5)$$

$$M_{17} = m_7 d_7 L_1 \cos(\theta_7 + \gamma_7 - \theta_1) \quad (6)$$

$$M_{33} = m_3 d_3^{\ 2} + I_3 + m_4 L_3^{\ 2} \tag{7}$$

$$M_{34} = m_4 d_4 L_3 \cos\left(\theta_4 + \gamma_4 - \theta_3\right) \tag{8}$$

$$M_{55} = m_5 d_5^{\ 2} + I_5 + m_6 L_5^{\ 2} \tag{9}$$

$$M_{56} = m_6 d_6 L_5 \cos\left(\theta_6 + \gamma_6 - \theta_5\right) \tag{10}$$

$$C_1 = -L_1 \dot{\theta}_2^{\ 2} \left(m_2 d_2 \sin\left(\theta_2 + \gamma_2 - \theta_1\right) + m_7 L_2 \sin\left(\theta_2 - \theta_1\right)\right) - m_7 d_7 L_1 \dot{\theta}_7^{\ 2} \sin\left(\theta_7 + \gamma_7 - \theta_1\right) \tag{11}$$

$$C_3 = -m_4 d_4 L_3 \dot{\theta}_4^{\ 2} \sin\left(\theta_4 + \gamma_4 - \theta_3\right) \tag{12}$$

$$C_5 = -m_6 d_6 L_5 \dot{\theta}_6^{\ 2} \sin\left(\theta_6 + \gamma_6 - \theta_5\right) \tag{13}$$

$$\Gamma_1 = m_1 d_1 \cos\left(\theta_1 + \gamma_1\right) + \left(m_2 + m_7\right) L_1 \cos\left(\theta_1\right) \tag{14}$$

$$\Gamma_3 = m_3 d_3 \cos\left(\theta_3 + \gamma_3\right) + m_4 L_3 \cos\left(\theta_3\right) \tag{15}$$

$$\Gamma_5 = m_5 d_5 \cos\left(\theta_5 + \gamma_5\right) + m_6 L_5 \cos\left(\theta_5\right) \tag{16}$$

The Lagrange multipliers λ_1, λ_2, λ_3 and λ_4 are related to each other through the following equations:

$$\mathbf{D} \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \end{bmatrix} = \begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \\ \tau_4 \end{bmatrix} \tag{17}$$

where

$$\mathbf{D} = \begin{bmatrix} -L_2 \sin\left(\theta_2\right) & L_2 \cos\left(\theta_2\right) & -L_2 \sin\left(\theta_2\right) & L_2 \cos\left(\theta_2\right) \\ L_4 \sin\left(\theta_4\right) & -L_4 \cos\left(\theta_4\right) & 0 & 0 \\ 0 & 0 & L_6 \sin\left(\theta_6\right) & -L_6 \cos\left(\theta_6\right) \\ -L_7 \sin\left(\theta_7\right) & L_7 \cos\left(\theta_7\right) & -L_7' \sin\left(\theta_7 + \alpha\right) & L_7' \cos\left(\theta_7 + \alpha\right) \end{bmatrix} \tag{18}$$

$$\tau_1 = M_{12} \ddot{\theta}_1 + M_{22} \ddot{\theta}_2 + M_{27} \ddot{\theta}_7 + C_2 + g\Gamma_2 \tag{19}$$

$$\tau_2 = M_{34} \ddot{\theta}_3 + M_{44} \ddot{\theta}_4 + C_4 + g\Gamma_4 \tag{20}$$

$$\tau_3 = M_{56} \ddot{\theta}_5 + M_{66} \ddot{\theta}_6 + C_6 + g\Gamma_6 \tag{21}$$

$$\tau_4 = M_{17} \ddot{\theta}_1 + M_{27} \ddot{\theta}_2 + M_{77} \ddot{\theta}_7 + C_7 + g\Gamma_7 \tag{22}$$

with

$$M_{22} = m_2 d_2^{\ 2} + I_2 + m_7 L_2^{\ 2} \tag{23}$$

$$M_{27} = m_7 d_7 L_2 \cos\left(\theta_7 + \gamma_7 - \theta_2\right) \tag{24}$$

$$M_{44} = m_4 d_4^{\;2} + I_4 \tag{25}$$

$$M_{66} = m_6 d_6^{\;2} + I_6 \tag{26}$$

$$M_{77} = m_7 d_7^{\;2} + I_7 \tag{27}$$

$$C_2 = L_1 \dot{\theta}_1^{\;2} \left(m_2 d_2 \sin\left(\theta_2 + \gamma_2 - \theta_1\right) + m_7 L_2 \sin\left(\theta_2 - \theta_1\right)\right) - m_7 d_7 L_2 \dot{\theta}_7^{\;2} \sin\left(\theta_7 + \gamma_7 - \theta_2\right) \tag{28}$$

$$C_4 = m_4 d_4 L_3 \dot{\theta}_3^{\;2} \sin\left(\theta_4 + \gamma_4 - \theta_3\right) \tag{29}$$

$$C_6 = m_6 d_6 L_5 \dot{\theta}_5^{\;2} \sin\left(\theta_6 + \gamma_6 - \theta_5\right) \tag{30}$$

$$C_7 = m_7 d_7 \left(L_1 \dot{\theta}_1^{\;2} \sin\left(\theta_7 + \gamma_7 - \theta_1\right) + L_2 \dot{\theta}_2^{\;2} \sin\left(\theta_7 + \gamma_7 - \theta_2\right)\right) \tag{31}$$

$$\Gamma_2 = m_2 d_2 \cos\left(\theta_2 + \gamma_2\right) + m_7 L_2 \cos\left(\theta_2\right) \tag{32}$$

$$\Gamma_4 = m_4 d_4 \cos\left(\theta_4 + \gamma_4\right) \tag{33}$$

$$\Gamma_6 = m_6 d_6 \cos\left(\theta_6 + \gamma_6\right) \tag{34}$$

$$\Gamma_7 = m_7 d_7 \cos\left(\theta_7 + \gamma_7\right) \tag{35}$$

Solving Eq. (17) for the Lagrange multipliers yields

$$\lambda_k = \frac{\mathrm{N}_k}{\Delta}, \quad k = 1,\ 2,\ 3,\ 4 \tag{36}$$

where, denoting the adjoint matrix of **D** by $\tilde{\mathbf{D}}$, N_k (k = 1, 2, 3, 4) are given by

$$\begin{bmatrix} \mathrm{N}_1 \\ \mathrm{N}_2 \\ \mathrm{N}_3 \\ \mathrm{N}_4 \end{bmatrix} = \tilde{\mathbf{D}} \begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \\ \tau_4 \end{bmatrix} \tag{37}$$

and Δ is the determinant of **D**, which can be expressed as

$$\Delta = L_2 L_4 L_6 \left(-L_7 \sin\left(\theta_6 - \theta_2\right) \sin\left(\theta_7 - \theta_4\right) + L_7' \sin\left(\theta_4 - \theta_2\right) \sin\left(\theta_7 + \alpha - \theta_6\right)\right) \tag{38}$$

As is evident from Eqs. (36), and as also explained in Ref. [11], the determinant Δ vanishes at type II singularities. Let us consider the general case where the centers of the moving platform joints R_3, R_6 and R_9 describe a triangle whose interior angle α can take any value in the open interval (0, 180°). It can be shown that for all values of α in this interval, the **D** matrix of the considered robot becomes rank deficient by only one at the singular configurations. By examining Eqs. (36), one can then conclude that for the Lagrange multipliers (and hence the required input torques) to have finite limits in the neighborhood of such a singularity, the following equations should hold:

$$\mathrm{N}_k\left(t_s\right) = 0, \quad k = 1,\ 2,\ 3,\ 4 \tag{39}$$

where t represents time, and t_s is the singularity time. Recall that if a matrix is rank deficient by one, then the rank of its adjoint matrix is one [42]. It follows from this that three of the four equations in the system of Eqs. (39) are redundant, and the linearly independent equation in that system gives the unique necessary condition for consistency of Eq. (17) at the singularity time. This “consistency condition” can be alternatively written using the linear dependence relations among the rows of the associated force coefficient matrix at the singular configuration [11].

However, although necessary, maintaining the consistency is not sufficient alone, and, as shown in Ref. [14] through the application of L’Hôpital’s Rule, the following condition must also be fulfilled to establish the boundedness of the inverse dynamics solution near the singularity:

$$\left.\frac{\mathrm{d}\Delta}{\mathrm{d}t}\right|_{t=t_s} \neq 0 \tag{40}$$

Otherwise, additional derivative conditions must be satisfied in this regard, which can be determined based on Ref. [14] as follows: If the first n time derivatives of Δ simultaneously vanish at $t = t_s$, it follows from successive applications of L’Hôpital’s Rule that, in addition to Eqs. (39), the following equations are also necessary for desingularizing the robot of interest to the present paper:

$$\left.\frac{\mathrm{d}^r \mathrm{N}_k}{\mathrm{d}t^r}\right|_{t=t_s} = 0, \quad k = 1,\ 2,\ 3,\ 4, \quad r = 1,\ 2,\ \ldots,\ n \tag{41}$$

It should be emphasized at this point that an increase in the number and complexity of the conditions to be met for removing a singularity will undoubtably lead to difficulties in practice. For this reason, hypersingularities, which are characterized by requirement of additional conditions in their removal other than those for maintaining consistency, should be identified in the trajectory planning stage. In the next section, a condition that can be used within this context is derived.

3. A CONDITION FOR THE OCCURRENCE OF HYPERSINGULARITIES

As explained in Section 2, for any α in the open interval (0, 180°), the 3-RRR planar parallel robot will suffer from a hypersingularity when

$$\left.\frac{\mathrm{d}\Delta}{\mathrm{d}t}\right|_{t=t_s} = 0 \tag{42}$$

By differentiating Eq. (38) with respect to time and simplifying, it can be shown that the holding of Eq. (42) is equivalent to the satisfaction of the following equation at the singularity time:

$$\mu_1\dot{\theta}_2 + \mu_2\dot{\theta}_4 + \mu_3\dot{\theta}_6 + \mu_4\dot{\theta}_7 = 0 \tag{43}$$

where

$$\mu_1 = L_7\cos(\theta_6-\theta_2)\sin(\theta_7-\theta_4) - L_7'\cos(\theta_4-\theta_2)\sin(\theta_7+\alpha-\theta_6) \tag{44}$$

$$\mu_2 = L_7\sin(\theta_6-\theta_2)\cos(\theta_7-\theta_4) + L_7'\cos(\theta_4-\theta_2)\sin(\theta_7+\alpha-\theta_6) \tag{45}$$

$$\mu_3 = -L_7\cos(\theta_6-\theta_2)\sin(\theta_7-\theta_4) - L_7'\sin(\theta_4-\theta_2)\cos(\theta_7+\alpha-\theta_6) \tag{46}$$

$$\mu_4 = -L_7\sin(\theta_6-\theta_2)\cos(\theta_7-\theta_4) + L_7'\sin(\theta_4-\theta_2)\cos(\theta_7+\alpha-\theta_6) \tag{47}$$

The velocity constraints due to the closed-loop structure of the robot can be written as

$$-L_1\dot{\theta}_1\sin(\theta_1) - L_2\dot{\theta}_2\sin(\theta_2) + L_3\dot{\theta}_3\sin(\theta_3) + L_4\dot{\theta}_4\sin(\theta_4) - L_7\dot{\theta}_7\sin(\theta_7) = 0 \tag{48}$$

$$L_1\dot{\theta}_1\cos(\theta_1) + L_2\dot{\theta}_2\cos(\theta_2) - L_3\dot{\theta}_3\cos(\theta_3) - L_4\dot{\theta}_4\cos(\theta_4) + L_7\dot{\theta}_7\cos(\theta_7) = 0 \tag{49}$$

$$-L_1\dot{\theta}_1\sin(\theta_1) - L_2\dot{\theta}_2\sin(\theta_2) + L_5\dot{\theta}_5\sin(\theta_5) + L_6\dot{\theta}_6\sin(\theta_6) - L_7'\dot{\theta}_7\sin(\theta_7+\alpha) = 0 \quad (50)$$

$$L_1\dot{\theta}_1\cos(\theta_1) + L_2\dot{\theta}_2\cos(\theta_2) - L_5\dot{\theta}_5\cos(\theta_5) - L_6\dot{\theta}_6\cos(\theta_6) + L_7'\dot{\theta}_7\cos(\theta_7+\alpha) = 0 \quad (51)$$

The Cartesian components of the velocity vector of the end point in the fixed coordinate system *xy* can be expressed as

$$\dot{x}_{\mathrm{E}} = -L_1\dot{\theta}_1\sin(\theta_1) - L_2\dot{\theta}_2\sin(\theta_2) - d_7'\dot{\theta}_7\sin(\theta_7+\beta) \quad (52)$$

$$\dot{y}_{\mathrm{E}} = L_1\dot{\theta}_1\cos(\theta_1) + L_2\dot{\theta}_2\cos(\theta_2) + d_7'\dot{\theta}_7\cos(\theta_7+\beta) \quad (53)$$

Equations (48)–(53) constitute a system of six equations that can be solved for $\dot{\theta}_1$, $\dot{\theta}_2$, $\dot{\theta}_3$, $\dot{\theta}_4$, $\dot{\theta}_5$ and $\dot{\theta}_6$ in terms of $\dot{x}_{\mathrm{E}}$, $\dot{y}_{\mathrm{E}}$ and $\dot{\theta}_7$. By doing so and substituting the resulting expressions for $\dot{\theta}_2$, $\dot{\theta}_4$ and $\dot{\theta}_6$ into Eq. (43), it can be concluded that a type II singular configuration of the 3-RRR planar parallel robot becomes a hypersingularity when the end-effector velocities satisfy the following equation at that configuration:

$$\kappa_1\dot{x}_{\mathrm{E}} + \kappa_2\dot{y}_{\mathrm{E}} + \kappa_3\dot{\theta}_7 = 0 \quad (54)$$

where

$$\kappa_1 = \frac{\mu_1\cos(\theta_1)}{L_2\sin(\theta_1-\theta_2)} + \frac{\mu_2\cos(\theta_3)}{L_4\sin(\theta_3-\theta_4)} + \frac{\mu_3\cos(\theta_5)}{L_6\sin(\theta_5-\theta_6)} \quad (55)$$

$$\kappa_2 = \frac{\mu_1\sin(\theta_1)}{L_2\sin(\theta_1-\theta_2)} + \frac{\mu_2\sin(\theta_3)}{L_4\sin(\theta_3-\theta_4)} + \frac{\mu_3\sin(\theta_5)}{L_6\sin(\theta_5-\theta_6)} \quad (56)$$

$$\begin{aligned}\kappa_3 = {} & \frac{-\mu_1 d_7'\sin(\theta_1-\theta_7-\beta)}{L_2\sin(\theta_1-\theta_2)} + \frac{\mu_2\left(L_7\sin(\theta_3-\theta_7) - d_7'\sin(\theta_3-\theta_7-\beta)\right)}{L_4\sin(\theta_3-\theta_4)} \\ & + \frac{\mu_3\left(L_7'\sin(\theta_5-\theta_7-\alpha) - d_7'\sin(\theta_5-\theta_7-\beta)\right)}{L_6\sin(\theta_5-\theta_6)} + \mu_4\end{aligned} \quad (57)$$

Notice that the inverse kinematic singularity condition yields $\sin(\theta_{2j-1} - \theta_{2j}) = 0$ for each j = 1, 2, 3, and these configurations are left out of scope of this paper, as also mentioned above.

Equation (54) describes a plane in the end-effector velocity space of the robot. If the end-effector velocities $\dot{x}_{\mathrm{E}}$, $\dot{y}_{\mathrm{E}}$ and $\dot{\theta}_7$ satisfy this plane equation at a type II singular configuration, that configuration becomes a hypersingularity. It is worth noting that the coefficients κ_1, κ_2 and κ_3 depend on the values of the joint variables θ_i (i = 1, 2, …, 7) at the singular configuration of interest. This implies that the orientation of the hypersingularity plane in the end-effector velocity space changes according to the singular configuration. A vector that is normal to the hypersingularity plane of a type II singular configuration is $\kappa_1\mathbf{e}_1 + \kappa_2\mathbf{e}_2 + \kappa_3\mathbf{e}_3$ where $\mathbf{e}_1$, $\mathbf{e}_2$ and $\mathbf{e}_3$ are the unit vectors along the $\dot{x}_{\mathrm{E}}$, $\dot{y}_{\mathrm{E}}$ and $\dot{\theta}_7$ axes of the end-effector velocity space, respectively. Another important remark is that the hypersingularity plane of any type II singular configuration passes through the origin of the end-effector velocity space. Thus, if the considered robot is to be brought instantaneously to rest at a type II singular configuration, that configuration transforms into a hypersingularity. As a final remark, if a type II singular configuration is to be passed through with a constant orientation angle or with instantaneously zero angular speed of the moving platform, then the hypersingularity condition given by Eq. (54) reduces to the fulfillment of the following slope condition at that configuration:

$$\frac{\mathrm{d}y_{\mathrm{E}}}{\mathrm{d}x_{\mathrm{E}}} = -\frac{\kappa_1}{\kappa_2} \quad (58)$$

4. CONCLUSIONS

Hypersingularities, which are characterized by requirement of additional conditions other than the ones for consistency in their removal from the inverse dynamics solution, constitute an emerging research topic in the area of parallel robots. This paper presents an analysis of the hypersingularities of the 3-RRR planar parallel robot, which is a widely used robot in applications. The main findings of this analysis are as follows:

- Any type II singular configuration of the considered robot becomes a hypersingularity if the end effector velocities at that configuration lie on a certain plane in the end-effector velocity space. This is called the hypersingularity plane of that type II singular configuration.
- The equation of the hypersingularity plane of a type II singular configuration depends on the values of the joint variables at that configuration. Hence, the orientation of the hypersingularity plane in the end-effector velocity space changes according to the singular configuration.
- Whatever the singular configuration is, its hypersingularity plane always passes through the origin of the end-effector velocity space. It follows from this fact that if the said robot is to be brought to instantaneous rest at a type II singular configuration, that configuration transforms into a hypersingularity.
- When a type II singular configuration is to be passed through with a constant orientation angle or with instantaneously zero angular speed of the moving platform, the hypersingularity condition reduces to a condition in terms of the slope of the path of the end point at that configuration.

These findings can be used for avoiding hypersingularities in the trajectory planning stage. By this means, the number of conditions to be satisfied for enabling the robot to pass through type II singular configurations is minimized. The analysis and results presented here can be easily extended to other 3-DOF planar parallel robots with different architectures and actuation schemes. It is believed that the present paper will stimulate further work on the applications of its theoretical findings.

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