THE PUBLISHING HOUSE
OF THE ROMANIAN ACADEMY

PROCEEDINGS OF THE ROMANIAN ACADEMY, Series A,
Volume 21, Number 2/2020, pp. 179–186

APPARENT FRONT RANKING: A NOVEL POPULATION RANKING METHOD FOR GENETIC MULTI-OBJECTIVE ALGORITHMS

Mihai NEGHINĂ[1], Lucian VINŢAN [1,2 †]

[1] "Lucian Blaga" University, Bd. Victoriei, no. 10, 550024, Sibiu, Romania
[2] Technical Sciences Academy of Romania
[†] Deceased November 5, 2019
Corresponding author: Mihai NEGHINĂ, E-mail: mihai.neghina@ulbsibiu.ro

Abstract. This article proposes a novel ranking method based on the individual distances to the so-called Apparent Front, computed for each population with the help of the correspondingly selected support vectors and template functions. The developed ranking algorithm is compared to Fuzzy Pareto Domination and Analytic Hierarchy Process rankings using test cases of randomly generated sets of vectors with up to 10 dimensions, although the method itself works with an arbitrarily large number of objectives (dimensions) and test set vectors. The developed method is shown to be more flexible and computationally efficient than both Fuzzy Pareto Domination and Analytic Hierarchy Process rankings, since it avoids pairwise comparisons, but still strongly correlates with them for particular Apparent Front templates.

***Key words*:** ranking methods, analytic hierarchy process, fuzzy Pareto domination, Pareto optimality, multi-objective optimization methods.

1. INTRODUCTION

The field of multi-objective optimization greatly benefited from the introduction of the concept of Pareto-optimality [1]. Early approaches attempt to solve multi-dimensional optimization problems (MOPs) directly, by aggregating the objectives into a single combined function, analogous to the decision making before the search, and evaluating the aggregation result in order to identify a local or global extremum or a solution fitted for a set of constraints [2,3]. Another approach for solving MOPs attempts to generate a representative set of solutions for the Pareto set, especially through evolutionary methods [4]. For such methods, the most critical operation is the selection of candidates at each epoch, where the number of objectives effectively assures that there will be a great number of non-dominated solutions and it would therefore be extremely difficult to distinguish individuals in a Pareto population.

A multitude of ingenious methods for the selection of individuals has been attempted, including dimensionality reduction through principal component analysis [5], attempts to emphasize diversity over dominance of the candidate individuals [6,7,8,9,10], as well as attempts to re-define the Pareto dominance in terms of fuzzy logic [11,12,13,14]. These methods infer the ranking of individuals. Farina et al. [14] generalize the concepts of Pareto-optimality to k-optimality and Pareto-dominance to fuzzy dominance, based on the number of objectives dominated. Köppen et al. [12] use concepts from fuzzy fusion theory to achieve a remarkable full ranking of the individuals in a population, called Fuzzy Pareto Domination (FPD) ranking.

The proposed ranking method estimates a continuous hypersurface, the Apparent Front (AF), subject to the constraints of a template function, which best approximates a subset of support vectors from the current population and then ranks the individuals based on their proximity to it, hence receiving the name Apparent Front Ranking (AFR). The AFR method may resemble the aggregation methods of the classical approaches, but it has some distinct features. Unlike those classical approaches, the aggregation is dynamic and adaptive: the aggregation is not decided beforehand, but determined by an elite subset of support vectors from the current population. Any change to the population may lead to the identification of different support vectors and, therefore, to a different AF. Furthermore, the proposed AFR method does not have the goal to determine an extremum, like classical methods, but rather to determine the Apparent Front against which all individuals of the population can be tested and ranked. In contrast with other ranking systems, such as Analytic Hierarchy Process (AHP) [15] or Fuzzy Pareto Domination (FPD) [12] ranking, which are endogenous methods computing nuanced dominance values within the set of available vectors, this method is partly exogenous, as it attempts to establish an AF using support vectors from within the population as well as external template function constraints.

The remainder of this article is structured as follows: Section 2 gives an overview of the proposed ranking method, detailing the steps and providing information on both the general method and the particular implementation used in experiments; Section 3 mentions the FPD and AHP versions used for comparison of the proposed ranking method, describes the experimental setup and the performance metrics used, the Kendall [16] and Pearson [17] correlations; Section 4 provides the obtained results, both as ranking correlations against the previously mentioned methods and against an objective ideal ranking; finally, Section 5 presents conclusions and perspectives for further research of the proposed ranking method.

2. THE PROPOSED APPROACH

The AFR method proposed in this article could be used in the selection part of any multi-objective Genetic Algorithm (GA). Since the selection step is performed by evaluating distances to the AF in the space of objectives, the individuals in a population can be described only by their evaluations in the space of objectives, effectively becoming vectors in that space. Therefore, they may be referred to as vectors, foregoing their original meaning of individuals in a population capable of performing genetic operations like crossovers and mutations. Furthermore, this article deals only with the case where all objective functions need to be maximized, although the method can be easily adapted to minimization as well.

The successive steps for the proposed AFR algorithm are the following:

1. The choice of the AF template (which in the case of usage with a GA can be done either only once at the beginning, or for each generation);
2. The selection of support vectors from the current generation;
3. The estimation of the AF under constraints of the template function;
4. The computation of proximities to the AF and subsequent ranking.

The steps of the AFR method will be covered in detail in the following subsections, emphasizing the general concept but also explaining the particular variation used to generate the results in Section 5.

2.1. Choice of the AF template

General AFR: The AF templates are families of functions from which the AF will be computed.

Particular AFR: As an example of suitable function templates for the task of maximizing objectives in a N-dimensional space ($N \geq 2$), the families described by the equation (1) would be worth of consideration. Each power n generates a family of functions, where coefficients a_k need to be found at step 3 of the method.

$$\sum_{k=1}^{N} a_k x_k^n - 1 = 0 \text{, with } n \in \mathrm{R} - \{0\} \text{ and } a_k > 0 \text{, } \forall k \in \{1, 2, ..., N\}. \quad (1)$$

Considering the particular case for two objectives (thus $N = 2$), the template is $a_1 x_1^n + a_2 x_2^n - 1 = 0$, with $a_1, a_2 > 0$. Examples of functions from this template are shown in Figure 1. The quadrants II, III and IV are greyed out because they are not part of the space of objectives, but are represented nevertheless in order to convey the general shape of the template. The power n may be any real positive value, although only even integers have the added significance of generalized ellipses.

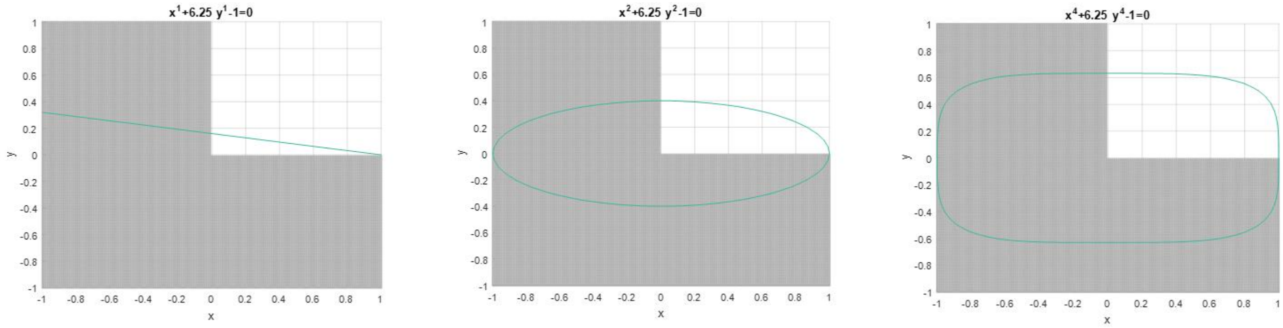

Fig. 1 – Examples of AFs for various powers n of the template function.

2.2. Selection of support vectors

General AFR: Ideally, the support vectors should be all non-dominated vectors of the current population. Selecting all non-dominated vectors could quickly become overly inclusive, considering that a large number of dimensions generates a high percentage of non-dominated solutions, as well as computationally expensive, since it requires a pairwise domination test between all the vectors. Considering that the template function may require only a limited number of support vectors, there are several possible ways of selecting enough support vectors and, at the same time, avoiding pairwise comparisons, such as re-partitioning the quadrant into a large enough number of regions (or solid angles) and selecting one (or multiple) vector(s) from each region. Such selection methods become more computationally efficient than pairwise comparisons.

Particular AFR: The particular version used for the experiments identifies all the non-dominated vectors because the ranking method is applied only once on randomly generated test sets, in order to be compared with other classical ranking methods.

2.3. Estimation of the template-based AF

General AFR: The estimation of the hypersurface that best fits the support vectors can be done through a number of statistical techniques, e.g. least-square minimization.

Particular AFR: In the chosen example of template, the power n is pre-selected as part of the template, not optimized (although the method does not prevent the choice of the more general template, having n as a parameter as well). Because the template is similar to ellipses, generalized in the sense of the number of dimensions and also generalized in the sense of the power n, fast least square fitting techniques could be used for estimating the AF [18,19]. In this article the method of Fitzgibbon et al. [18] has been adapted for fitting multidimensional general ellipses.

2.4. Computation of proximity to the AF

General AFR: The proximity to the AF for each vector $\boldsymbol{P}$, noted $d(\boldsymbol{P})$, would simply be the evaluation of the determined AF function at point $\boldsymbol{P}$. It could be also interpreted as the free term adjustment of the AF in order to go through $\boldsymbol{P}$.

Particular AFR: $d(P)=\sum_{k=1}^{N} a_k p_k^n - 1$, with coefficients a_k determined in the previous step.

For points lying on the estimated AF, the distance is $d=0$. Points beyond the estimated AF would have positive distances $(d>0)$, and the points below the estimated AF would have negative values $(-1 \le d < 0)$.

3. EXPERIMENTAL SETUP

3.1. Test sets

The developed test cases contain between 200 and 400 randomly generated vectors $\boldsymbol{P}=[p_1,\ldots,p_N]^{\mathrm{T}}$, constrained to the inequation (2) and represented in spaces with $N=2$ to $N=10$ dimensions. Figures 2 and 3 show the sets for two and three dimensions, with magenta points being the support vectors and the green line and surface being the computed AFs for template power $n=2$. It should be noted that the template function chosen (and the subsequent AF) cannot perfectly approximate the real Pareto front, which may happen with real problems as well.

$$(-1)^N \prod_{k=1}^{N}(p_k-2)\geq 1,\ \text{where}\ \ 0\leq p_k\leq 2\ \ \text{for all}\ k. \quad (2)$$

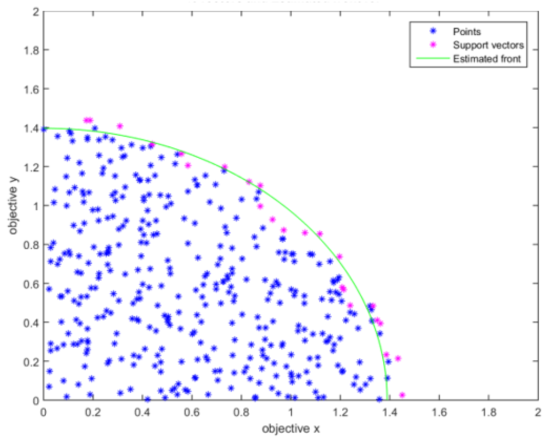

Fig. 2 – 2-D test set, support vectors and estimated AF for $n=2$.

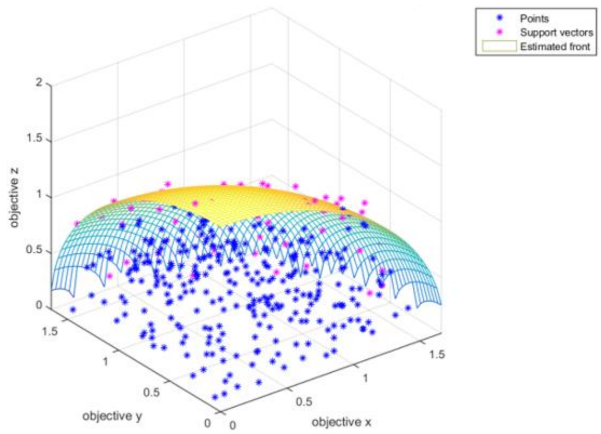

Fig. 3 – 3-D test set, support vectors and estimated AF for $n=2$.

3.2. Algorithms for comparison

The proposed AFR method is compared to two ranking algorithms from the literature: Köppen et al.'s algorithm based on Fuzzy Pareto Dominance (FPD) [12] and Saaty's Analytic Hierarchy Process (AHP) [12], each of them implemented in two versions.

Classical AHP computes the vector of criteria weights, then the pairwise comparison matrix, then the matrix of option scores and finally the ranking options. The versions of AHP used differ only in the way that the elements of the pairwise comparison matrix $\boldsymbol{M}$ (for any two vectors $\mathbf{p}$ and $\mathbf{q}$) are computed:

- $m_{pq}^{(k)}=p_k/q_k$ (further referred to as AHP_{v1});
- $m_{pq}^{(k)}=\left(8\cdot\dfrac{|p_k-q_k|}{\text{MAX}(k)-\text{MIN}(k)}+1\right)^{\text{sign}(p_k-q_k)}$ (further referred to as AHP_{v2}),

where $\text{MAX}(k)/\text{MIN}(k)$ represent the maximum/minimum value of the current set on the k-th dimension.

On the other hand, FPD uses an asymmetric measure for fuzzy domination, by which vector $\mathbf{p}$ is said to dominate vector $\mathbf{q}$ by degree $\mu_{pq}=\prod_k \min(p_k,q_k)/\prod_k p_k$, considering that all objectives need to be minimized. For the cases used here, when the objectives are maximized, the fuzzy membership needs to be changed to $\mu_{pq}=\prod_k \min(p_k,q_k)/\prod_k q_k$. The two versions of FPD ranking compute the ranking values by averaging (further referred to as FPD_{ave}) or taking the minimum (further referred to as FPD_{min}) fuzzy dominance of each vector over all others within the population.

3.3. Performance metrics

In order to evaluate the similarity of the rankings, the following measures are used:

Kendall similarity measure [16] between two rankings ($\mathbf{R_1},\mathbf{R_2}$) related to a given set $\mathbf{S}$ and its ordered pairs O_{S,R_1},O_{S,R_2} : the Kendall correlation coefficient τ in equation (3) can be shown to be between 0 (no correlation) and 1 (perfect correlation, i.e. identical ranking).

$$\tau = \frac{0.5\cdot N\cdot(N-1)-\Delta(O_{S,R_1},O_{S,R_2})}{0.5\cdot N\cdot(N-1)} \tag{3}$$

Pearson correlation [17]: It is easy to prove that the Pearson correlation coefficient ρ defined in equation (4) is between -1 and $+1$. Zero means that there is no linear correlation, whereas values close to $+/-1$ signify perfect direct/inverse correlation between **X** and **Y**. The Pearson correlation coefficient is computed on the ranks as well as on the values generated by the method for deciding the ranks. The following values are considered: the AHP values, FPD average and minimum dominance, and the AFR distances to the AF.

$$\rho = \frac{\mathrm{cov}(X,Y)}{\sigma_X\cdot\sigma_Y} = \frac{\sum_{k=1}^{N}\left(x_k-\bar{x}\right)\left(y_k-\bar{y}\right)}{\sqrt{\sum_{k=1}^{N}\left(x_k-\bar{x}\right)^2}\sqrt{\sum_{k=1}^{N}\left(y_k-\bar{y}\right)^2}} \tag{4}$$

4. RESULTS AND DISCUSSIONS

Figure 4 shows the correlation ranges of AFR to the literature methods (FPD and AHP) for up to 10 dimensions. It could be noted that the correlation to FPD drops quite rapidly, whereas the correlation remains high with $\mathrm{AHP_{v1}}$ and especially with $\mathrm{AHP_{v2}}$ for a larger range of powers n.

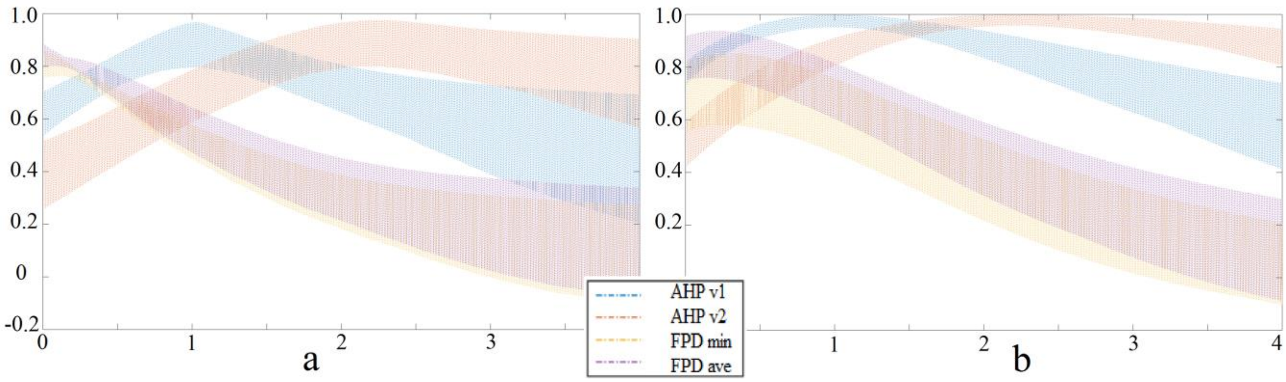

Fig. 4 – The Kendall rank similarity range (a) and the Pearson value similarity range (b) across all test cases (up to 10 dimensions)

Regardless of the number of dimensions, for all correlation coefficients, the best AFR correlation against $\mathrm{AHP_{v1}}$ was found with a template $n=1\pm0.05$, the best correlation against $\mathrm{AHP_{v2}}$ was found with template $n=2.2\pm0.11$, while the best correlation against FPD variants was found with templates $n<0.35$. This pattern is observed for test sets with any number of dimensions, showing thus intrinsic qualitative behaviours of these methods under certain circumstances. The Kendall correlation varies between 0.79 and 0.98 in case of the comparison with AHP variants, and between 0.76 and 0.89 in the case of the comparison with FPD variants. The Pearson correlation on values is a good indicator of how similar the ranking decisions are, even at high Kendall correlation coefficients. When comparing AFR to AHP, the Pearson correlation on values is high (>0.95), but when comparing AFR to FPD, the Pearson correlation on values drops in some cases even to 0.59, with a pronounced drop for a larger number of dimensions. Considering the Kendall correlation on ranks and Pearson correlation on values, it seems that AFR is able to produce similar results to AHP by also keeping the decision values similarly clustered, but is also able to produce similar results to FPD despite having different relations between decision values. The idea behind AFR seems to be much more relatable to the ideas behind AHP than to those behind FPD, while still covering both.

While for other test cases (with different real Pareto fronts) these observations may not hold so strictly, it is remarkable that the proposed method can generate similar results to two completely different methods at the same time, depending on one parameter of the template function (the power n). As already discussed in the theoretical description of the proposed ranking method, the general AFR is much more flexible than this particular version used for experiments, since the templates are not restricted only to generalized ellipses.

For the artificially generated test sets used in this article, the vectors are bounded by equation (2). The distances to the real Pareto front (and therefore an objective ideal ranking) could be computed in a similar manner to the way that distances were already computed for AFR (described in Section 2), i.e. by evaluating the boundary condition $(-1)^N \prod_{k=1}^{N} (p_k - 2) - 1$ at the considered points.

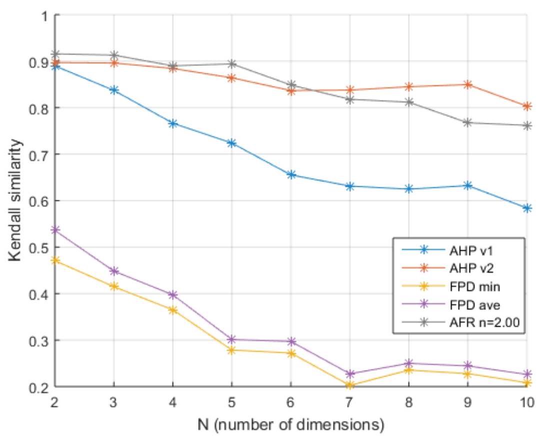

Fig. 5 – Kendall correlation to the objective ideal ranking.

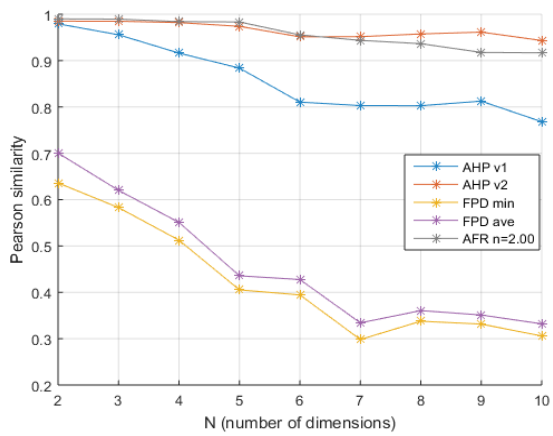

Fig. 6 – Pearson correlation to the objective ideal ranking

Figures 5 and 6 show the Kendall and Pearson rank correlations between the objective ideal ranking and each of the other considered methods. For AFR only the results for $n = 2$ are shown. While other templates (with powers different from $n = 2$) could score a better correlation to the objective ideal ranking, as shown in Table 1, template $n = 2$ is usually close to the maximum possible correlation to the objective ideal ranking that this particular AFR implementation could achieve for these particular test sets.

Table 1

Best correlation coefficients of AFR versions to the objective ideal ranking

Test set	Correlation between objective Ideal ranking and AFR ($n = 2$)		Correlation between objective ideal ranking and AFR (best power n)	
	Kendall	Pearson	Kendall	Pearson
Test set 2-D	0.9157	0.9900	0.9655 (n=1.47)	0.9981 (n=1.50)
Test set 3-D	0.9129	0.9895	0.9464 (n=1.59)	0.9951 (n=1.64)
Test set 4-D	0.8899	0.9840	0.8956 (n=1.80)	0.9846 (n=1.86)
Test set 5-D	0.8941	0.9834	0.9018 (n=1.80)	0.9845 (n=1.87)
Test set 6-D	0.8486	0.9555	0.8497 (n=2.06)	0.9631 (n=2.36)
Test set 7-D	0.8180	0.9436	0.8227 (n=2.22)	0.9520 (n=2.32)
Test set 8-D	0.8117	0.9365	0.8175 (n=2.14)	0.9441 (n=2.24)
Test set 9-D	0.7674	0.9177	0.7723 (n=2.14)	0.9256 (n=2.26)
Test set 10-D	0.7620	0.9168	0.7696 (n=2.17)	0.9260 (n=2.32)

For the test cases considered, the best correlations to the objective ideal ranking are either AFR (for up to 6 dimensions) or AHP_{v2} (for 7 to 10 dimensions). However, other concrete variants of AFR are possible and simply considering other templates or other fitting techniques may lead to different, hopefully better results. The proposed method outperforms even the powerful AHP method for relatively low-dimensional spaces, making AFR an alternative worth considering, with potential fertile applications in developing new efficient multi-objective optimization algorithms. Of course, further detailed investigations are necessary.

5. CONCLUSIONS AND PERSPECTIVES

The proposed new ranking method, called Apparent-Front Ranking, represents a flexible, scale-independent and computationally efficient method of arranging a set of vectors in the multidimensional space objectives, with resulting rankings comparable to other well-known methods presented in the literature. For the test cases considered, artificially generated sets of points with up to 10 dimensions, AFR displayed the capacity of ranking vectors similarly to both FPD and AHP, depending on one parameter of a particular family of template functions (the power n), with the best Kendall ranking similarity above 0.75 in all cases and usually greater than 0.9. AFR displayed rankings similar to the FPD variants for very low values of n, and similar to AHP variants for values around $n=1$ and $n=2.2$ respectively. As observed with all test sets, low template powers n emphasize central points, whereas bigger values for n improve the ranks of points near the edges.

Regarding the computational efficiency, unlike pairwise comparison methods such as FPD and AHP, AFR only needs to go through all vectors just twice:

- Before computing the AF, in order to select the support vectors
- After computing the AF, in order to compute the ranking values

The flexibility of the method is given by the possibility to choose any template function. For this article, the family of templates in equation (1) was chosen due to the properties of those functions. Because the proximities to the AF are evaluated against a continuous hypersurface determined on support vectors from the population, if the support vectors are well distributed, all regions of the quadrant should have points close to (or even exceeding) the AF, leading to a balanced spread of highly ranked points. Unlike the other methods presented in literature, where a change in the ranking algorithm is difficult, AFR offers an easy way to adjust the template function (or a parameter of the template as, for example, the power n in the case of this article) to fit the problem at hand. The flexibility of the AF may become a useful asset in genetic algorithms who need to adapt their selection.

Comparing AFR, AHP and FPD ranking methods with an objective ideal ranking, that takes into consideration the real Pareto front, it was shown that the AFR algorithm represents a better alternative to other methods, with potential fertile applications in developing new evolutionary efficient multi-objective optimization algorithms.

As a perspective, the suggested ranking method will be included in multi-objective genetic algorithms in order to compare the presented AFR-based selection to ranking selections such as in FDD-GA [12] or HypE [20], as well as non-ranking selections such as NSGA-II [21], SPEA2 [22], PESA [23], grid domination [24] or θ-domination [25]. Rather than using artificially generated test sets, the performance of the AFR-based genetic algorithms will be evaluated in the context of well established benchmarks such as ZDT [26] or DTLZ [27] test suites. After the intended inclusion in the framework for automatic design space exploration (FADSE) [28], the AFR-based genetic algorithms will also be applied to the grid ALU processor (GAP) micro-architecture hardware optimization problem [28,29,30]. Furthermore, multi-objectives optimization problems related to cyber-physical production systems that are investigated in the DiFiCiL project [31] can be addressed using AFR-based genetic algorithms as well.

ACKNOWLEDGEMENT

This work is supported through the DiFiCIL project (contract no. 69/08.09.2016, ID P_37_771, web: http://dificil.grants.ulbsibiu.ro) cofunded by ERDF through the Competitiveness Operational Programme 2014-2020, as well as "Lucian Blaga" University of Sibiu and Hasso Plattner Foundation research grant LBUS-IRG-2019-05.

REFERENCES

1. V. PARETO, *Cours d'economique politique*. Vol. I/II. Macmillan, Paris/London, 1897.
2. E. ZITZLER, *Evolutionary algorithms for multiobjective optimization: Methods and applications*, Swiss Federal Institute of Technology Zurich, Switzerland, 1999.
3. K. MIETTINEN, *Nonlinear multiobjective optimization*, vol. 12. Springer, US, 1999.
4. K. DEB, *Multi-objective optimization using evolutionary algorithms*. Chichester, Wiley, UK, 2001.
5. K. DEB, D. SAXENA, *On finding Pareto-optimal solutions through dimensionality reduction for certain large-dimensional multi-objective optimization problems*, Kangal report 2005011, Indian Institute of Technology, 2005.
6. M. LAUMANNS, L. THIELE, K. DEB, E. ZITZLER, *Combining convergence and diversity in evolutionary multiobjective optimization*, Evol. Comp., **10**, *3*, pp. 263-282, 2002.
7. S.F. ADRA, P.J. FLEMING, *Diversity management in evolutionary many-objective optimization*, Evol. Comp., **15**, *2*, pp. 183-195, 2011.
8. S. JIANG, S. YANG, *Convergence versus diversity in multiobjective optimization*, Proceedings of the 14th International Conference on Parallel Problem Solving from Nature (PPSN XIV), Lecture Notes in Computer Science, vol. 9921, Springer, Cham, pp. 984-993, 2016.
9. S. ROSTAMI, F. NERI, *A fast hypervolume driven selection mechanism for many-objective optimisation problems*, Swarm and Evol. Comp., **34**, pp. 50-67, 2017.
10. Z. LIU, Y. WANG, P. HUANG, *A many-objective evolutionary algorithm with angle-based selection and shift-based density estimation*, Information Sciences, **509**, pp. 400-419, 2020.
11. M. FARINA, P AMATO, *Fuzzy optimality and evolutionary multiobjective optimization*, Evolutionary Multi-Criterion Optimization EMO2003, Lecture Notes in Computer Science, vol. 2632, pp. 58-72, 2003.
12. M. KÖPPEN, R. VICENTE-GARCIA, B. NICKOLAY, *Fuzzy-Pareto-dominance and its application in evolutionary multi-objective optimization*, Evolutionary Multi-Criterion Optimization, Lecture Notes in Computer Science, vol. 3410, pp. 399-412, 2005.
13. R.C. SILVA, A. YAMAKAMI, *Definition of fuzzy Pareto-optimality by using possibility theory*, International Fuzzy Systems Association & European Society for Fuzzy Logic and Technology Conference, IFSA/EUSFLAT, pp. 1234-1239, 2009.
14. M. FARINA, P. AMATO, *On the optimal solution definition for many-criteria optimization problems*, 2002 Annual Meeting of the North American Fuzzy Information Processing Society Proceedings, NAFIPS-FLINT, pp. 233-238, 2002.
15. T.L. SAATY, *The analytical hierarchy process*, McGraw-Hill, New York, 1980.
16. M.G. KENDALL, *Rank correlation methods*, Hafner Publishing Co., New York, 1955.
17. K. PEARSON, *Notes on regression and inheritance in the case of two parents*, Proc. Royal Society London, **58**, pp. 240-242, 1895.
18. A. FITZGIBBON, M. PILU, R. FISCHER, *Direct least square fitting of ellipses*, Pattern and Analysis Machine Intelligence, **21**, *5*, pp. 476-480, 1999.
19. P. O'LEARY, P.Z. MURRAY, *Direct and specific least square fitting of hyperbolae and ellipses*, Electronic Imaging, **13**, *5*, pp. 492–503, 2004.
20. J. BADER, E. ZITZLER, *HypE an algorithm for fast hypervolume-based many-objective optimization*, Evol. Comp., **19**, *1*, pp. 45-76, 2011.
21. K. DEB, A. PRATAP, S. AGARWAL, T. MEYARIVAN, *A fast and elitist multiobjective genetic algorithm: NSGA-II*, Evol. Comp., **6**, *2*, pp. 182-197, 2002.
22. E. ZITZLER, M. LAUMANNS, L. THIELE, *SPEA2: Improving the strength pareto evolutionary algorithm for multiobjective optimization*, Proc. of Evolutionary Methods for Design, Optimization and Control, EUROGEN, pp. 95-100, 2001.
23. D.W. CORNE, J.D. KNOWLES, M.J. OATES, *The Pareto envelope-based selection algorithm for multiobjective optimization*, Parallel Problem Solving from Nature (PPSN VI), pp. 839-848, 2000.
24. S. YANG, M. LI, X. LIU, J. ZHENG, *A grid-based evolutionary algorithm for many-objective optimization*, Evol. Comp., **17**, *5*, pp. 721-736, 2013.
25. Y. YUAN, H. XU, B. WANG, X. YAO, *A new dominance relation based evolutionary algorithm for many-objective optimization*, Evol. Comp., **20**, *1*, pp.16-37, 2016.
26. E. ZITZLER, K. DEB, L. THIELE, *Comparison of multiobjective evolutionary algorithms*, Evol. Comp., **8**, *2*, pp. 173-195, 2000.
27. K. DEB, L. THIELE, M. LAUMANNS, E. ZITZLER, *Scalable test problems for evolutionary multi-objective optimization*, Kangal report 2001001, Indian Institute of Technology, 2001.
28. R. JAHR, H. CALBOREAN, L. VINŢAN, T. UNGERER, *Finding near-perfect parameters for hardware and code optimizations with automatic multi-objective design space explorations*, Concurrency and Computation: Practice and Experience, **27**, *9*, pp. 2196-2214, 2015.
29. L. VINŢAN, R. CHIŞ, M.A. ISMAIL, C. COŢOFANĂ, *Improving computing systems automatic multi-objective optimization through meta-optimization*, IEEE Trans. on Computer-Aided Design of Integrated Circuits and Systems, **35**, *7*, pp. 1125-1129, 2016.
30. A. GELLERT, A. FLOREA, U. FIORE, P. ZANETTI, L. VINŢAN, *Performance and energy optimisation in CPUs through fuzzy knowledge representation*, Information Sciences, **476**, pp. 375-391, 2019.
31. DiFiCIL: Development of Cyber-Physical Social Systems based on the IoT for the Factory of the Future, http://dificil.grants.ulbsibiu.ro/en/, accessed on January 2020.

Received September 10, 2019