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EVIDENCE OF DIRAC LARGE NUMBERS HYPOTHESIS

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Abstract. In the first stage of calculations using dimensional analysis, we have found the approximate value of a large number of the order of 10^{61} connecting cosmological parameters (mass of the Hubble sphere, Hubble distance, age of the universe, density of the universe, and minimal measurable temperature of the universe) and the respective fundamental microscopic properties of the matter (Planck mass, Planck length, Planck time, Planck density, and Planck temperature). In the final stage of calculations, we have recalculated precise Planck units with the physical definition of Planck mass as a mass whose Compton wavelength and gravitational radius are equals. In result, exact equation of the large number 5.73×10^{60} has been found, connecting cosmological parameters and Planck units. Thus, a precise formulation and evidence of Dirac large numbers hypothesis has been found, connecting the microworld and the macroworld.

***Key words**:* Dirac large numbers hypothesis, Planck units, dimensional analysis, cosmological parameters.

1. INTRODUCTION

The history of large numbers 'coincidences' began with Weyl [1], who showed that the hypothetical particle whose rest energy $m_x c^2$ is equal to the gravitational self-energy of the electron $\frac{Gm_e^2}{r_e}$ have mass $m_x = \frac{Gm_e^2}{c^2 r_e}$ and electrostatic radius $r_x = \frac{e^2}{m_x c^2} = \frac{e^2 r_e}{Gm_e^2}$. Thus he found that the ratio of r_x and classical electron radius $r_e = \frac{e^2}{m_e c^2}$ is the large number:

$$\frac{r_x}{r_e} = \frac{e^2}{Gm_e^2} \approx 4.2\times10^{42}, \tag{1}$$

where m_e and e are the mass and charge of electron, G – gravitational constant and c – speed of light in vacuum.

This coincidence was further developed by Eddington [2] who related the above ratio to the estimated number of charged particles in the universe N_e:

$$\frac{r_x}{r_e} \sim \sqrt{N_e} \sim 10^{40}. \tag{2}$$

Dirac [3] suggested the large numbers hypothesis (LNH) pointing out that the ratio of the age of the universe H^{-1} and the strong time scale $\tau = e^2/(m_e c^3) \sim 10^{-23}$ s is a large number of the order of 10^{40}. Besides, the ratio of electrostatic e^2/r^2 and gravitational forces $Gm_e m_p/r^2$ between proton and electron in a hydrogen atom is of the order of 10^{39} and the ratio of mass of the observable universe M and nucleon mass roughly is of the order of 10^{80}. That is to say:

$$\frac{H^{-1}}{\tau} \sim \frac{e^2}{Gm_e m_p} \sim \sqrt{\frac{M}{m_p}} \sim N_D \sim 10^{40}, \tag{3}$$

where m_p is the proton mass and $N_D \sim 10^{40}$ is the Dirac large number.

Relying on the ratios (3), he proposed that as a consequence of causal connections between macro and micro physical world, gravitational constant G slowly decreases with time.

Many other interesting ratios have been found approximately relating some cosmological parameters and microscopic properties of the matter. For example, the ratio of radius of the observable universe and classical radius of the electron $e^2/(m_e c^2)$ has been found of the order of 10^{40} [4]. Also, the ratio of the electron mass m_e and Hubble scale mass $m_H = \hbar H/c^2$ approximates to 10^{39} [5]. The number of nucleons in the universe $\overline{\rho}(cH^{-1})^3/m_p$ was found of the order of magnitude of $10^{80} \sim N_D^2$ [6]. The mass ratio for a typical star and an electron has been found of the order of 10^{60} [7]. The ratio of mass of the observable universe and Planck mass is of the order of 10^{61} [8]. The ratio of Hubble distance and Planck length has been found of the order of 10^{60} [9]. Finally, the ratio of Planck density ρ_P and recent critical density of the universe ρ_c is of the order of 10^{121} [10]. Most of these large numbers are rough ratios of astrophysical parameters and microscopic properties of the matter determined with accuracy of the order of magnitude. The comprehensive review on Dirac LNH is available in [11].

The Planck mass m_P has been derived in [12] by dimensional analysis using three fundamental constants – the speed of light in vacuum (c), the gravitational constant (G), and the reduced Planck constant ($\hbar$):

$$m_P \sim \sqrt{\frac{\hbar c}{G}} \approx 2.17\times10^{-8}\ \text{kg}. \tag{4}$$

Also, the Planck mass can be derived by setting it as a mass, whose Compton wavelength and gravitational radius are equal [13]. Analogously, formulae for Planck length l_P, Planck time $t_P = l_P/c$ and Planck density ρ_P were derived by dimensional analysis [12]. The energy equivalent of Planck mass $E_P = m_P c^2 \sim 10^{19}$ GeV represents unification energy of the fundamental interactions [14].

The Planck temperature T_P is defined as:

$$T_P \sim \frac{m_P c^2}{k_B} = \sqrt{\frac{\hbar c^5}{G k_B^2}} \approx 1.42\times10^{32}\ \text{K}, \tag{5}$$

where k_B is the Boltzmann constant.

Although, the deep nature of Planck units yet is unrevealed, they are a subject of theoretical research of modern quantum cosmology, string theory and quantum gravity. Apparently, the Planck length sets the fundamental limits on the accuracy of length measurement. In some forms of quantum gravity, the Planck length is the length scale at which the structure of spacetime becomes dominated by quantum effects, and it is impossible to determine the difference between two locations less than one Planck length apart. The precise effects of quantum gravity are unknown, but it is theorized that spacetime might have a discrete or foamy structure at a Planck length scale.

The dimensional analysis is a conceptual tool often applied in physics to understand physical situations involving certain physical quantities [15–18]. When it is known that quantities should be connected, but the form of this connection is unknown, a dimensional equation is formulated. Most often, dimensional analysis is applied in mechanics and other fields of modern physics, where problems have few determinative quantities. Many interesting and important problems related to the fundamental constants have been considered [19-22].

The discovery of the linear relationship between recessional velocity of distant galaxies, and distance $v = Hr$ introduces new fundamental quantity in physics and cosmology – the famous Hubble constant H [23]. The Hubble constant (parameter) determines the age of the universe $H^{-1} \sim 13.8$ billion years, the Hubble distance $cH^{-1} \sim 13.8$ billion light years, and the critical density of the universe $\rho_c = 3H^2/(8\pi G) \approx$ $\approx 9.47\times10^{-27}$ kg m^{-3} [24]. According to the contemporary cosmology, the Hubble constant slowly decreases with the age of the universe $\dot{H}/H \sim -H = -2.3\times10^{-18}$ s^{-1}.

2. APPROXIMATE ESTIMATION OF THE LARGE NUMBER CONNECTING COSMOLOGICAL PARAMETERS AND PLANCK UNITS

Because of the importance of the Hubble constant, we have included H in the dimensional analysis together with c, G and $\hbar$, and thus three new triads of constants besides (c, G, $\hbar$) have been created – (c, $\hbar$, H), (c, G, H) and (G, $\hbar$, H) [25]. There it has been shown that a unique mass m_i can be deduced from every mentioned triad. The first derived mass m_1 is the Hubble scale mass m_H:

$$m_1 \sim \frac{\hbar H}{c^2} = m_H \sim 10^{-33}\ \text{eV}. \tag{6}$$

This exceptionally small mass coincides with the minimal measurable gravitational self-energy of a particle [26] which is accepted as minimum quantum of energy $E_{\min} = \hbar H \sim 10^{-33}$ eV [27]. This energy takes substantial place in the estimations of total information and entropy of the observable universe [28–30]. Thus, the mass m_1 seems close to the graviton mass obtained by different methods [31–34]. The mass m_1 is several orders of magnitude smaller than the upper limit of graviton mass, obtained by astrophysical constraints [35]. Therefore, this value doesn't contradict of astrophysical observations.

The presence of a small nonzero mass of the graviton should involve Yukawa type potential of gravitational field $V(r) = -(Gm/r)\exp(-m_H cr/\hbar)$ [36] that set a finite range of the gravity close to the Hubble distance $cH^{-1} \approx 1.38\times10^{10}$ light years. If this case takes a place, the Hubble distance should determines the size of gravitationally connected universe for an arbitrary observer.

Evidently, the minimum quantum of energy $E_{\min} = \hbar H$ set a lowest limit of measurable temperature T_H:

$$T_H = \frac{\hbar H}{k_B} \approx 1.75\times10^{-29}\ \text{K}. \tag{7}$$

This temperature is of the order of Hawking temperature for a black hole having mass of the Hubble sphere $T_{BH} = \dfrac{hc^3}{8\pi G k_B M_H} \sim 10^{-29}$ K [37] and the inverse temperature of the universe found by the quantum tunneling [38].

The second derived mass m_2 [25] is close to the mass of the Hubble sphere:

$$m_2 \sim \frac{c^3}{GH} = M \sim 10^{53}\ \text{kg}. \tag{8}$$

The Hubble sphere is a spherical region of the universe surrounding an observer beyond which objects recede from that observer at a rate greater than the speed of light. The radius of the Hubble sphere is equal to the Hubble distance $cH^{-1} \approx 13.8$ billions light years and the average density of the Hubble sphere (including dark matter and dark energy) is $\bar{\rho} \approx \rho_c = 3H^2/(8\pi G)$. Therefore, the mass of the Hubble sphere is:

$$M_H = \frac{4\pi}{3}\frac{c^3}{H^3}\frac{3H^2}{8\pi G} = \frac{c^3}{2GH} \approx 8.8\times10^{52}\ \text{kg}. \tag{9}$$

The observable universe consists of the galaxies and other matter that can, in principle, be observed at the present time because light and other signals from these objects have had time to reach Earth since the beginning of the cosmological expansion. The comoving distance from Earth to the edge of the observable universe is about 46.5 billion light years in any direction. Therefore, the observable universe is a three dimensional sphere with a diameter of about 8.8×10^{26} m [39,40].

The approximate equation for total density of the universe $\tilde{\rho} \approx \rho_c$ has been deduced by means of dimensional analysis [41]:

$$\tilde{\rho} \sim \frac{H^2}{G} \approx 7.93\times10^{-26}\ \text{kg}\cdot\text{m}^{-3}. \tag{10}$$

Thus, the equations of Hubble scale mass (6), mass of the Hubble sphere (8) and total density of the universe (10) have been derived approximately by dimensional analysis with the fundamental constants c, G, $\hbar$ and H. The Planck mass (4), Planck temperature (5), Planck length (11), Planck time (12) and Planck density (13) also have been deduced by dimensional analysis by means of constants c, G and $\hbar$ [12]:

$$l_P \sim \sqrt{\frac{G\hbar}{c^3}} \approx 1.61\times10^{-35}\ \text{m} \tag{11}$$

$$t_P = l_P / c \sim \sqrt{\frac{G\hbar}{c^5}} \approx 5.37\times10^{-44}\ \text{s} \tag{12}$$

$$\rho_P \sim \frac{c^5}{\hbar G^2} \approx 5.2\times10^{96}\ \text{kg}\cdot\text{m}^{-3} \tag{13}$$

Taking into account equations (4–8, 10–13), as well as Hubble distance cH^{-1} and Hubble time H^{-1} we find remarkable ratios:

$$\frac{M}{m_P} = \frac{cH^{-1}}{l_P} = \frac{H^{-1}}{t_P} = \sqrt{\frac{\rho_P}{\tilde{\rho}}} = \frac{T_P}{T_H} = \frac{m_P}{m_H} = \sqrt{\frac{M}{m_H}} = \sqrt{\frac{c^5}{G\hbar H^2}} = N \approx 8.1\times10^{60}. \tag{14}$$

Therefore, the ratio of the mass of the Hubble sphere M and the Planck mass m_P is equal to the large number N defined from the equation $N = \sqrt{c^5/(G\hbar H^2)} \approx 8.1\times10^{60}$ Besides, the large number N defines the ratio of the Hubble distance cH^{-1} and the Planck length l_P, the ratio of Hubble time (age of the universe) H^{-1} and the Planck time t_P, the square root of the ratio of the Planck density ρ_P and the approximate density of the universe $\tilde{\rho}$, the ratio of Planck temperature T_P and minimal measurable temperature T_H, and the ratio of Planck mass m_P and the Hubble scale mass m_H. These ratios are very important because they connect cosmological parameters (mass of the Hubble sphere, Hubble distance, age of the universe, density of the universe, and minimal measurable temperature of the universe) and the respective fundamental microscopic properties of the matter (Planck mass, Planck length, Planck time, Planck density, and Planck temperature). In recent quantum gravity models, the Planck units imply quantization of spacetime at extremely short range. Thus, the ratios (14) represent connection between cosmological parameters and quantum properties of spacetime. Obviously, the ratios (14) represent an approximate formulation of Dirac LNH because according recent CMB observations the total density of the universe $\overline{\rho}$ (including dark matter and dark energy) is close to the critical one [42-44]:

$$\overline{\rho} = \rho_c = \frac{3H^2}{8\pi G} \approx 9.47\times10^{-27}\ \text{kg}\cdot\text{m}^{-3}. \tag{15}$$

Replacing experimental density of the universe $\overline{\rho}$ instead $\tilde{\rho}$ and mass of the Hubble sphere $M_H = c^3/(2GH)$ instead M in ratios (14) latter become approximate. In Section 3, we show that the reasons of these small discrepancies of ratios (14) are approximate values of Planck units obtained by dimensional analysis.

3. PRECISE DETERMINATION OF THE LARGE NUMBER CONNECTING COSMOLOGICAL PARAMETERS AND PLANCK UNITS

It is known that the dimensional analysis allows findings unknown quantities with accuracy of the dimensionless parameter k, unit order of magnitude [14]. Therefore, the "standard" values of Planck units derived by dimensional analysis are approximate. Below, we recalculate the ratios (14) using experimental value of total density of the universe $\overline{\rho}$, mass of the Hubble sphere M_H and recalculated (precise) values of the Plank units with a definition of Planck mass as a mass whose Compton wavelength and gravitational radius are equal. We mark these values of Planck mass and other Planck units by asterisk to differentiate them from standard Planck units approximately derived by dimensional analysis. Therefore, the precise value of Planck mass m_P^* is the mass, whose reduced Compton wavelength $\lambda\!\!\!^{-}$ and gravitational (Schwarzschild) radius r_S are equal:

$$\lambda\!\!\!^{-} = \frac{\hbar}{mc} = r_S = \frac{2Gm}{c^2}. \tag{16}$$

We find the recalculated value of Planck mass from (16):

$$m_P^* = \sqrt{\frac{\hbar c}{2G}} = m_P / \sqrt{2} \approx 1.54\times10^{-8}\ \text{kg}. \tag{17}$$

The precise value of Planck length l_P^* follows from (16) and (17):

$$l_P^* = r_S = \frac{2G}{c^2} m_P^* = \sqrt{\frac{2G\hbar}{c^3}} = \sqrt{2} l_P \approx 2.28\times10^{-35}\ \text{m}. \tag{18}$$

Clearly, the recalculated value of Planck time is:

$$t_P^* = l_P^* / c = \sqrt{\frac{2G\hbar}{c^5}} = \sqrt{2} t_P \approx 7.59\times10^{-44}\ \text{s}. \tag{19}$$

The precise value of Planck density ρ_P^* is determined as the density of a sphere possessing mass m_P^* and radius l_P^*:

$$\rho_P^* = \frac{3m_P^*}{4\pi l_P^{*3}} = \frac{3}{16\pi}\frac{c^5}{\hbar G^2} = \frac{3}{16\pi}\rho_P \approx 3.1\times10^{95}\ \text{kg}\cdot\text{m}^{-3}. \tag{20}$$

Finally, the recalculated value of Planck temperature T_P^* is:

$$T_P^* = \frac{m_P^* c^2}{k_B} = \sqrt{\frac{\hbar c^5}{2Gk_B^2}} = t_P / \sqrt{2} \approx 10^{32}\ \text{K}. \tag{21}$$

Taking into account equations (6), (7), (15), (17–21) and $M_H = c^3/(2GH)$, as well as Hubble distance cH^{-1} and Hubble time H^{-1}, we find the exact ratios (22):

$$\frac{M_H}{m_P^*} = \frac{cH^{-1}}{l_P^*} = \frac{H^{-1}}{t_P^*} = \sqrt{\frac{\rho_P^*}{\bar{\rho}}} = \frac{T_P^*}{T_H} = \frac{m_P^*}{m_H} = \sqrt{\frac{M_H}{m_H}} = \sqrt{\frac{c^5}{2G\hbar H^2}} = N^* = N/\sqrt{2} \approx 5.73\times10^{60}. \tag{22}$$

Therefore, the large number N^* exactly connects cosmological parameters (mass of the Hubble sphere, Hubble distance, age of the universe, density of the universe, and minimal measurable temperature of the universe) and the respective fundamental microscopic properties of the matter (Planck mass, Planck length, Planck time, Planck density, and Planck temperature). Since the most of parameters entering equation (22) are characterized by the maximum or minimum possible values in the universe, it can be roughly claimed that the Dirac LNH shows that the largest and the smallest in the universe are related with a number of the order of 5×10^{60}.

Clearly, the ratios (22) where Planck units are obtained by definition of Planck mass as a mass whose Compton wavelength and gravitational radius are equal perfectly fits with experimental value of total density of the universe $\bar{\rho} \approx \rho_c$ and mass of the Hubble sphere M_H. That reinforces the trust in the recalculated (precise) Planck units by means of this approach. Since, the total density of the universe $\bar{\rho} \approx \rho_c = 3H^2/(8\pi G)$ $\approx 9.47\times10^{-27}\ \text{kg}\,\text{m}^{-3}$ is experimentally determinate by experiment WMAP with relative error < 0.4 % [45], this experiment should be considered as crucial evidence of the found formulation of Dirac LNH represented by equation (22). The recent values of all physical constants were taken from [46] excluding Hubble constant ($H \approx 70\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$) taken from [45].

Therefore, the recalculated equations (17-21) for the Planck mass, length, time, density and temperature are exact whereas Planck units obtained by dimensional analysis (4, 5, 11–13) are approximate. Besides, the large number $N^* = \sqrt{c^5/(2G\hbar H^2)} \approx 5.73\times10^{60}$ is not simply ratio of two quantities but it is an exact formula expressed by means of the fundamental constants c, G, $\hbar$ and H. Therefore, the ratios (22) represent a precise formulation of Dirac LNH.

The following thought experiment shows that the Planck length sets the fundamental limits on the accuracy of length measurement: Suppose we want to determine the position of an object using electromagnetic radiation (photons). The greater is the energy of photons, the shorter is their wavelength and the more accurate the measurement. When the wavelength reaches $\lambda/(2\pi) = l_P^* = \sqrt{2G\hbar/c^3}$ the photon has enough energy $E = h\nu = \hbar c/(\lambda/2\pi) = \sqrt{\hbar c^5/(2G)} = m_P^* c^2$ to measure objects the size of the Planck length l_P^*. But the photon would collapse into a black hole having mass $m = E/c^2 \equiv m_P^*$ and Schwarzschild radius $r_S = 2G\,m_P^*/c^2 = \sqrt{2G\hbar/c^3} \equiv l_P^*$, and the measurement would be impossible.

It is very interesting that the Planck mass represents the geometric mean of Hubble scale mass and mass of the Hubble sphere:

$$\sqrt{m_H M_H} = \sqrt{\frac{\hbar H}{c^2}\frac{c^3}{2GH}} = \sqrt{\frac{\hbar c}{2G}} \equiv m_P^*. \tag{23}$$

Taking in consideration equations (6), (9), (15) and (20) we find ratios (24):

$$\frac{M_H}{\rho_P^*} = \frac{m_H}{\bar{\rho}} = \frac{8\pi}{3}\frac{G\hbar}{Hc^2} = V_0 \approx 2.83\times10^{-43}\ \mathrm{m^3}. \tag{24}$$

Obviously, the radius of the sphere having volume V_0 is $r_0 = [2G\hbar/(Hc^2)]^{1/3} \approx 4.1\times10^{-15}$ m, i.e. of the order of size of the atomic nucleus. Therefore, the equation (24) shows that when the matter containing in the current Hubble sphere was concentrated in a small area of size of the atomic nucleus, the density was close to the Planck density ρ_P^*. Besides, the volume V_0 of the recent universe (having average density $\bar{\rho} \approx \rho_c \sim$ $\sim 10^{-26}\ \mathrm{kg\cdot m^{-3}}$) holds matter and energy equivalent to the Hubble scale mass $m_H \sim 10^{-33}$ eV.

It is important to note that the constants c, G and $\hbar$ as well as the Planck units built by means of these constants are time independent in the suggested form of LNH described from equation (22). This form of LNH avoids the problems of classical form of Dirac LNH given from equation (3) and requiring decrease of G with cosmological time, and controversial modifications of Einstein's equations of General Relativity [47-50].

As the large number N^* is inverse proportional to H, the former increases during cosmological expansion. Apparently, the total density of the universe $\bar{\rho} \approx \rho_c = 3H^2/(8\pi G)$ and the Hubble scale mass $m_H = \hbar H/c^2$ decrease with the age of the universe H^{-1}, whereas the mass of the Hubble sphere $M_H = c^3/(2GH)$ increases. Nevertheless, the equations (22) and (23) continue to be in force during the extension. Furthermore, the time variations of these quantities are negligible:

$$\frac{\dot{M}}{M} = -\frac{\dot{m}_H}{m_H} = -\frac{1}{2}\frac{\dot{\bar{\rho}}}{\bar{\rho}} = \frac{\dot{N}^*}{N^*} \sim H \approx 7.26\times10^{-11}\ \mathrm{yr^{-1}}. \tag{25}$$

Clearly, the large number N^* and Dirac large number N_D are connected by the approximate formula (26):

$$N_D \sim N^{*2/3} = [c^5/(2G\hbar H^2)]^{1/3} \approx 3.2\times10^{40}. \tag{26}$$

4. DISCUSSIONS AND CONCLUSIONS

The deep nature of Planck state of matter (PSM) featuring of enormous temperature and density and extremely short space-time intervals of the order of Planckian is not sufficiently clear yet. The contemporary Lambda cold dark matter (ΛCDM) model states that in moment close to the Planck time ~ 10^{-43} s after the Big Bang, the universe density and temperature had been close to the Planckian, respectively $\rho_c \sim 10^{96}$ kg m^{-3} and $T_P \sim 10^{32}$ K [51,52]. Apparently, PSM coincides with the strongly symmetric matter existing till $t_P \sim 10^{-43}$ s after the Big Bang when the gravity freezes out and the symmetry of forces breaks up.

The ratio of the Hubble sphere mass M_H and the Planck mass m_P^* was found equal to the large number N^* definite from the equation $N^* = \sqrt{c^5/(2G\hbar H^2)} \approx 5.73\times10^{60}$. Besides, the large number N^* defines the ratio of the Hubble distance cH^{-1} and the Planck length l_P^*, ratio of Hubble time (age of the universe) H^{-1} and the Planck time t_P^*, the square root of the ratio of the Planck density ρ_P^* and actual total density of the universe $\overline{\rho}$, and the ratio of Planck temperature T_P^* and minimal measurable temperature T_H. Therefore, the large number N^* connects cosmological parameters (mass of the Hubble sphere, Hubble distance, age of the universe, density of the universe, and minimal measurable temperature of the universe) and the respective fundamental microscopic properties of the matter (Planck mass, Planck length, Planck time, Planck density, and Planck temperature). Thus, a precise formulation and evidence of Dirac LNH has been found connecting the microworld and the macroworld. It is worth noting that the derived ratios (22) are not simply numbers of the same order of magnitude but a single large number N^*, represented by an exact equation by means of fundamental constants – c, G, $\hbar$ and H. Besides, it has been found that the Planck mass represents the geometric mean of Hubble scale mass and mass of the Hubble sphere $m_P^* = \sqrt{m_H M_H}$.

Since the most of parameters entering equation (22) are characterized by the maximum or minimum possible values in the universe, it can be roughly claimed that the Dirac LNH shows that the largest and the smallest in the universe are related with a number of the order of 5×10^{60}. Although, the reason of found ratios connecting cosmological parameters and respective Planck units yet is unclear, the very fact of existence of these relations is extremely interesting and significant and deserves attention and further investigations.

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