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QUANTUM FIDELITY OF TWO-MODE GAUSSIAN STATES IN A THERMAL RESERVOIR

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Abstract. The dynamics of quantum fidelity of two non-coupled bosonic modes interacting with a common thermal reservoir is studied in terms of the covariance matrix, in the framework of the theory of open systems based on completely positive quantum dynamical semigroups. As initial states there are considered an entangled squeezed vacuum state and a squeezed thermal state. We study the dependence of quantum fidelity on time, temperature, squeezing parameter and average numbers of thermal photons. We deduce the asymptotic expression of quantum fidelity and prove that it takes a non-zero value in the limit of large times.

Key words: Uhlmann fidelity, Gaussian states, open quantum systems, squeezed thermal states.

1. INTRODUCTION

In quantum information theory a special importance presents distinguishability between a pair of quantum states. There are a few distance measures generally accepted to study the closeness of the quantum states, for instance the trace distance and quantum fidelity [1], which is used to define the Bures distance [2]. The Bures metric can be used to measure entanglement in the system [2,3], nonclassicality [4], and polarization [5]. For pure states the Uhlmann fidelity represents the probability of transition between two studied states. Jozsa [6] proposed a definition of quantum fidelity for mixed quantum states using Uhlmann's transition probability [7], which was later expressed for Gaussian states in terms of covariance matrices [8].

The detailed study of fidelity for quantum systems placed in a noisy environment paves the way for experimental implementation of highly efficient Gaussian quantum channels for quantum cryptography [9] and quantum teleportation [10]. As well, it can be useful for choosing the optimal strategy to clone and copy selected states [11,12]. Recent experiments allow experimental measurements of fidelity by the use of quantum logic gates implemented in different media from ion traps [13,14] to solid state solitons [15]. Implementation of ion traps requests special measures for noise and error control, like coating by exotic materials. As an option in Ref. [16] it was proposed graphene, due to its excellent electron mobility [17].

A special interest presents calculation of fidelity for mixed states, pure states being less likely to be obtained in experiments due to decoherence phenomenon produced during the interaction with the environment. For example, in the evaluation of a quantum protocol, one might be interested to analyse the similarity between the desired output state and the actually obtained output state. The Gaussian states play an important role in quantum information [1], since they naturally occur in quantum optics, nanomechanical oscillators and ion traps [18]. For Gaussian states of continuous variable systems, the Uhlmann fidelity has already been calculated in the case of a one-dimensional harmonic oscillator coupled to an environment [19,20] and for the system of two bosonic modes interacting with two thermal reservoirs [21].

In the present paper we consider a system of two bosonic modes interacting with a common thermal reservoir. Our goal is to study, in the framework of the theory of open systems based on completely positive quantum dynamical semigroups, the evolution of quantum fidelity as a function of time, environment temperature, squeezing parameter, average number of thermal photons and frequencies of the two modes. In Section 2 it is introduced the general expression for Uhlmann fidelity and its exact analytic formula for the two-mode case in terms of covariance matrices. In Section 3 it is written the Kossakowski-Lindblad quantum Markovian master equation for an open system in contact with a thermal reservoir and the temporal evolution of the corresponding covariance matrix. Taking as initial states a squeezed thermal state and an entangled squeezed vacuum state, we describe the behaviour of the quantum fidelity and discuss the obtained results. Conclusions are presented in Section 5.

2. QUANTUM FIDELITY

The quantum Uhlmann fidelity of two states characterized by the density operators ρ_1 and ρ_2 is defined as the maximal transition probability between these two states and is given by:

$$\mathcal{F}(\rho_1,\rho_2)=\left[\mathrm{Tr}\left(\sqrt{\sqrt{\rho_2}\,\rho_1\sqrt{\rho_2}}\right)\right]^2 . \tag{1}$$

According to Ref. [6], quantum fidelity has to satisfy the following axioms:

1. $0 \le \mathcal{F}(\rho_1,\rho_2) \le 1$ and $\mathcal{F}(\rho_1,\rho_2)=1 \leftrightarrow \rho_1=\rho_2$;
2. $\mathcal{F}(\rho_1,\rho_2)=\mathcal{F}(\rho_2,\rho_1)$ (symmetry);
3. $\mathcal{F}(\rho_1,\rho_2)=\mathrm{Tr}(\rho_1\rho_2)$ if either ρ_1 or ρ_2 is a pure state;
4. $\mathcal{F}(U\rho_1U^\dagger,U\rho_2U^\dagger)=\mathcal{F}(\rho_1,\rho_2)$, for all unitary transformations U.

We note that the fidelity is 0 if and only if the two states are orthogonal: $\mathcal{F}(\rho_1,\rho_2)=0 \leftrightarrow \rho_1\rho_2=0$.

Fidelity measure is very useful as an indicator of security in teleportation protocols [22], and can be used for estimation of the success of a logic operation in quantum gates [23]. Since the general formula (1) for fidelity includes square roots of density matrices, it is not easy to estimate the fidelity value for mixed multi-mode states. The authors of Ref. [8] have been able to obtain an analytical expression of the fidelity for two-mode Gaussian states in terms of their covariance matrices:

$$\mathcal{F}(\rho_1,\rho_2)=\frac{\exp\left[-\frac{1}{2}\delta^{\mathrm{T}}\left(\sigma(0)+\sigma(t)\right)^{-1}\delta\right]}{\left(\sqrt{\Gamma}+\sqrt{\Lambda}\right)-\sqrt{\left(\sqrt{\Gamma}+\sqrt{\Lambda}\right)^2-\Delta}}, \tag{2}$$

where $\sigma(0)$ is the covariance matrix at time $t=0$, $\sigma(t)$ is the covariance matrix at estimation time, δ is the average two-mode displacement, and

$$\begin{aligned}
&\Delta := \det(\sigma(0)+\sigma(t)) \geqq 1,\\
&\Gamma := 16\det[(J\sigma(0))(J\sigma(t))-\frac{1}{4}I] \geqq \Delta,\\
&\Lambda := 16\det(\sigma(0)+\frac{i}{2}J)\,\det(\sigma(t)+\frac{i}{2}J) \geqq 0,
\end{aligned} \tag{3}$$

and $J=\bigoplus_{k=1}^{2}J_k$ with $J_k=\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}$. In this work we consider undisplaced modes, therefore the exponential expression in Eq. (2) becomes 1.

3. EQUATIONS OF MOTION FOR TWO-MODE BOSONIC MODES

We study the dynamics of a system composed of two uncoupled bosonic modes placed in contact with a thermal reservoir, in the framework of the theory of open systems based on completely positive quantum dynamical semigroups. The temporal evolution of the open system is described by the Kossakowski-Lindblad quantum Markovian master equation, which in the Schrödinger representation for a density operator $\rho(t)$ has the form:

$$\frac{\mathrm{d}\rho(t)}{\mathrm{d}t}=-\frac{i}{\hbar}\left[H,\rho(t)\right]+\frac{1}{2\hbar}\sum_j\left(\left[V_j\rho(t),V_j^\dagger\right]+\left[V_j,\rho(t)V_j^\dagger\right]\right). \tag{4}$$

H denotes the Hamiltonian of the two uncoupled bosonic modes (harmonic oscillators) of identical mass m and frequencies ω_1 and ω_2:

$$H=\frac{1}{2m}\left(p_x^2+p_y^2\right)+\frac{m}{2}\left(\omega_1^2x^2+\omega_2^2y^2\right) \tag{5}$$

and Lindblad operators $V_{j=1,2,3,4}$ are taken as polynomials of the first degree in the canonical variables of coordinates and momenta x, y, p_x and p_y of the system.

The equation of motion for the covariance matrix is the following [24–30]:

$$\frac{\mathrm{d}\sigma(t)}{\mathrm{d}t}=Y\sigma(t)+\sigma(t)Y^{\mathrm{T}}+2D, \tag{6}$$

where

$$Y=\begin{pmatrix}-\lambda & 1/m & 0 & 0\\ -m\omega_1^2 & -\lambda & 0 & 0\\ 0 & 0 & -\lambda & 1/m\\ & 0 & -m\omega_2^2 & -\lambda\end{pmatrix},\quad D=\begin{pmatrix}D_{xx} & D_{xp_x} & D_{xy} & D_{xp_y}\\ D_{xp_x} & D_{p_xp_x} & D_{yp_x} & D_{p_xp_y}\\ D_{xy} & D_{yp_x} & D_{yy} & D_{yp_y}\\ D_{xp_y} & D_{p_xp_y} & D_{yp_y} & D_{p_yp_y}\end{pmatrix}, \tag{7}$$

and D is the matrix of diffusion coefficients. The solution of Eq. (6) is given by [24,25]:

$$\sigma(t)=M(t)[\sigma(0)-\sigma(\infty)]M^{\mathrm{T}}(t)+\sigma(\infty), \tag{8}$$

with $M=\exp(Yt)$. The covariance matrix at large times $\sigma(\infty)$ corresponding to an asymptotic Gibbs state has the following form [25], where T denotes the temperature of the thermal reservoir (we set $\hbar=1$, $k=1$, $m=1$):

$$\sigma(\infty)=\mathrm{diag}\left(\frac{1}{2\omega_1}\coth\frac{\omega_1}{2T},\ \frac{\omega_1}{2}\coth\frac{\omega_1}{2T},\ \frac{1}{2\omega_2}\coth\frac{\omega_2}{2T},\ \frac{\omega_2}{2}\coth\frac{\omega_2}{2T}\right). \tag{9}$$

As initial states we shall choose an entangled squeezed vacuum state (SVS) and a non-symmetric squeezed thermal state (STS), whose covariance matrices have the following form:

$$\sigma_{SVS}(0)=\frac{1}{2}\begin{pmatrix}\cosh 2r & 0 & \sinh 2r & 0\\ 0 & \cosh 2r & 0 & -\sinh 2r\\ \sinh 2r & 0 & \cosh 2r0 & 0\\ 0 & -\sinh 2r & 0 & \cosh 2r\end{pmatrix},\quad \sigma_{STS}(0)=\begin{pmatrix}a & 0 & c & 0\\ 0 & a & 0 & -c\\ c & 0 & b & 0\\ 0 & -c & 0 & b\end{pmatrix}, \tag{10}$$

where

$$a = n_1 \cosh^2 r + n_2 \sinh^2 r + \frac{1}{2}\cosh 2r,$$
$$b = n_1 \sinh^2 r + n_2 \cosh^2 r + \frac{1}{2}\cosh 2r, \tag{11}$$
$$c = \frac{1}{2}(n_1 + n_2 + 1)\sinh 2r.$$

Here r denotes the squeezing parameter of the initial states, and n_1, n_2 are average numbers of thermal photons. A two-mode STS is entangled when [25]

$$r > r_s, \quad \cosh^2 r_s = \frac{(n_1+1)(n_2+1)}{n_1+n_2+1}. \tag{12}$$

4. TIME EVOLUTION OF QUANTUM FIDELITY

In the following we describe the dependence of the Uhlmann quantum fidelity on time, temperature of the thermal reservoir, squeezing parameter, and frequencies of the two bosonic modes in two cases: a) the initial state is an entangled squeezed vacuum state; b) the initial state is a squeezed thermal state. For the initial squeezed thermal state was also investigated the influence of the average thermal photon numbers of the modes. Moreover, in both cases we analyze the behaviour of quantum fidelity in the limit of large times.

4.1. Initial squeezed vacuum state

For an initial entangled squeezed vacuum state with the covariance matrix having the form (10), $\Lambda = 0$ and $\Gamma = \Delta$ and therefore, the expression of fidelity (2) reduces to:

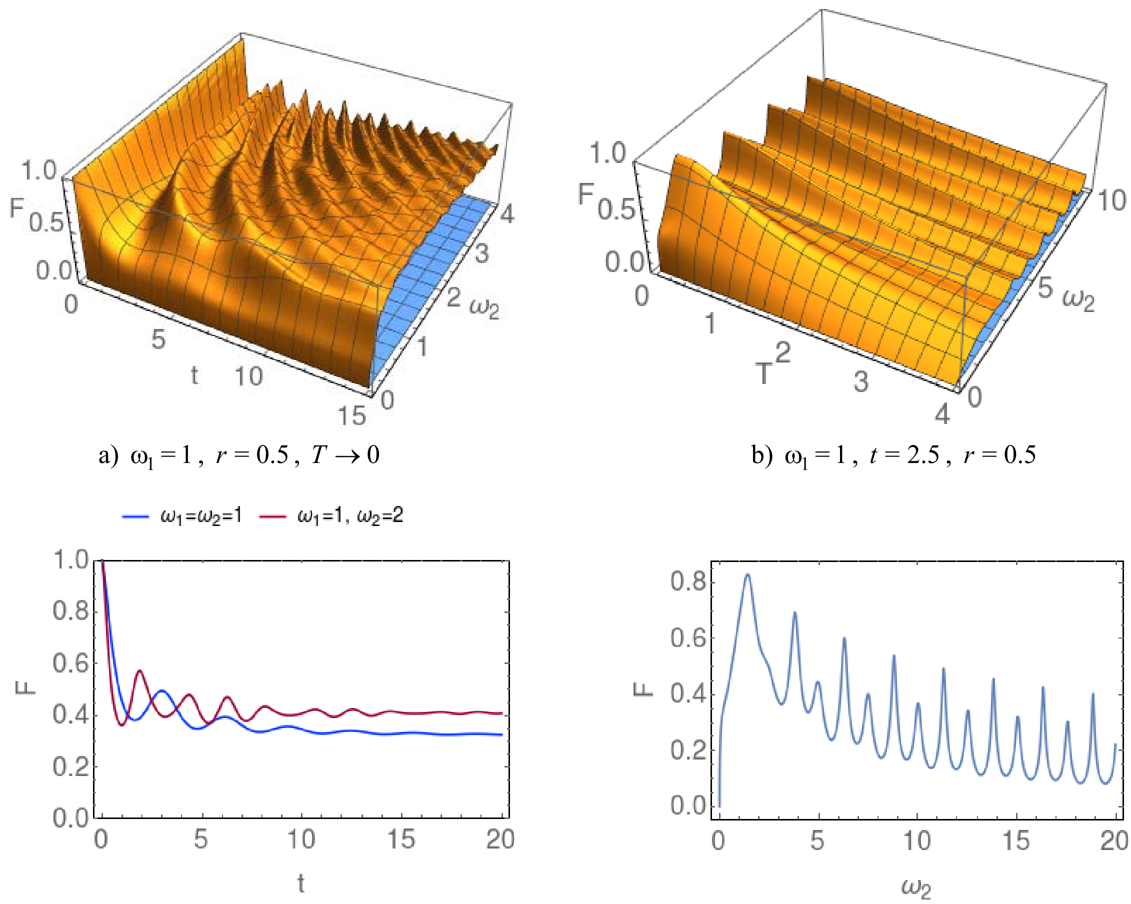
$$\mathcal{F} = \frac{1}{\sqrt{\Gamma}}. \tag{13}$$

The fidelity dependence on the frequency of one of the bosonic modes and time is shown in Fig. 1a. The temporal dependences of the fidelity for the resonant case ($\omega_1 = \omega_2$) and the non-resonant case ($\omega_1 \neq \omega_2$) are compared in Fig. 1. From Figs. 1a,c we can observe that the fidelity is oscillating in time and tends asymptotically to a definite value that depends on the chosen values of frequencies. In Fig. 1b it is represented the dependence of quantum fidelity on the squeezing parameter and the frequency of one of the modes, at some fixed moment of time, and for a given temperature and frequency of the other mode. By increasing temperature, the quantum fidelity decreases monotonically. The behaviour of the fidelity as a function of the frequency of the second mode for zero temperature is illustrated in Fig. 1d. Furthermore, we estimate the fidelity when the time tends to infinity. In this case it is given by

$$F_\infty = \frac{4\sqrt{\omega_1\omega_2}}{\sqrt{\omega_1\omega_2 + \cosh 2r\left(\omega_2 \coth\frac{\omega_1}{2T} + \omega_1 \coth\frac{\omega_2}{2T}\right) + \coth\frac{\omega_1}{2T}\coth\frac{\omega_2}{2T}}} \times$$
$$\times \frac{1}{\sqrt{1 + \cosh 2r\left(\omega_1 \coth\frac{\omega_1}{2T} + \omega_2 \coth\frac{\omega_2}{2T}\right) + \omega_1\omega_2 \coth\frac{\omega_1}{2T}\coth\frac{\omega_2}{2T}}}. \tag{14}$$

In the particular resonant case ($\omega_1 = \omega_2 = 1$) the asymptotic fidelity only depends on the squeezing parameter and the temperature of the reservoir:

$$F_\infty = \frac{4}{1 + 2(\cosh 2r)\left(\coth\frac{1}{2T}\right) + \coth^2\frac{1}{2T}}. \tag{15}$$



a) $\omega_1 = 1$, $r = 0.5$, $T \to 0$ b) $\omega_1 = 1$, $t = 2.5$, $r = 0.5$

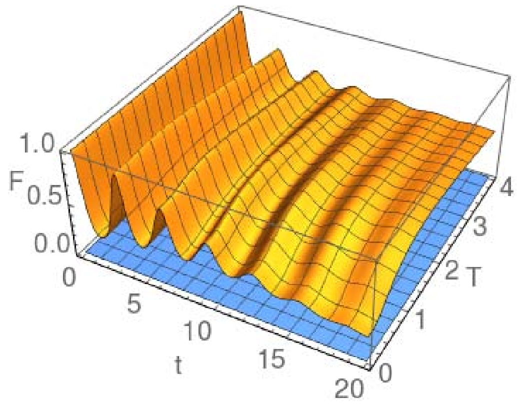
c) $r = 0.5$, $T = 1$ d) $\omega_1 = 1$, $t = 2.5$, $r = 0.5$, $T \to 0$

Fig. 1 – Variation of fidelity with: a) frequency of the second mode and time; b) frequency of the second mode and the temperature of the reservoir; c) time; d) frequency of the second mode. The dissipation parameter is chosen $\lambda = 0.1$.

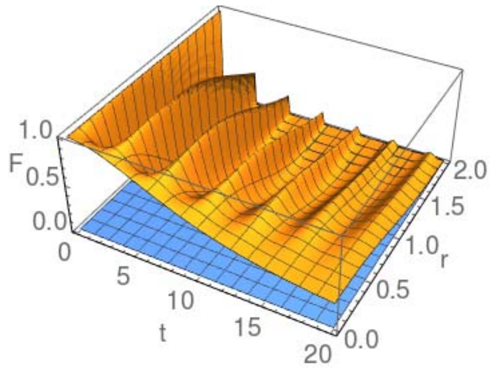
4.2. Initial squeezed thermal state

For the initial squeezed thermal state, the fidelity dependence on time and temperature is illustrated in Figs. 2a,c. From Figs. 2b,d we can observe the dependence of quantum fidelity on time and the squeezing parameter of the initial state. The increase of the squeezing parameter reduces the fidelity, whose values decrease rapidly to zero, so that the two Gaussian states tend to become orthogonal. Figs. 2a,b depict a resonant case, while Figs. 2c,d present a non-resonant case. In all the considered situations, the quantum fidelity manifests an oscillatory, non-monotonically decreasing behaviour in time.

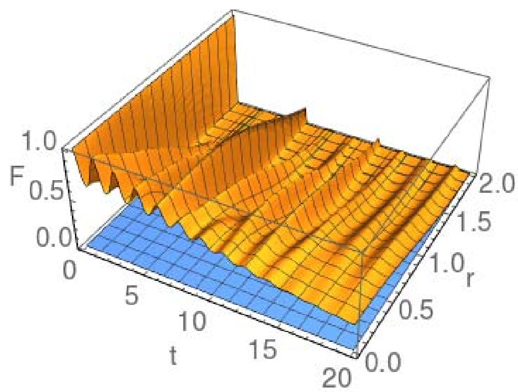
For the initial squeezed thermal state, the dependence of the quantum fidelity on time and frequency of the second mode, represented in Fig. 3a, is similar to that for an initial squeezed vacuum state, represented in Fig. 1a. However, the secondary maxima of the fidelity are more evident in the case of the initial squeezed thermal state. The behaviour of quantum fidelity as a function of time and average thermal photon number of the second mode is presented in Fig. 3b. The general tendency of the fidelity is to monotonically decrease by increasing the thermal photon number.



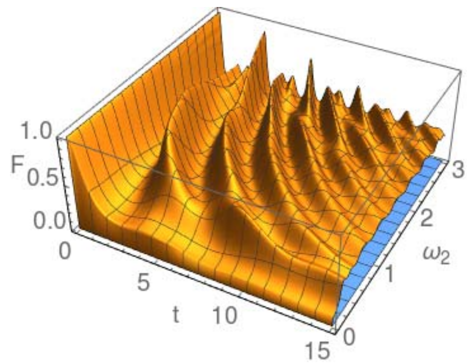
a) $\omega_1 = \omega_2 = 1$, $r = 0.5$



b) $\omega_1 = \omega_2 = 1$, $T \to 0$



c) $\omega_1 = 1$, $\omega_2 = 2$, $r = 0.5$



d) $\omega_1 = 1$, $\omega_2 = 2$, $T \to 0$

Fig. 2 – Evolution of quantum fidelity with: a) temperature and time for a resonant case $\omega_1 = \omega_2$; b) the squeezing parameter and time for a resonant case $\omega_1 = \omega_2$; c) temperature and time for a non-resonant case $\omega_1 \neq \omega_2$; d) the squeezing parameter and time for a non-resonant case $\omega_1 \neq \omega_2$. The dissipation parameter is chosen $\lambda = 0.1$, and the average numbers of thermal photons are $n_1 = 1$, $n_2 = 2$.

We have also analyzed the expressions of quantum fidelity when time approaches infinity. If the average numbers of thermal photons are equal ($n_1 = n_2 = n$), then for the resonant case ($\omega_1 = \omega_2 = 1$) the expression of fidelity becomes:

$$F_\infty = \frac{4}{2 + 4n(1+n)(\cosh 2r)\left(\coth\frac{1}{2T}\right) + \operatorname{csch}^2\frac{1}{2T}}. \tag{16}$$

The graphical representation of the fidelity (16) is presented in Fig. 4. The asymptotic fidelity monotonically decreases with the increase of the squeezing parameter. However, with the increase of temperature the fidelity first increases and after that it monotonically decreases.

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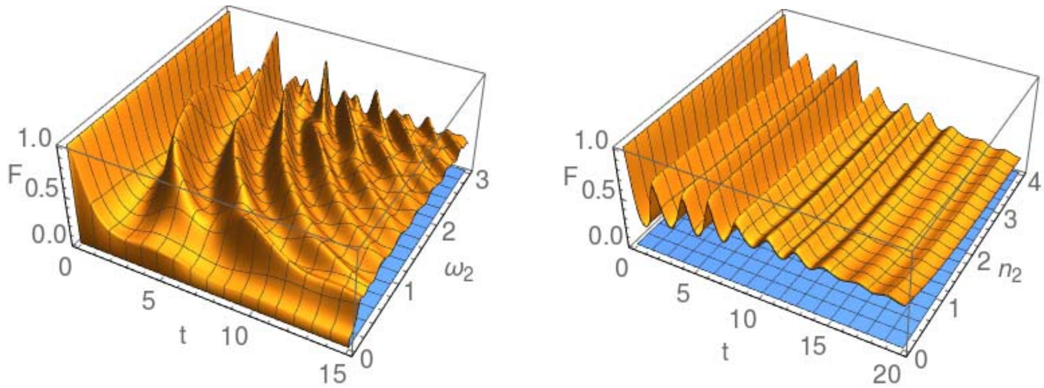

a) $r = 0.5$, $n_1 = n_2 = 1$, $T \to 0$ b) $\omega_2 = 2$, $r = 0.5$, $n_1 = 1$, $T \to 0$

Fig. 3 – Evolution of quantum fidelity with: a) time and frequency of the second mode; b) time and average number of thermal photons of the second mode. The dissipation parameter is chosen $\lambda = 0.1$, and the frequency of the first mode $\omega_1 = 1$.

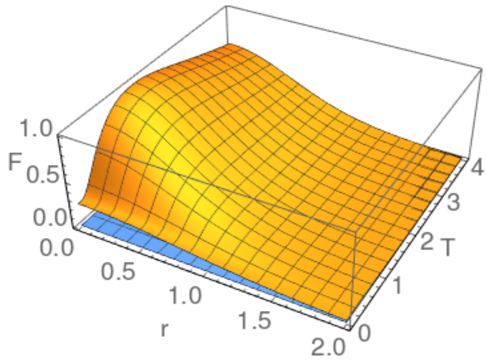

Fig. 4 – Variation of asymptotic fidelity $(t \to \infty)$ with the squeezing parameter and the temperature for $\lambda = 0.1$, $\omega_1 = \omega_2 = 1$, $n_1 = n_2 = 1$.

5. CONCLUSIONS

We investigated the Markovian dynamics of the Uhlmann fidelity for a system consisting of two non-interacting bosonic modes, embedded in a thermal bath. The study was implemented in the framework of the theory of open systems based on completely positive quantum dynamical semigroups. We have studied the influence of the thermal bath, frequencies of the two modes, and squeezing on the temporal dependence of the quantum fidelity in terms of covariance matrices in the case when as the initial state was chosen the entangled squeezed vacuum state. For initial entangled squeezed thermal state, together with the other properties, we have examined the influence of the average number of thermal photons on the Uhlmann fidelity evolution. We have shown that the fidelity value strongly decreases with the increase of temperature of the thermal reservoir, and with the increase of squeezing parameter. However, in the case of a squeezed thermal state the fidelity increases with temperature for very low temperatures. As well, the frequency of bosonic modes has a strong influence on fidelity value. Increase of the average number of thermal photons decreases the fidelity. We also studied the fidelity behaviour for large times in both cases and have shown that the asymptotic fidelity has nonzero values for all values of the temperature and squeezing parameter. The time evolution of the quantum fidelity of Gaussian mixed states can present a great interest in connection with the efficiency of the quantum information protocols, like quantum cryptography and quantum teleportation.

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