3 STAGE HELICAL SPEED REDUCER PARTIAL GEAR RATIOS OPTIMAL DETERMINATION USING GENETIC ALGORITHMS

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Abstract. Designing compact multi-stage speed reducers are challenging demands of nowadays mechanical power transmission manufacturer. In this paper a Genetic Algorithm (GA) was used for obtaining optimal models for partial gear ratios in order to achieve a 3 stage helical speed reducer with minimum mass or length. The objectives were described by a set of 18 design variables of mixed nature and were subjected to a highly non-linear set of 57 engineering constraints. The proposed methodology automates the design process and the results obtained by using GA conduct to expressions for partial gear ratios, which offer an even distribution on all 3 stages for the total transmission ratio with better design solutions for both the objectives as compared with the ones given in literature.

Key words: optimal determination of partial gear ratios, Genetic Algorithms, 3 stage helical speed reducer, automated optimal design.

1. INTRODUCTION

Gears are nowadays widely used in various mechanical engineering systems application, from different fields and working conditions. Their complex shape and geometry require a large number of design variables (typically, well over ten [19] – of different types: integers, discrete and real), resulting a complicated and difficult design process. Furthermore, gearing design is invariably based on iterations and making decisions– which unfortunately are always compromises. Considering all these remarks, it is obviously that the manual design (i.e. a trial and error type method) of gearing is very difficult and there is a need for computer-aided design. Moreover, continuously increasing and challenging demands of compact and reliable gears, force the mechanical designers to consider more and more the optimal design methodology [16]. In the last decades many researchers have paid attention on this problem of gear optimization. Ramamurti et al. in [14] presented a design methodology for two-speed gearbox. Huang et al. developed an interactive physical programming in order to optimize a three-stage spur gear reduction unit [11]. Abersek et al. in [1] developed an expert system to design and manufacture a gearbox. Yokota et al. in [20] solved an optimal weight design problem of a gear with an improved GA. Deb and Sachin, in [6] used a non-dominated sorting genetic algorithm (NSGA-II) in order to solve a multi-objective optimization of a multi-speed gearbox. Thompson et al. [18] presented a generalized optimal design of two stage and three stage spur gear reduction units in a formulation with multiple objectives. Gologlu and Zeyveli in [8] applied GA to minimize the volume of a two-stage helical gear train.

All the above studies were mentioned to highlight the importance of using modern global optimization techniques in mechanical power transmission design. In here the author extends the technology to the broader design space of a 3 stage helical speed reducer gearings (Fig.2) whose every defining element is subject to change throughout the optimal design process. This discussion represents only the first part from a broader study (in order to build up a generic transmission system design tool based on the evolutionary optimization concepts [5]) at the end of which the author aims to optimize the complete 3 stage speed reducer. The large complexity of the design problem and the author’s experience in this field [4,5,19] conducts to a step-by-step procedure which allow finding out which are the dimensional tendencies of the speed reducer’s components and how the functional and structural interdependencies affect them. As you
will see in Section 4, there is a need of 18 design variables for describing the optimal design problem! Assuming their definition domains (Table 1), there could be obtained a number of possible helical gearings designs of the order of $4 \times 10^{31}$! Once this preliminary phase is completed the virtual design space of the helical gearings will be generated and the problem formulation will be extended with the shafts and the housing subsystems. Therefore, a systematic approach [5] in solving this optimal design problem is essential to achieve the desired level of reliability.

Into the next Section, the current procedure for designing speed reducers is introduced, after which, is presented a short description of the general principle of the proposed Genetic Algorithm (Section 3), followed by a detailed discussion regarding the statement of the optimal design problem (Section 4). The fifth Section contains an effective example and a detailed presentation and comparison of the numerical results solutions. Eventually, discussion is concluded with some reflections and suggestions regarding the possible extensions of the present study.

2. THE CURRENT PROCEDURE FOR SPEED REDUCERS DESIGN

Commonly methods (trial and error) for designing mechanical gear transmissions involve some difficulties considering the multiple interactions between its subsystems. Furthermore, design is an iterative and decision-make process [17]. The whole speed reducer design process start from a set of input data with information regarding the input power $P_m$ (kW), the rotational frequency of the pinion rotation $n_m$ (rpm), the total transmission ratio $i_T$, the working time of the gearings $L_h$ (h), and the layout drawing (Fig. 2). The first phase of the design process ($A$) consists in making two important preliminary selections, regarding the partial gear ratios for all the stages ($A.1$) and about the materials (including here the hardness and the thermal treatment) for the gears ($A.2$). As concerns the total transmission ratio splitting, it is known that represents a key decision which has a major impact over the entire design process with remarkable influence over the mass and the cost of the gear transmission [21]. There are a few studies which tackle this difficult problem. Some of them use different graphics to determine the partial gear ratios. See for example the studies of Kudreavtev et al. [12] (Fig. 1a), and Niemann [13] (Fig. 1b).

![Fig. 1 – Determination of partial ratios of 3 stage gearboxes: a) from to Kudreavtev et al. [12]; b) according to Niemann [13].]

Another category uses modelling methods [21]. From here, there should be reminded the study of Romhild and Linke [15], which developed the following equations:

$$u_1 = 0.4643 \cdot i_T^{0.609} , \quad u_2 = 1.205 \cdot i_T^{0.262}.$$  \hspace{1cm} (1)

Also, from this class two other interesting studies are those made by Vu, firstly described in [21], where are shown the following expressions for the gear ratio:

$$u_2 \approx 1.3104 \cdot i_T^{0.2533} \cdot k_{c2}^{0.3714} / k_{c3}^{0.0977} , \quad u_1 \approx 2.3417 \cdot i_T^{0.088} \cdot k_{c2}^{0.3455} / k_{c3}^{0.2492} ,$$  \hspace{1cm} (2)

and, secondly, the study from [22]:

$$u_1 = 0.314 \cdot \sqrt{i_T^2} , \quad u_2 = 1.33 \cdot \sqrt{i_T}.$$  \hspace{1cm} (3)
Next, the number of teeth on pinion and helical gear on each stage are determined (A.3). Then the error of the actual gear ratio is checked (according to National Romanian Standard, i.e. STAS 6012 the error should be ±2.5% when \( u < 4 \); or ±3% when \( u > 4 \)). Once this preliminary phase is completed the process continues with the helical gears design phase – B. In a briefly description, at this level of design process there should be made several computations regarding the estimated allowable contact (\( \sigma_{HP1,2} \), \( \sigma_{HP} \)) and bending (\( \sigma_{FP1,2} \)) stresses, the preliminary center distance \( a_{w} \), the normal module \( m_n \), the elementary center distance \( a \), the elements of the helical gears and of the equivalent spur gears. Finally, the helical gearing is checked on bending and contact stresses. The last two sections concern with the shafts subassembly (section C) and housing design (section D). They will be included into a further study, when the complete optimization of the entire speed reducer will be considered. However, until then, let us return to our present discussion whereas already was pointed out, a simple GA is proposed for developing appropriate models for determine the values of the partial gear ratios for achieving speed reducers with minimum mass or length.

3. GENETIC ALGORITHM

Genetic Algorithms (GAs) are a subclass of Evolutionary Algorithms (EAs). They are a computer based search technique, which mimics the biological evolution as a problem-solving strategy. The basic concepts of GAs were developed by John Holland [10]. Algorithmically, the basic GAs is outlined as below:

(a) the genotype (i.e. the search space of coded solutions) of every chromosome (individual) in the population is randomly initialized. (b) The phenotype (i.e. collections of parameters such as: the number of teeth on pinions and wheels, the standardized center distances, etc.) of every chromosome from the initial population is evaluated using the fitness function (i.e. the mass and the total length of the helical gearings). Next, (c) two parent chromosomes are randomly selected (using the roulette wheel method) for reproduction according to their fitness (the higher the fitness, the more chances of selection). Offsprings are created (d,e) by applying the genetic operators: crossover (merges information from two parent chromosomes into one or two offsprings) and mutation (acts on a single offspring and works by applying some variation to one or more genes in the offspring’s chromosome). The new generated individuals are then evaluated (f) using the fitness measure. After the evaluation, the offspring replaces some/all of the chromosomes in the current population (g). This full process of evaluation and reproduction continues until either a satisfactory solution emerges or the GA has run for a specified number of generations.

4. STATEMENT OF THE OPTIMAL DESIGN PROBLEM

4.1. The ‘genotype’ of the 3 stage helical speed reducer

The 18 genes that uniquely describe the optimization problem (Fig. 2) are detailed in Table 1.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Range</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( z_1, z_2, z_5 )</td>
<td>{14, ..., 21}</td>
<td>Number of teeth of the pinions. Integer values.</td>
</tr>
<tr>
<td>( z_2, z_4, z_6 )</td>
<td>{45, ..., 172}</td>
<td>Number of teeth of the wheels. Integer values.</td>
</tr>
<tr>
<td>( a_{w11}, a_{w12}, a_{w13} ) (mm)</td>
<td>{56, ..., 315}</td>
<td>Center distance of each stage. Standardized, discrete, real values</td>
</tr>
<tr>
<td>( x_{n1}, x_{n3}, x_{n5} )</td>
<td>{−0.5, ..., 1}</td>
<td>Normal tooth addendum coefficients of the pinions. Discrete, real values.</td>
</tr>
<tr>
<td>( \beta_{11}, \beta_{12}, \beta_{13} ) (( ^\circ ))</td>
<td>{4, 19.75}</td>
<td>Helix angles measured at the pitch diameters. Discrete real values.</td>
</tr>
<tr>
<td>( \psi_{11}, \psi_{12}, \psi_{13} )</td>
<td>{0.2, ..., 0.5}</td>
<td>Gear width to center distance ratio coefficients. Real values.</td>
</tr>
</tbody>
</table>

4.2. The objective functions

Two separately objectives were considered for minimization, i.e. the mass (Eq.4) and the length (Eq.5) of the helical gearings in order to obtain optimal models for partial gear ratios. The expressions for the objectives functions are:
**4.3. The constraints**

In here, a set of 57 engineering design constraints (involving strength, geometrical and structural considerations) typically encountered in practical design of a power transmission were considered. All these constraints are of inequality type, defined as \( g_i = a_i/b_i - 1 \leq 0 \), where a constraint is of the form \( a_i \leq b_i \), with \( a_i, b_i > 0 \). It is obviously that the values of all these constraints have to be negative or at last zero (case in which the solution is feasible). For the sake of conciseness, we shall not dwell on the details regarding their calculation. It should be mentioned here that all the details of the gearings calculations may be founded in the relevant industrial standard document DIN 3990 [7]. These 57 constraints should be with reference to the sketch presented in Fig. 2. C1–4 The relative error of the total and partial gear ratios (on each stage) should be ±2.5% when \( u < 4 \); or ±3% when \( u > 4 \). C5–7 The numbers of teeth on all stages must be relative primes. C8–10 The Hertzian contact pressure on the teeth of gears on each stage must not exceed the allowable Hertzian contact pressure. C11–16 The bending stress on the teeth of helical gears on all three stages must not exceed a specified value. C17–22 The teeth of all helical gears must not be undercut. C23–28 The top land on the teeth on gears 1 through 6 must not vanish. C29–31 The contact ratio of each stage must be greater than a specified value (i.e. \( \varepsilon_{\min} = 1 \)). C32–34 The addendum coefficient of the gears (2), (4) and (6) should be in the range of \([-0.6, ..., 1]\). C35–52 A set of measurability constraints for all the helical gears. C53–55 The shearing stresses on the key and keyway for mounting the wheels (2), (4) and (6) must not exceed a specified value (i.e. the allowable shearing stress 70 MPa; corresponding to the material of the keys i.e. steel grades E355 [9]). Is worth noting here, that there are a few papers [1,6,8,16,18,21,22] where,
at this stage of problem formulation (when only the gears are considered—without shafts sub-assembly, housing, etc.) the authors do not consider the shaft—gear hub connections. The key joints have a major impact over the shape and the mass of the driven helical gears. Depending on the key length (computed from bearing strength condition) a gear could or not have a salient hub [9]. In the following rows a short explanation on how this problem was tackled in here. Firstly, we computed the diameters of the shafts on which are mounted the helical gears (2), (4) and (6) (Fig. 3). These dimensions are determined with: \( d = (T/0.2 \cdot \tau)^{1/3} \), where: \( T \) is the torque acting in the rated shaft cross section, in [N-mm]; \( \tau \) is the allowable stress on the torsion, in [MPa] [9]. Next depending on the values obtained with the above equation, from standard tables are chosen the key cross-section \((b \times h)\) dimensions. Once these values are known, the rated key length from the strength condition \((l_c = 4 \cdot T / (\sigma_{st} / d / h)) [9]\), where \( \sigma_{st} \) is the allowable bearing stress [MPa]; \( h \) is the height of the key cross-section) is determined. Now, that the key length is known a decision can be made as to whether the helical driven gears will have or not a salient hub. At this moment all the dimensions of the driven helical gears (2), (4) and (6) are complete defined. \textbf{C56} Gear (2) and (5) must not interfere. \textbf{C57} Gear (4) must not interfere with the output shaft.

5. A 3 STAGE HELICAL SPEED REDUCER OPTIMAL DESIGN EXAMPLE

This is probably a good time to consider the practical example of a 3 stage helical speed reducer optimal design (Fig. 2). In order to obtain optimal models for the partial gear ratios for which the speed reducer has a minimum mass or a minimum length were considered 7 input data sets (InDS).

<table>
<thead>
<tr>
<th>InDS – 1</th>
<th>InDS – 2</th>
<th>InDS – 3</th>
<th>InDS – 4</th>
<th>InDS – 5</th>
<th>InDS – 6</th>
<th>InDS – 7</th>
</tr>
</thead>
<tbody>
<tr>
<td>( i_T )</td>
<td>( n_m ) (rpm)</td>
<td>( P_m ) (kW)</td>
<td>( i_T )</td>
<td>( n_m ) (rpm)</td>
<td>( P_m ) (kW)</td>
<td>( i_T )</td>
</tr>
<tr>
<td>40</td>
<td>750</td>
<td>6.5</td>
<td>50</td>
<td>750</td>
<td>5.2</td>
<td>63</td>
</tr>
<tr>
<td>1000</td>
<td>8</td>
<td></td>
<td>1000</td>
<td>7</td>
<td></td>
<td>1000</td>
</tr>
<tr>
<td>1500</td>
<td>12.5</td>
<td></td>
<td>1500</td>
<td>10.5</td>
<td></td>
<td>1500</td>
</tr>
</tbody>
</table>

Each InDS [23] is defined by a total transmission ratio – \( i_T \), an input speed – \( n_m \) (rpm) and the corresponding input power – \( P_m \) (kW). The helical gears of the speed reducer should be based on an ISO 53 basic rack profile \((a_w = 20°, h_{int} = 1, c_{ad} = 0.4)\) with the pinions and wheels made of case hardened alloy steel 17CrNiMo6 and 17Cr3, respectively. The values of all considered genes, after optimization, for each InDS are given in Table 3 (for minimum mass) and Table 4 (for minimum length of the helical gears).

<table>
<thead>
<tr>
<th>( z_1 )</th>
<th>( z_2 )</th>
<th>( z_3 )</th>
<th>( z_4 )</th>
<th>( a_{w(1)} ) (mm)</th>
<th>( a_{w(2)} ) (mm)</th>
<th>( a_{w(3)} ) (mm)</th>
<th>( x_{n1} )</th>
<th>( x_{n2} )</th>
<th>( x_{n5} )</th>
<th>( \beta_{[1]} ) (°)</th>
<th>( \beta_{[2]} ) (°)</th>
<th>( \beta_{[3]} ) (°)</th>
<th>( \psi_{u[1]} )</th>
<th>( \psi_{u[2]} )</th>
<th>( \psi_{u[3]} )</th>
<th>Mass (kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>InDS – 1: (1.1): ( i_T = 40 ): ( n_m = 750 ) rpm, ( P_m = 6.5 ) kW; (1.2): ( n_m = 1000 ) rpm, ( P_m = 8 ) kW; (1.3): ( n_m = 1500 ) rpm, ( P_m = 12.5 ) kW</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>19</td>
<td>19</td>
<td>21</td>
<td>66</td>
<td>67</td>
<td>67</td>
<td>80</td>
<td>125</td>
<td>160</td>
<td>0.5758</td>
<td>0.9224</td>
<td>0.7431</td>
<td>16.5</td>
<td>11</td>
<td>14.5</td>
<td>0.3873</td>
<td>0.3445</td>
</tr>
<tr>
<td>17</td>
<td>19</td>
<td>20</td>
<td>60</td>
<td>68</td>
<td>65</td>
<td>80</td>
<td>125</td>
<td>180</td>
<td>0.937</td>
<td>0.763</td>
<td>0.7615</td>
<td>4</td>
<td>13.25</td>
<td>16</td>
<td>0.4025</td>
<td>0.3425</td>
</tr>
<tr>
<td>17</td>
<td>19</td>
<td>20</td>
<td>60</td>
<td>68</td>
<td>65</td>
<td>80</td>
<td>140</td>
<td>160</td>
<td>0.58</td>
<td>0.9935</td>
<td>0.7885</td>
<td>13.5</td>
<td>16</td>
<td>18.75</td>
<td>0.4025</td>
<td>0.3255</td>
</tr>
</tbody>
</table>

| InDS – 2: (2.1): \( i_T = 50 \): \( n_m = 750 \) rpm, \( P_m = 5.2 \) kW; (2.2): \( n_m = 1000 \) rpm, \( P_m = 7 \) kW; (2.3): \( n_m = 1500 \) rpm, \( P_m = 10 \) kW |
| 18       | 16       | 21       | 71       | 65       | 65       | 90       | 125      | 160     | 0.586   | 0.5545  | 0.781   | 7.75    | 12       | 18.75    | 0.2915    | 0.3775    | 0.4325    | 23.837   |
| 18       | 16       | 21       | 71       | 65       | 65       | 90       | 125      | 160     | 0.586   | 0.586   | 0.799   | 7.75    | 10       | 18.75    | 0.3025    | 0.368    | 0.4425    | 23.954   |
| 17       | 21       | 67       | 85       | 65       | 90       | 140      | 200      | 0.6176  | 0.6953  | 0.5758  | 19.75    | 18.25    | 14.25    | 0.245     | 0.2425    | 0.2075    | 23.446   |

| InDS – 3: (3.1): \( i_T = 63 \): \( n_m = 750 \) rpm, \( P_m = 4.5 \) kW; (3.2): \( n_m = 1000 \) rpm, \( P_m = 6 \) kW; (3.3): \( n_m = 1500 \) rpm, \( P_m = 9 \) kW |
| 18       | 19       | 19       | 79       | 77       | 66       | 100      | 140      | 180     | 0.6116  | 0.5459  | 0.779   | 13.5    | 16       | 16.25    | 0.2       | 0.33      | 0.3285    | 26.203   |
| 18       | 19       | 19       | 79       | 77       | 66       | 100      | 140      | 180     | 0.4024  | 0.4383  | 0.6176  | 14.5    | 16.85    | 0.2       | 0.3295    | 0.3125    | 25.446   |
| 18       | 19       | 19       | 79       | 77       | 66       | 100      | 140      | 180     | 0.6116  | 0.5459  | 0.689   | 13.5    | 16       | 17       | 0.2       | 0.3295    | 0.3245    | 26.049   |
The proposed GA (as it could be seen from the above tables) led to helical gearings which weights from ~22 kg (for InDS 4 – (4.1)) to ~26 kg (for InDS 7 (7.1)) and lengths between ~490 mm (for InDS 1 (1.2)) and ~558 mm (for InDS 7 (7.3)). The following remark on speed reducers with minimum volume/length generated by genetic simulations refers to the chaotic behaviour of the value of the helix angle measured at the pitch diameter for each stage. Despite this fact, the objective functions have very similar values. This means that \( \beta_{1,2,3} \) does not affect significantly the overall goal of the optimization. Another important note about the optimization result emerges by analysing the values of the helix gear width to
center distance ratio coefficient $\psi_{a(1,2,3)}$. When the objective function was the mass, the values of these coefficients vary as follows: $0.2–0.4$ (for the 1st stage); $0.2175–0.33$ (for the 2nd stage); $0.2–0.49$ (for the 3rd stage). For the second case when was minimized the length of the gearings, the values of $\psi_{a(1,2,3)}$ ranged between: $0.34–0.49$ (for the 1st stage); $0.29–0.56$ (for the 2nd stage); $0.29–0.57$ (for the 3rd stage). It is obviously that the genetic simulations for this case had generated optimal solutions with significantly higher values of $\psi_{a(1,2,3)}$. Now, once these preliminary conclusions were pointed out, based on these result sets models for determine the optimum values for gear ratios for each stage of the mechanical transmission will be further on developed. Firstly, the values of the gear ratios are computed (Table 5). Next, the Curve Fitting Toolbox™ (Fig.3) from MATLAB is used to perform an exploratory data analysis. From this toolbox is selected the ‘cftool’ function for conducting the regression analysis.

The regression analysis conducts to the following expressions for the helical gear ratios:

a) for minimum mass:

$$u_{(1)} = 0.8184 \cdot i_T^{0.3996}, \quad u_{(2)} = 1.302 \cdot i_T^{0.2809}, \quad u_{(3)} = 0.9194 \cdot i_T^{0.3208} \quad (6)$$

b) for minimum length:

$$u_{(1)} = 0.9126 \cdot i_T^{0.3731}, \quad u_{(2)} = 0.7414 \cdot i_T^{0.4188}, \quad u_{(3)} = 1.486 \cdot i_T^{0.2023} \quad (7)$$

Table 6

| Gear ratios comparison ($u_{(1)} \times u_{(2)} \times u_{(3)}$ and $i_{(1)} \times i_{(2)} \times i_{(3)}$) |
|---|---|---|
| 40 | $3.574 \times 3.6697 \times 3.0022$ | $3.5729 \times 3.424 \times 3.0022$ | $3.6726 \times 3.3448 \times 3.2563$ | $4.3776 \times 3.1676 \times 2.8847$ |
| 50 | $3.55 \times 3.55 \times 3.15$ | $3.55 \times 3.55 \times 3.15$ | $3.55 \times 3.55 \times 3.15$ | $4.5 \times 3.15 \times 2.8$ |
| 63 | $3.9073 \times 3.9071 \times 3.225$ | $4.1387 \times 3.6231 \times 3.345$ | $4.2616 \times 3.5367 \times 3.3174$ | $5.0289 \times 3.583 \times 2.9606$ |
| 80 | $4.243 \times 4.1691 \times 3.4732$ | $4.8827 \times 3.8415 \times 3.403$ | $4.9715 \times 3.747 \times 3.3819$ | $5.7889 \times 3.5679 \times 3.0502$ |
| 100 | $4.5 \times 4 \times 3.55$ | $5 \times 4 \times 3.55$ | $5 \times 4 \times 3.55$ | $5.6 \times 3.55 \times 3.15$ |
| 125 | $4.7146 \times 4.4589 \times 3.7499$ | $5.6405 \times 4.0812 \times 3.4753$ | $5.8298 \times 3.9776 \times 3.4499$ | $6.6955 \times 3.7984 \times 3.1457$ |
| 160 | $5 \times 4.5 \times 3.55$ | $5.6 \times 4.5 \times 3.55$ | $5.6 \times 4.5 \times 3.55$ | $6 \times 4.5 \times 3.15$ |
| 200 | $5.1543 \times 4.7469 \times 4.0281$ | $6.5336 \times 4.3185 \times 3.5442$ | $6.7649 \times 4.2058 \times 3.5147$ | $7.6701 \times 4.0271 \times 3.2375$ |
| 250 | $5.45 \times 4$ | $6.3 \times 4.5 \times 3.55$ | $7.1 \times 4.5 \times 3.55$ | $8.4 \times 3.15$ |
| 300 | $5.635 \times 5.054 \times 4.3271$ | $7.5681 \times 4.5696 \times 3.6145$ | $7.85 \times 4.4471 \times 3.5806$ | $8.7865 \times 4.2695 \times 3.3321$ |
| 400 | $6.2192 \times 5.4169 \times 4.6837$ | $8.9044 \times 4.8645 \times 3.6939$ | $9.2543 \times 4.7302 \times 3.6551$ | $10.2119 \times 4.5548 \times 3.4399$ |
| 500 | $6.3 \times 5.6 \times 4.5$ | $9 \times 5.5 \times 3.55$ | $9 \times 4.5 \times 3.55$ | $10 \times 4.5 \times 3.55$ |
Next, for validating these results the values of the gear ratios computed with the above equations are compared with the ones determined with the current models from literature (Section 2, Eqs. 1-3). For a better analysis and interpretation, the traditional and optimal designs’ gear ratios are compared side-by-side in Table 6. Also, in here are suggested the standardized values – $i_{(1,2,3)}$ that should be selected from the STAS 6012 for the actual gear ratios – $u_{(1,2,3)}$. It can be seen from Table 6 that all 3 models from literature, conduct to greater values of the gear ratios for the first stage and to an uneven distribution of these on speed reducer’s stages. Furthermore, when the total transmission ratio increases (above 80) current models offer for the first stage larger values (i.e. 8, 9 or 10) than the recommended ones. The results obtained by using GA show significant improvement over the results obtained by traditional design.

6. CONCLUSIONS

Speed reducers are the most common ways of transmitting power. Designing mechanical power transmissions is not an easy task, considering the iterative nature of the whole process. Furthermore, designing compact multi-stage speed reducers are challenging demands of nowadays mechanical power transmission manufacturer. Considering these aspects in this paper a GA was used to solve the complex structural design problem of multi-stage helical speed reducer. In here two objectives (the mass and the length of the helical gearings) and a set of 7 input data sets (InDS) [23] were used for obtaining appropriate optimal values for the partial gear ratios. The design variables considered in the optimization problem are of mixed nature i.e., integer (e.g. the gears number of teeth), discrete (e.g. normal tooth addendum coefficients) and real (e.g. gears width), in a total of 18. The objective functions were subjected to a highly non-linear set of 57 constraints. The results obtained by using GA conduct to mathematical expressions for optimal values of the partial ratios (for achieving minimum mass Eq. 6, and minimum length of the helical gearings Eq. 7), which offer an even distribution of the gear ratios on all 3 stages as compared with the values given by [15,21,22]. Once this preliminary phase is completed the virtual design space of the helical gearings will be generated and the problem formulation will be extended with the shafts and the housing subsystems. After that, when the complete optimization of the speed reducer will be done a generic transmission system design tool based on the evolutionary optimization concepts [5] will start to be developed. This optimization example illustrates the effectiveness of the proposed approach and also serves as further evidence of the power and versatility of GAs in designing mechanical power transmissions. The proposed GA could be easily modified to suit multi-objective design optimization.

REFERENCES


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