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ABUNDANT MULTIWAVE SOLUTIONS TO THE (3+1)-DIMENSIONAL SHARMA-TASSO-OLVER-LIKE EQUATION

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Abstract. The paper aims to present an application of the three-wave method and the homoclinic test method to the (3+1)-dimensional Sharma-Tasso-Olver-like equation. As a consequence, abundant novel types of analytical solutions involving multiple arbitrary parameters to the equation are revealed. Moreover, by choosing special values for the parameters, a few plots of the presented solutions are made to exhibit localized structures and dynamic behaviors.

Key words: (3+1)-dimensional Sharma-Tasso-Olver-like equation, symbolic calculation, exact solutions, dynamic behaviors.

1. INTRODUCTION

As is well-known, a great many of real physical features and properties of nonlinear complex phenomena can be characterized by nonlinear partial differential equations. Due to the fact that the analysis of exact solutions to nonlinear partial differential equations provides more insight into interpreting these nonlinear physical phenomena and dynamical processes, it is a significant subject for researchers to seek novel exact solutions to nonlinear partial differential equations. During the past decades, a number of fruitful algorithmic methods and their extensions have been presented, such as the Hirota's bilinear method [1,2], the Darboux transformation method [3], the (G'/G)-expansion method [4], the sub-ODE method [5], the multiple exp-function method [6,7], the transformed rational function method [8], the algebra-geometric method [9-12] and so forth. Whereas among the methods of solving nonlinear differential equations, the Hirota's bilinear method is one of the most direct and powerful approaches. The key step in this method is to transform the equation under consideration into its related bilinear differential form, based on which one can construct one-solitary-wave, two-solitary-wave as well as N-solitary-wave solutions. Inspired by this idea, plenty of research work was done on the interaction phenonmena among solitary waves, periodic waves and others. For example, Dai et al. [13] proposed an extended homoclinic test approach and obtained two types of exact periodic solitary-wave and kinky periodic-wave solutions of the Jimbo-Miwa equation. Wang et al. [14] handled the (2+1)-dimensional and (3+1)-dimensional KdV-type equations via generalized three-wave type ansatz approach, and acquired periodic type three-wave solutions. Recently, various classes of interaction solutions between lumps and kinks to the (2+1)-dimensional BKP equation [15] and KP equation [16] were presented through combining quadratic functions and exponential functions. Diverse interaction phenomena between lumps and solitons [17,18] were explored as well by using quadratic functions and hyperbolic cosine functions.

The (1+1)-dimensional classical Sharma-Tasso-Olver equation [19] reads

$$u_t + \alpha(u^3)_x + \frac{3}{2}\alpha(u^2)_{xx} + \alpha u_{xxx} = 0\,, \tag{1}$$

where u is an unknown function of the variables x and t. This equation is a prominent double nonlinear dispersive model. Since the significance of scientific applications, systematical investigations [20-23] have been carried out on equation (1). Recently, based on the idea of Lax pair generating function, a new (3+1)-dimensional Sharma-Tasso-Olver-like equation [24]

$$\begin{aligned} u_t + \alpha[(3uu_x + u^3)_x + u_{xxx}] + \beta[(2uu_y + u_x\partial_x^{-1}u_y + u^2\partial_x^{-1}u_y)_x + u_{xxy}] \\ + \gamma[(2uu_z + u_x\partial_x^{-1}u_z + u^2\partial_x^{-1}u_z)_x + u_{xxz}] = 0, \end{aligned} \tag{2}$$

with α, β, γ being real constants, was proposed. Here ∂_x^{-1} denotes the inverse operator of ∂_x defined by

$$(\partial_x^{-1} f)(x) = \int_{-\infty}^{x} f(t)\,\mathrm{d}t,$$

and $\partial_x^{-1}\partial_x = \partial_x\partial_x^{-1} = 1$. It is clear that Eq. (1) can be regarded as the special case of Eq. (2) when $\beta = \gamma = 0$. If only set $\gamma = 0$, then Eq. (2) reduces to the (2+1)-dimensional Sharma-Tasso-Olver-like equation. By virtue of the simplified Hirota's approach, multiple-soliton solutions for Eq. (2) were gained [24]. It will be our main concern to construct more novel exact solutions to Eq. (2) in the rest of this paper.

2. THREE-WAVE METHOD

In this section, we are interested in studying Eq. (2) by applying the three-wave method [14,25] and find out diverse exact solutions. Equation (2) can be mapped into the following equation in f:

$$\alpha ff_{xxxx} + \beta ff_{xxxy} + \gamma ff_{xxxz} - \alpha f_x f_{xxx} - \beta f_x f_{xxy} - \gamma f_x f_{xxz} + ff_{xt} - f_x f_t = 0 \tag{3}$$

via employing a dependent variable transformation

$$u = (\ln f)_x$$

with $f = f(x, y, z, t)$ as an auxiliary function. Obviously, if f satisfies Eq. (3), then $u = (\ln f)_x$ directly generates a solution of the original equation (2).

In order to determine f explicitly, we set an auxiliary function of such form

$$\begin{cases} f = a_2\cos\xi_2 + a_3\cosh\xi_3 + a_4\mathrm{e}^{\xi_4} + a_5\mathrm{e}^{-\xi_4}, \\ \xi_i = k_i x + l_i y + m_i z + c_i t, \quad i = 2,3,4, \end{cases} \tag{5}$$

where a_i, k_i, l_i, m_i, c_i and a_5 are some constants to be determined below. Carrying (5) into (3) yields a system of determining equations about the unknowns. However, for the sake of simplicity, we omit to list them. Then, under the condition of a_2, a_3, a_4, a_5 being all not zero, we solve the resulting system to find that

$$c_2 = k_2^2(\alpha k_2 + \beta l_2 + \gamma m_2), \quad c_3 = -k_3^2(\alpha k_3 + \beta l_3 + \gamma m_3), \quad c_4 = -\alpha k_4^3 - \beta k_4^2 l_4 - \gamma k_4^2 m_4,$$

where $k_2, k_3, k_4, l_2, l_3, l_4, m_2, m_3, m_4$ are all arbitrary constants. Accordingly, we acquire the expression of solutions as follows

$$u = \frac{-a_2k_2\sin\xi_2 + a_3k_3\sinh\xi_3 + a_4k_4\mathrm{e}^{\xi_4} - a_5k_4\mathrm{e}^{-\xi_4}}{a_2\cos\xi_2 + a_3\cosh\xi_3 + a_4\mathrm{e}^{\xi_4} + a_5\mathrm{e}^{-\xi_4}}, \tag{6}$$

in which

$$\begin{cases} \xi_2 = k_2x + l_2y + m_2z + k_2^2(\alpha k_2 + \beta l_2 + \gamma m_2)t, \\ \xi_3 = k_3x + l_3y + m_3z - k_3^2(\alpha k_3 + \beta l_3 + \gamma m_3)t, \\ \xi_4 = k_4x + l_4y + m_4z - (\alpha k_4^3 + \beta k_4^2 l_4 + \gamma k_4^2 m_4)t. \end{cases}$$

Setting $a_4 > 0$ and $a_5 = 1$ in (6), therefore we arrive at the kinky periodic soliton solutions to (2)

$$u = \frac{-a_2 k_2 \sin\xi_2 + a_3 k_3 \sinh\xi_3 + 2k_4\sqrt{a_4}\sinh\left(\xi_4 + \frac{1}{2}\ln a_4\right)}{a_2\cos\xi_2 + a_3\cosh\xi_3 + 2\sqrt{a_4}\cosh\left(\xi_4 + \frac{1}{2}\ln a_4\right)}.$$

Next, we are going to consider some special cases associated with (5) and present a series of exact solutions to Eq. (2).

● Case 1. If $a_2 = a_3 = 0$ and

$$c_4 = -\alpha k_4^3 - \beta k_4^2 l_4 - \gamma k_4^2 m_4,$$

where $a_4, a_5, k_2, k_3, k_4, l_2, l_3, l_4, c_2, c_3, m_2, m_3, m_4$ are free constants, then (5) can be abbreviated as

$$f = a_4 e^{\xi_4} + a_5 e^{-\xi_4},$$

which in turn gives the soliton solution of Eq. (2)

$$u = \frac{a_4 k_4 e^{\xi_4} - a_5 k_4 e^{-\xi_4}}{a_4 e^{\xi_4} + a_5 e^{-\xi_4}}$$

with

$$\xi_4 = k_4 x + l_4 y + m_4 z - (\alpha k_4^3 + \beta k_4^2 l_4 + \gamma k_4^2 m_4)t.$$

This solution is similar as the result appeared in [24].

● Case 2. If $a_2 = 0$ and

$$a_5 = \frac{a_3^2}{4a_4},\ \ k_3 = \varepsilon k_4,\ \ c_3 = -2\varepsilon\alpha k_4^3 - \beta k_4^2 l_3 - \varepsilon\beta k_4^2 l_4 - \gamma k_4^2 m_3 - \varepsilon\gamma k_4^2 m_4 - \varepsilon c_4,\ \ \varepsilon = \pm 1,$$

where $a_3, k_2, k_4, l_2, l_3, l_4, c_2, c_4, m_2, m_3, m_4$ and $a_4 \neq 0$ are free constants, then (5) can be expressed by

$$f = a_3\cosh\xi_3 + a_4 e^{\xi_4} + \frac{a_3^2}{4a_4} e^{-\xi_4}.$$

Substituting the results into (4) gives rise to

$$u = \frac{\varepsilon a_3 k_4 \sinh\xi_3 + a_4 k_4 e^{\xi_4} - \frac{a_3^2}{4a_4} k_4 e^{-\xi_4}}{a_3\cosh\xi_3 + a_4 e^{\xi_4} + \frac{a_3^2}{4a_4} e^{-\xi_4}}$$

with

$$\begin{cases} \xi_3 = \varepsilon k_4 x + l_3 y + m_3 z + (-2\varepsilon\alpha k_4^3 - \beta k_4^2 l_3 - \varepsilon\beta k_4^2 l_4 - \gamma k_4^2 m_3 - \varepsilon\gamma k_4^2 m_4 - \varepsilon c_4)t, \\ \xi_4 = k_4 x + l_4 y + m_4 z + c_4 t. \end{cases}$$

● Case 3. If $a_3 = 0$ and

$$c_2 = \alpha k_2^3 + \beta k_2^2 l_2 + \gamma k_2^2 m_2,\ \ c_4 = -k_4^2(\alpha k_4 + \beta l_4 + \gamma m_4),$$

where $a_2, a_4, a_5, k_2, k_3, k_4, l_2, l_3, l_4, c_3, m_2, m_3, m_4$ are free constants, then we obtain the auxiliary function

$$f = a_2\cos\xi_2 + a_4 e^{\xi_4} + a_5 e^{-\xi_4}, \tag{7}$$

and the kinky breather wave solution to Eq. (2)

$$u = \frac{-a_2 k_2 \sin\xi_2 + a_4 k_4 e^{\xi_4} - a_5 k_4 e^{-\xi_4}}{a_2\cos\xi_2 + a_4 e^{\xi_4} + a_5 e^{-\xi_4}},$$

where

$$\begin{cases}\xi_2 = k_2x + l_2y + m_2z + (\alpha k_2^3 + \beta k_2^2 l_2 + \gamma k_2^2 m_2)t,\\ \xi_4 = k_4x + l_4y + m_4z - k_4^2(\alpha k_4 + \beta l_4 + \gamma m_4)t.\end{cases}$$

Under the constraints of $k_2 = k_4 \neq 0$ and $a_5 = 1$, expression (7) is rewritten as

$$f = a_2 \cos\xi_2 + 2\sqrt{a_4}\cosh\left(\xi_4 + \frac{1}{2}\ln a_4\right).$$

The corresponding kinky breather wave solution for Eq. (2) takes the form

$$u = \frac{-a_2k_4 \sin\xi_2 + 2k_4\sqrt{a_4}\sinh\left(\xi_4 + \frac{1}{2}\ln a_4\right)}{a_2\cos\xi_2 + 2\sqrt{a_4}\cosh\left(\xi_4 + \frac{1}{2}\ln a_4\right)} \tag{8}$$

with

$$\begin{cases}\xi_2 = k_4x + l_2y + m_2z + (\alpha k_4^3 + \beta k_4^2 l_2 + \gamma k_4^2 m_2)t,\\ \xi_4 = k_4x + l_4y + m_4z - k_4^2(\alpha k_4 + \beta l_4 + \gamma m_4)t.\end{cases}$$

● Case 4. If $a_4 = k_4 = c_4 = 0$ and

$$c_2 = \alpha k_2^3 + \beta k_2^2 l_2 + \gamma k_2^2 m_2,\quad c_3 = -\alpha k_3^3 - \beta k_3^2 l_3 - \gamma k_3^2 m_3, \tag{9}$$

where $a_2, a_3, a_5, k_2, k_3, l_2, l_3, l_4, m_2, m_3, m_4$ are free constants, then (5) becomes

$$f = a_2\cos\xi_2 + a_3\cosh\xi_3 + a_5 \mathrm{e}^{-\xi_4}.$$

As a result, the expression of solution reads

$$u = \frac{-a_2k_2\sin\xi_2 + a_3k_3\sinh\xi_3}{a_2\cos\xi_2 + a_3\cosh\xi_3 + a_5\mathrm{e}^{-\xi_4}},$$

where

$$\begin{cases}\xi_2 = k_2x + l_2y + m_2z + (\alpha k_2^3 + \beta k_2^2 l_2 + \gamma k_2^2 m_2)t,\\ \xi_3 = k_3x + l_3y + m_3z - (\alpha k_3^3 + \beta k_3^2 l_3 + \gamma k_3^2 m_3)t,\\ \xi_4 = l_4y + m_4z.\end{cases}$$

In particular, if $a_4 = a_5 = 0$ and c_2, c_3 satisfy (9), then the auxiliary function is of the form

$$f = a_2\cos\xi_2 + a_3\cosh\xi_3,$$

which leads to the kinky breather wave solution

$$u = \frac{-a_2k_2\sin\xi_2 + a_3k_3\sinh\xi_3}{a_2\cos\xi_2 + a_3\cosh\xi_3},$$

where

$$\begin{cases}\xi_2 = k_2x + l_2y + m_2z + (\alpha k_2^3 + \beta k_2^2 l_2 + \gamma k_2^2 m_2)t,\\ \xi_3 = k_3x + l_3y + m_3z - (\alpha k_3^3 + \beta k_3^2 l_3 + \gamma k_3^2 m_3)t.\end{cases}$$

3. HOMOCLINIC TEST METHOD

In what follows, we proceed to look for novel solutions of Eq. (2) by virtue of the homoclinic test method [26], which has the following assumption

$$\begin{cases} f = 1 + b_1 e^{\xi_2} \cos\xi_1 + b_2 e^{2\xi_2}, \\ \xi_i = k_i x + l_i y + m_i z + c_i t + d_i, \quad i = 1, 2, \end{cases} \tag{10}$$

where $k_i, l_i, m_i, c_i, d_i, b_i$ are some undetermined parameters. And then substitution of (10) into (3) leads to a set of algebraic equations with respect to the unknowns. Setting b_1, b_2 being all not zero, the solutions that follow these equations can be given below:

• Case 1. When $k_2 = c_2 = 0$ and

$$c_1 = \alpha k_1^3 + \beta k_1^2 l_1 + \gamma k_1^2 m_1, \; l_2 = -\frac{\gamma m_2}{\beta},$$

where $b_1, b_2, k_1, l_1, m_1, m_2, d_1, d_2$ are some free parameters, the exact periodic soliton solution for Eq. (2) is

$$u = -\frac{b_1 k_1 e^{\xi_2} \sin\xi_1}{1 + b_1 e^{\xi_2} \cos\xi_1 + b_2 e^{2\xi_2}}, \tag{11}$$

in which

$$\begin{cases} \xi_1 = k_1 x + l_1 y + m_1 z + (\alpha k_1^3 + \beta k_1^2 l_1 + \gamma k_1^2 m_1) t + d_1, \\ \xi_2 = -\frac{\gamma m_2}{\beta} y + m_2 z + d_2. \end{cases}$$

With regard to expression (11), by taking account of $b_2 > 0$, it is then turned into

$$u = -\frac{b_1 k_1 \sin\xi_1}{b_1 \cos\xi_1 + 2\sqrt{b_2} \cosh\left(\xi_2 + \frac{1}{2}\ln b_2\right)}. \tag{12}$$

• Case 2. When $k_1 = c_2 = 0$ and

$$c_1 = -\beta k_2^2 l_1 - \gamma k_2^2 m_1, \; l_2 = -\frac{\alpha k_2 + \gamma m_2}{\beta},$$

where $b_1, b_2, k_2, l_1, m_1, m_2, d_1, d_2$ are free parameters, the corresponding solution reads

$$u = \frac{b_1 k_2 e^{\xi_2} \cos\xi_1 + 2 b_2 k_2 e^{2\xi_2}}{1 + b_1 e^{\xi_2} \cos\xi_1 + b_2 e^{2\xi_2}}$$

with

$$\begin{cases} \xi_1 = l_1 y + m_1 z - (\beta k_2^2 l_1 + \gamma k_2^2 m_1) t + d_1, \\ \xi_2 = k_2 x - \frac{\alpha k_2 + \gamma m_2}{\beta} y + m_2 z + d_2. \end{cases}$$

• Case 3. When

$$m_1 = \frac{-1}{2\gamma k_1 k_2}(3\alpha k_1^2 k_2 + 3\alpha k_2^3 + \beta k_1^2 l_2 + 2\beta k_1 k_2 l_1 + 3\beta k_2^2 l_2 + \gamma k_1^2 m_2 + 3\gamma k_2^2 m_2),$$

$$c_1 = \frac{-1}{2k_1 k_2}(\alpha k_1^4 k_2 + 6\alpha k_1^2 k_2^3 - 3\alpha k_2^5 + \beta k_1^4 l_2 + 6\beta k_1^2 k_2^2 l_2 - 3\beta k_2^4 l_2 + \gamma k_1^4 m_2 + 6\gamma k_1^2 k_2^2 m_2 - 3\gamma k_2^4 m_2),$$

$$c_2 = -4k_2^2(\alpha k_2 + \beta l_2 + \gamma m_2),$$

where $b_1, b_2, k_1, k_2, l_1, l_2, m_2, d_1, d_2$ are free parameters, hence Eq. (2) possesses the solution

$$u = \frac{-b_1 k_1 e^{\xi_2} \sin\xi_1 + b_1 k_2 e^{\xi_2} \cos\xi_1 + 2 b_2 k_2 e^{2\xi_2}}{1 + b_1 e^{\xi_2} \cos\xi_1 + b_2 e^{2\xi_2}}$$

with

$$\begin{cases} \xi_1 = k_1x + l_1y - \dfrac{1}{2\gamma k_1k_2}(3\alpha k_1^2k_2 + 3\alpha k_2^3 + \beta k_1^2l_2 + 2\beta k_1k_2l_1 + 3\beta k_2^2l_2 + \gamma k_1^2m_2 + 3\gamma k_2^2m_2)z \\ \quad -\dfrac{1}{2k_1k_2}(\alpha k_1^4k_2 + 6\alpha k_1^2k_2^3 - 3\alpha k_2^5 + \beta k_1^4l_2 + 6\beta k_1^2k_2^2l_2 - 3\beta k_2^4l_2 + \gamma k_1^4m_2 + 6\gamma k_1^2k_2^2m_2 - 3\gamma k_2^4m_2)t + d_1, \\ \xi_2 = k_2x + l_2y + m_2z - 4k_2^2(\alpha k_2 + \beta l_2 + \gamma m_2)t + d_2. \end{cases}$$

● Case 4. When

$$b_2 = \frac{b_1^2}{4},\ \ k_1 = \varepsilon i k_2,\ \ m_1 = \frac{1}{\gamma}(\varepsilon i\gamma m_2 + \varepsilon i\beta l_2 - \beta l_1),\ \ c_2 = -8\gamma k_2^2m_2 - 8\alpha k_2^3 - 8\beta k_2^2l_2 + \varepsilon i c_1,\ \ \varepsilon = \pm 1,$$

where $b_1, c_1, k_2, l_1, l_2, m_2, d_1, d_2$ are free parameters, Eq. (2) admits the solution

$$u = \frac{-\varepsilon i b_1k_2\mathrm{e}^{\xi_2}\sin\xi_1 + b_1k_2\mathrm{e}^{\xi_2}\cos\xi_1 + \frac{1}{2}b_1^2k_2\mathrm{e}^{2\xi_2}}{1 + b_1\mathrm{e}^{\xi_2}\cos\xi_1 + \frac{1}{4}b_1^2\mathrm{e}^{2\xi_2}}$$

with

$$\begin{cases} \xi_1 = \varepsilon i k_2x + l_1y + \dfrac{1}{\gamma}(\varepsilon i\gamma m_2 + \varepsilon i\beta l_2 - \beta l_1)z + c_1t + d_1, \\ \xi_2 = k_2x + l_2y + m_2z - (8\gamma k_2^2m_2 + 8\alpha k_2^3 + 8\beta k_2^2l_2 - \varepsilon i c_1)t + d_2. \end{cases}$$

4. DISCUSSIONS AND CONCLUSIONS

We choose some of the obtained solutions to display their characteristics of localized structures and dynamic behaviors by depicting graphics in three dimensions. Figure 1 shows that the breather wave solution (8) stands in a straight line and propagates towards the negative direction of the t axis with increasing y. In this wave, there also exists a certain angle with the x axis and the t axis, which means that the breather wave possesses both spatial and temporal periodicities. When time t is fixed, it is found that the amplitude of the wave oscillates up and down, and the wave moves towards the negative direction of the x axis. Thus it can be seen that such a wave is generated by the interaction between the soliton and the periodic wave. In addition, the solution (12) is plotted in Fig. 2. Worthy to note that in Fig. 2b the wave stands in a straight line and possesses many adjacent humps in opposite directions: some are above the plane and others are underneath. As increasing the variable z, the wave travels towards the positive direction of the y axis.

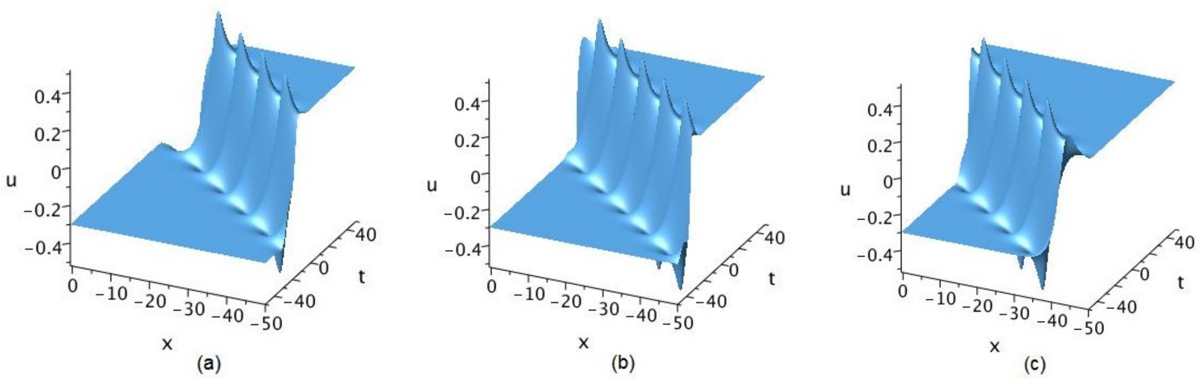

Fig. 1 – Kinky breather wave solution (8): a) $y = -2$; b) $y = 0$; c) $y = 3$.

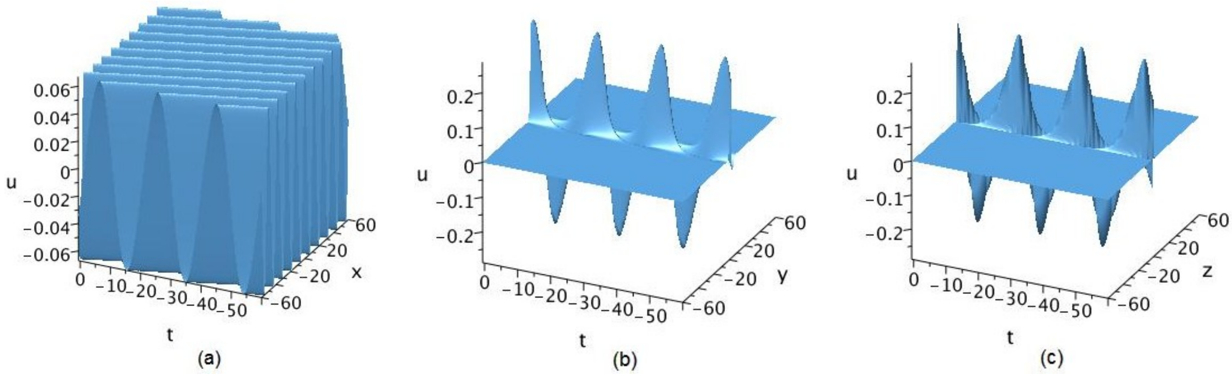

Fig. 2 – Periodic solitary wave solution (12): a) $y = z = 0$; b) $x = z = 0$; c) $x = y = 0$.

In conclusion, taking advantage of two direct constructive approaches, we have succeeded in presenting diverse new exact analytical solutions for the (3+1)-dimensional Sharma-Tasso-Olver-like equation, some of which include kinky periodic soliton solutions, kinky breather wave solutions and periodic solitary wave solutions. The obtained solutions contain multiple arbitrary parameters. Also the characteristics of localized structures and dynamic behaviors of some waves were shown graphically. The advantages of algorithms performed in this paper are straightforward and reliable in their applications and do not result in more complicated calculations.

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REFERENCES

1. A.M. WAZWAZ, *Multiple kink solutions for the (2+1)-dimensional integrable Gardner equation*, Proc. Romanian Acad. A, **15**, *3*, pp. 241–246, 2014.
2. A.M. WAZWAZ, *New (3+1)-dimensional nonlinear evolution equations with Burgers and Sharma-Tasso-Olver equations constituting the main parts*, Proc. Romanian Acad. A, **16**, *1*, pp. 32–40, 2015.
3. T.C. XIA, X.H. CHEN, D.Y. CHEN, *Darboux transformation and soliton-like solutions of nonlinear Schrödinger equation*, Chaos Solitons Fractals, **26**, *3*, pp. 889–896, 2005.
4. M.L. WANG, X.Z. LI, J.L. ZHANG, *The (G'/G)-expansion method and travelling wave solutions of nonlinear evolution equations in mathematical physics*, Phys. Lett. A, **372**, *4*, pp. 417–423, 2008.
5. X.Z. LI, M.L. WANG, *A sub-ODE method for finding exact solutions of a generalized KdV-mKdV equation with high-order nonlinear terms*, Phys. Lett. A, **361**, *1-2*, pp. 115–118, 2007.
6. W.X. Ma, T.W. Huang, Y. Zhang, *A multiple exp-function method for nonlinear differential equations and its application*, Phys. Scr., **82**, *6*, pp. 065003, 2010.
7. W.X. MA, Z.N. ZHU, *Solving the (3+1)-dimensional generalized KP and BKP equations by the multiple exp-function algorithm*, Appl. Math. Comput., **218**, *24*, pp. 11871–11879, 2012.
8. W.X. MA, J.H. LEE, *A transformed rational function method and exact solutions to the 3+1 dimensional Jimbo-Miwa equation*, Chaos Solitons Fractals, **42**, *3*, pp. 1356–1363, 2009.
9. C. YUE, T.C. XIA, *Algebro-geometric solutions for the complex Sharma-Tasso-Olver hierarchy*, J. Math. Phys., **55**, *8*, pp. 083511, 2014.
10. Q. LI, T.C. XIA, C. YUE, *Algebro-geometric solutions for the generalized nonlinear Schrödinger hierarchy*, J. Nonlinear Sci. Appl., **9**, *2*, pp. 661–676, 2016.
11. W.X. MA, *Trigonal curves and algebro-geometric solutions to soliton hierarchies I*, Proc. R. Soc. A, **473**, 20170232, 2017.
12. W.X. MA, *Trigonal curves and algebro-geometric solutions to soliton hierarchies II*, Proc. R. Soc. A, **473**, 20170233, 2017.

13. Z.D. DAI, J. LIU, X.P. ZENG, Z.J. LIU, *Periodic kink-wave and kinky periodic-wave solutions for the Jimbo-Miwa equation*, Phys. Lett. A, **372**, *38*, pp. 5984–5986, 2008.
14. C.J. WANG, Z.D. DAI, L. LIANG, *Exact three-wave solution for higher dimensional KdV-type equation*, Appl. Math. Comput., **216**, *2*, pp. 501–505, 2010.
15. J.B. ZHANG, W.X. MA, *Mixed lump-kink solutions to the BKP equation*, Comput. Math. Appl., **74**, *3*, pp. 591–596, 2017.
16. H.Q. ZHAO, W.X. MA, *Mixed lump-kink solutions to the KP equation*, Comput. Math. Appl., **74**, *6*, pp. 1399–1405, 2017.
17. J.Y. YANG,W.X. MA, Z.Y. QIN, *Lump and lump-soliton solutions to the (2+1)-dimensional Ito equation*, Anal. Math. Phys., **8**, *3*, pp. 427–436, 2017.
18. W.X. MA, X.L. YONG, H.Q. ZHANG, *Diversity of interaction solutions to the (2+1)-dimensional Ito equation*, Comput. Math. Appl., **75**, *1*, pp. 289–295, 2017.
19. Z.Y. YAN, *Integrability of two types of the (2+1)-dimensional generalized Sharma-Tasso-Olver integro-differential equations*, MM Res., **22**, pp. 302–324, 2003.
20. Z.J. LIAN, S.Y. LOU, *Symmetries and exact solutions of the Sharma-Tasso-Olver equation*, Nonlinear Anal., **63**, *5-7*, pp. 1167–1177, 2005.
21. S. WANG, X.Y. TANG, S.Y. LOU, *Soliton fission and fusion: Burgers equation and Sharma-Tasso-Olver equation*, Chaos Solitons Fractals, **21**, *1*, pp. 231–239, 2004.
22. W.X. MA, M. CHEN, *Do symmetry constraints yield exact solutions*?, Chaos Solitons Fractals, **32**, *4*, pp. 1513–1517, 2007.
23. A.H. CHEN, *Multi-kink solutions and soliton fission and fusion of Sharma-Tasso-Olver equation*, Phys. Lett. A, **374**, *23*, pp. 2340–2345, 2010.
24. A.M. WAZWAZ, S.A. EL-TANTAWY, *New (3+1)-dimensional equations of Burgers type and Sharma-Tasso-Olver type: multiple-soliton solutions*, Nonlinear Dyn., **87**, *4*, pp. 2457–2461, 2017.
25. H.Y. LUO, Z.D. DAI, J. LIU, G. MU, *Explicit doubly periodic soliton solutions for the (2+1)-dimensional Boussinesq equation*, Appl. Math. Comput., **219**, *12*, pp. 6618–6621, 2013.
26. Y.J. HU, H.L. CHEN, Z.D. DAI, *New kink multi-soliton solutions for the (3+1)-dimensional potential-Yu-Toda-Sasa-Fukuyama equation*, Appl. Math. Comput., **234**, pp. 548–556, 2014.

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