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MATHEMATICAL APPROACH OF THE FLAT ROLLING PROBLEM AND NEW ASPECTS CONCERNING THE GEOMETRY OF THE DEFORMATION ZONE

Cosmin Dănuț BARBU [1], Nicolae ȘANDRU [2]

[1]National Meteorological Administration, Department of Numerical Modelling,
Bucuresti-Ploiesti 97, 013686, Bucharest, Romania
[2]University of Bucharest, Department of Applied Mathematics, Faculty of Mathematics and Computer Science,
Academiei 14, 010014, Bucharest, Romania
Corresponding author: Cosmin Dănuț BARBU, E-mail: cosmin_barbu@ymail.com

Abstract. The mathematical formulation of the flat rolling problem in complex conjugate variables is considered. Based on an asymptotic expansion method with respect to a dimensionless group parameters – the Bingham and the Reynolds numbers - a model is proposed to analyse the behaviour of the strip during symmetrical rolling with a neutral point. The solution of the problem is represented by the first three terms of the perturbation expansion. The equations for the discontinuity surfaces located at the entrance and exit of the deformation zone, are also determined. From the numerical examples presented, where we neglect the inertial terms, we point that the geometry of the deformation zone is not a fixed one, as previously has been assumed in the literature. Furthermore, we show how it is influenced by various rolling parameters such as friction, working speed and thickness reduction of the strip.

Key words: flat rolling, asymptotic expansion, deformation zone geometry.

1. INTRODUCTION

Rolling is a major and a most widely used mechanical working technique. In the longitudinal rolling, only a small part of the strip – the part passing through the roll gap – is the subject of deformation at any moment of time. This part of the strip is known as deformation zone. The aim of the rolling is to reduce the thickness of the metal strip by permanent viscoplastic deformation resulting as a consequence of its passing through a pair of rigid rolls which rotate in opposite directions.

In the theory of metal forming, the rolling process has been studied both theoretically and experimentally for many decades. An extensive list of mathematical models for rolling using various analytical and numerical approaches, such as the slab method, the upper-bound theory and the finite element technique, is given in [1] and [2].

Approximate methods based on the asymptotic techniques have been used in metal forming process, see for instance [3–7]. A perturbation method with respect to two small dimensionless parameters – the Bingham number Bg and the Reynolds number Re – has been developed and applied for the strip and wire drawing problems in [3–5]. These parameters naturally emerge by considering the governing equations for a viscoplastic Bingham model. The effect of the high working speed in the forming process has been investigated in [6–8]. The originality of this asymptotic method consists of the fact that it is based on two dimensionless parameters, which represents a combination of mechanic and kinematic quantities characterising the metal forming process, like the viscosity, the shear yielding limit and the working speed.

In this paper, we use this asymptotic method of perturbation with respect to Bingham and Reynolds numbers. In Section 2, based on the principles of continuum mechanics, we summarise the relevant equations of the viscoplastic Bingham model and we specify the kinematic and boundary conditions associated to the rolling process. In our recent paper [11], by introducing the Stokes stream function and using the polar coordinates (r,θ), we have determined the velocity and stress fields in the deformation zone for a global friction Coulomb's law. The neutral point position and the equations of the discontinuity surfaces, delimiting the deformation zone, have been also found. Unlike [11], we consider here the mathematical formulation of the rolling problem in complex conjugate variables $(z,\bar{z})$.

The solution approximation of the rolling problem is presented in Section 3. According to the results obtained in [11], the solution of the problem is expressed by neglecting the Reynolds terms and by considering only the first two terms of the perturbation expansion. The first term of the solution gives the viscosity influence in the strip rolling process, while the second one captures the plasticity effects. The contribution of the inertial term will be presented elsewhere.

Instead of assuming that the boundary surfaces separating the deformation zone are a priori given as in [9] and [10], where these surfaces are supposed to be planes perpendicular to the rolling direction or circular curves, we determine simultaneous their equations with the solution of the problem. In Section 4, we investigate and illustrate how the thickness reduction, the velocity of the strip and the friction coefficient significantly influence the geometry of the deformation zone.

2. FORMULATION OF THE PROBLEM

We consider rigid rolls having the same size, constant angular velocity $\boldsymbol{\omega}$ and surface conditions. The flow that occurs in the rolling process is axially symmetric.

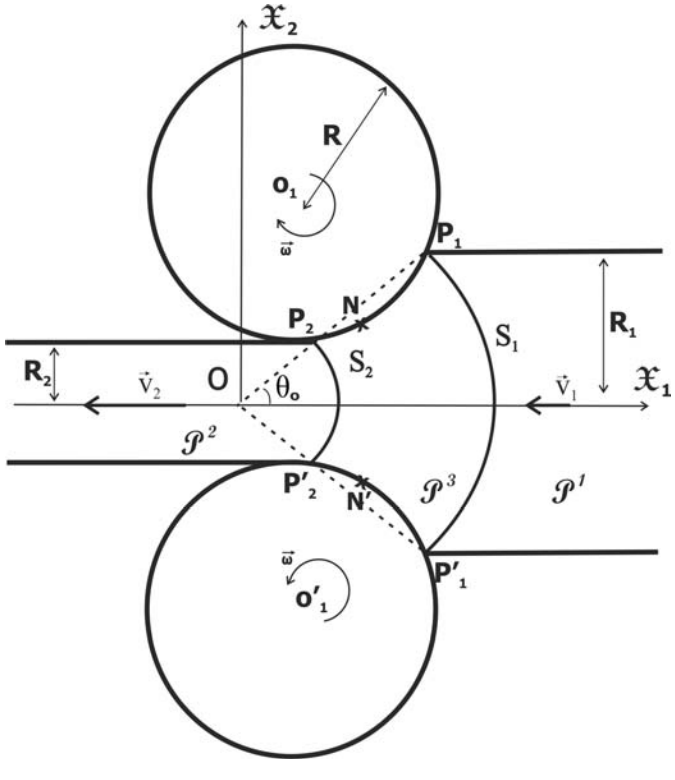


Fig. 1 – Side view of flat rolling process.

We assume that the mechanical properties of the material are described by a Bingham rigid-viscoplastic model. The strip (Fig.1) is divided in three zones. In $\mathcal{P}^1$ and $\mathcal{P}^2$ zones, we consider that the rigid body motion takes place in the negative Ox_1 direction. The viscoplastic deformation zone $\mathcal{P}^3$ is bounded by two biting arcs (replaced by two chords) and by two discontinuity surfaces S_1 and S_2 (which have to be determined). We suppose a stationary incompressible motion and the absence of body forces.

2.1. The model equations

Define a 2D control domain (as shown in Fig. 1) in which the following viscoplastic of Bingham type dynamical incompressible stationary equations are satisfied:

– the balance equations

$$\frac{\partial T_{11}}{\partial x_1}+\frac{\partial T_{12}}{\partial x_2}=\rho\left(v_1\frac{\partial v_1}{\partial x_1}+v_2\frac{\partial v_1}{\partial x_2}\right),\quad \frac{\partial T_{12}}{\partial x_1}+\frac{\partial T_{22}}{\partial x_2}=\rho\left(v_1\frac{\partial v_2}{\partial x_1}+v_2\frac{\partial v_2}{\partial x_2}\right), \tag{1}$$

– the continuity equation

$$\frac{\partial v_1}{\partial x_1}+\frac{\partial v_2}{\partial x_2}=0, \tag{2}$$

– the kinematical relations

$$D_{11}=\frac{\partial v_1}{\partial x_1},\ D_{22}=\frac{\partial v_2}{\partial x_2},\ D_{12}=\frac{1}{2}\left(\frac{\partial v_1}{\partial x_2}+\frac{\partial v_2}{\partial x_1}\right), \tag{3}$$

– the constitutive equations (of Bingham type)

$$T_{11}=-p+\left(2\eta+\frac{k}{\sqrt{II_{\mathbf{D}}}}\right)D_{11},\quad T_{22}=-p+\left(2\eta+\frac{k}{\sqrt{II_{\mathbf{D}}}}\right)D_{22},\quad T_{12}=\left(2\eta+\frac{k}{\sqrt{II_{\mathbf{D}}}}\right)D_{12}, \tag{4}$$

where T_{ij} are the components of the Cauchy stress tensor, v_i are the components of the velocity vector $\mathbf{v}$, D_{ij} are the components of the deformation rate tensor $\mathbf{D}$ (index $i,j=1, 2$). Here, ρ is the material density and p is the pressure. The material constants η and k are the dynamic viscosity coefficient and the shear yielding limit, respectively.

The global conservation of the mass is expressed by

$$\mathrm{V}_1 R_1=\mathrm{V}_2 R_2, \tag{5}$$

Across a stationary discontinuity surface, the dynamical jump conditions of compatibility

$$[v_n]=0, \tag{6}$$

$$[T_{kl}n_k]-\rho v_n[v_l]=0, \tag{7}$$

must be satisfied. Here, k is the muted indice. The normal component of the velocity on a discontinuity surface is denoted by v_n.

Introducing the Stokes stream function $\psi=\psi(x_1,x_2)$ by

$$v_1=\frac{\partial\psi(x_1,x_2)}{\partial x_2},\quad v_2=-\frac{\partial\psi(x_1,x_2)}{\partial x_1}, \tag{8}$$

the continuity equation (2) is fulfilled.

Introducing (8) in (3) one gets

$$D_{11}=\frac{\partial^2\psi}{\partial x_1\partial x_2}, D_{22}=-\frac{\partial^2\psi}{\partial x_1\partial x_2}, D_{12}=\frac{1}{2}\left(\frac{\partial^2\psi}{\partial x_2^2}-\frac{\partial^2\psi}{\partial x_1^2}\right). \tag{9}$$

Further on, by introducing (4) in (1) via (9) we obtain a system of partial differential equations for the unknown functions $p(x_1,x_2)$ and $\psi(x_1,x_2)$

$$\begin{aligned}
&-\frac{\partial p}{\partial x_1}+\eta\frac{\partial}{\partial x_2}(\Delta\psi)+\frac{k}{2}\left[\frac{\partial}{\partial x_2}\left(\frac{\frac{\partial^2\psi}{\partial x_2^2}-\frac{\partial^2\psi}{\partial x_1^2}}{\sqrt{II_{\mathbf{D}}}}\right)+\frac{\partial}{\partial x_1}\left(\frac{2\frac{\partial^2\psi}{\partial x_1\partial x_1}}{\sqrt{II_{\mathbf{D}}}}\right)\right]=\\
&=\frac{\rho}{2}\left[\frac{\partial}{\partial x_1}\left(v_1^2+v_2^2\right)+\frac{\partial}{\partial x_1}\left(v_1^2-v_2^2\right)+\frac{\partial}{\partial x_2}\left(2v_1v_2\right)\right],\\
&-\frac{\partial p}{\partial x_2}-\eta\frac{\partial}{\partial x_1}(\Delta\psi)-\frac{k}{2}\left[-\frac{\partial}{\partial x_1}\left(\frac{\frac{\partial^2\psi}{\partial x_2^2}-\frac{\partial^2\psi}{\partial x_1^2}}{\sqrt{II_{\mathbf{D}}}}\right)+\frac{\partial}{\partial x_2}\left(\frac{2\frac{\partial^2\psi}{\partial x_1\partial x_1}}{\sqrt{II_{\mathbf{D}}}}\right)\right]=\\
&=\frac{\rho}{2}\left[\frac{\partial}{\partial x_2}\left(v_1^2+v_2^2\right)-\frac{\partial}{\partial x_2}\left(v_1^2-v_2^2\right)+\frac{\partial}{\partial x_1}\left(2v_1v_2\right)\right],
\end{aligned}\tag{10}$$

where the second invariant of the deformation rate tensor $\mathbf{D}$ is

$$II_{\mathbf{D}}=\frac{1}{4}\left[\left(\frac{\partial^2\psi}{\partial x_2^2}-\frac{\partial^2\psi}{\partial x_1^2}\right)^2+4\left(\frac{\partial^2\psi}{\partial x_1\partial x_2}\right)^2\right].\tag{11}$$

Now, instead of using the Cartesian variables (x_1,x_2), we introduce the complex conjugate variables $z=x_1+i\,x_2$ and $\overline{z}=x_1-ix_2$. It is easy to find the following derivative expressions

$$\begin{aligned}
&\frac{\partial}{\partial z}=\frac{1}{2}\left(\frac{\partial}{\partial x_1}-i\frac{\partial}{\partial x_2}\right),\qquad \frac{\partial}{\partial\overline{z}}=\frac{1}{2}\left(\frac{\partial}{\partial x_1}+i\frac{\partial}{\partial x_2}\right),\\
&\frac{\partial^2}{\partial z\partial\overline{z}}=\frac{1}{4}\left(\frac{\partial^2}{\partial x_1^2}+\frac{\partial^2}{\partial x_2^2}\right),\quad \frac{\partial^2}{\partial\overline{z}^2}=\frac{1}{4}\left(\frac{\partial^2}{\partial x_1^2}-\frac{\partial^2}{\partial x_2^2}+2i\frac{\partial^2}{\partial x_1\partial x_2}\right).
\end{aligned}\tag{12}$$

Then, from (11) we get

$$\sqrt{II_{\mathbf{D}}}=2\left|\frac{\partial^2\psi}{\partial\overline{z}^2}\right|.\tag{13}$$

The system of equations (10) in complex form becomes

$$\frac{\partial}{\partial\overline{z}}\left(p+i\eta\Delta\psi\right)+ki\frac{\partial}{\partial z}\left(\frac{\frac{\partial^2\psi}{\partial\overline{z}^2}}{\left|\frac{\partial^2\psi}{\partial\overline{z}^2}\right|}\right)=-\frac{\rho}{2}\left\{\frac{\partial}{\partial\overline{z}}\left[\left(\frac{\partial\psi}{\partial x_1}\right)^2+\left(\frac{\partial\psi}{\partial x_2}\right)^2\right]+\frac{\partial}{\partial z}\left[\left(\frac{\partial\psi}{\partial x_2}\right)^2-\left(\frac{\partial\psi}{\partial x_1}\right)^2-2i\frac{\partial\psi}{\partial x_1}\frac{\partial\psi}{\partial x_2}\right]\right\}.\tag{14}$$

In the equation (14) only the unknown functions $p=p(x_1,x_2)$ and $\psi=\psi(x_1,x_2)$ are involved. We can rewrite (14) in polar coordinates (r,θ), by using the relations

$$\frac{\partial}{\partial z}=\frac{e^{-i\theta}}{2}\left(\frac{\partial}{\partial r}-\frac{i}{r}\frac{\partial}{\partial\theta}\right),\qquad \frac{\partial}{\partial\overline{z}}=\frac{e^{i\theta}}{2}\left(\frac{\partial}{\partial r}+\frac{i}{r}\frac{\partial}{\partial\theta}\right).\tag{15}$$

Let $z=z(s)$, $s\in[s_1,s_2]$ be the equation of a smooth curve S connecting two points on the arcs of contact and being included in the deformation zone $\mathcal{P}^3$. If $\mathbf{n}=(x_2'(s),-x_1'(s))$ is the unit normal vector to the curve, then the stress vector written in complex form is

$$T_{n1} + iT_{n2} = -\frac{i}{2}\Big[(T_{11} + T_{22})z'(s) + (T_{22} - T_{11} - 2i\,T_{12})\overline{z'(s)}\Big]. \tag{16}$$

From equations (4) and (9), we obtain

$$T_{11} + T_{22} = -2p\,, \tag{17}$$

$$T_{22} - T_{11} - 2iT_{12} = 4i\left(2\eta + \frac{k}{2\left|\dfrac{\partial^2\psi}{\partial\overline{z}^2}\right|}\right)\frac{\partial^2\psi}{\partial\overline{z}^2}\,. \tag{18}$$

Then (16) becomes

$$T_{n1} + iT_{n2} = i\left[pz'(s) - 4i\eta\frac{\partial^2\psi}{\partial z^2}\overline{z}'(s) - ik\frac{\dfrac{\partial^2\psi}{\partial\overline{z}^2}}{\left|\dfrac{\partial^2\psi}{\partial\overline{z}^2}\right|}\overline{z}'(s)\right]. \tag{19}$$

Let us consider the following differential form, which is related to the dynamical compatibility condition (7).

$$\begin{aligned} \mathrm{d}X + i\mathrm{d}Y = -\frac{i}{2}\Big\{&(T_{11} + T_{22})z'(s) + (T_{22} - T_{11} - 2iT_{12})\overline{z}'(s) - \rho\Big[v^2 z'(s) + (v_2^2 - v_1^2 - 2iv_1v_2)\overline{z}'(s)\Big] + \\ &+ \rho\Big[(v_1 - iv_2)z'(s) - (v_1 + iv_2)\overline{z}'(s)\Big]V\Big\}\,\mathrm{d}s\,, \end{aligned} \tag{20}$$

where $v^2 = v_1^{\,2} + v_2^{\,2}$ and V is a real constant.

By considering relations (14) and (2), it is easy to verify that (20) is a total differential form This allow an easy calculation of the stress resultant on the discontinuity surfaces S_1 (at entrance) and S_2 (at exit).

2.2. The kinematic and boundary conditions

In order to determine the parameters involved in the problem solution, the following conditions, are used, as in [11]

a) The component v_θ of the velocity is zero on the surfaces of the rolls (i.e. the normal velocity on P_1P_2 and $P'_1P'_2$)

$$v_\theta(r,\theta)\Big|_{\theta=\pm\theta_0} = 0\,. \tag{21}$$

b) The discontinuity surface S_1 passes through the points P_1 and P_1'. The volume flow rate of material passing through S_1 is constant and using (5), we get

$$\psi(r,\theta)\Big|_{\substack{\theta=\theta_0\\ r=r_1,r_2}} = -R_1V_1 = -R_2V_2\,. \tag{22}$$

c) The neutral point N lies on the roll surface, near the exit. The friction force makes the strip to advance on the zone between the neutral point and the entrance, while the friction force opposes the rolling process on the zone between the exit and the neutral point.

The friction condition on the surfaces of the rolls is prescribed by Coulomb's global law. We denote by T_1, T_2 and N_1, N_2 the tangential and normal resultant forces acting on the strip surfaces of the rolls, respectively. Thus, we have

$$T_1 = \mu N_1 \text{ and } T_2 = -\mu N_2\,, \tag{23}$$

where μ is a positive constant (the Coulomb friction coefficient) and

$$T_1 = \int_{r_N}^{r_1} T_{r\theta}(r,\theta_0)\mathrm{d}r,\ T_2 = \int_{r_2}^{r_N} T_{r\theta}(r,\theta_0)\mathrm{d}r,\quad N_1 = \int_{r_N}^{r_1} T_{\theta\theta}(r,\theta_0)\mathrm{d}r,\ N_2 = \int_{r_2}^{r_N} T_{\theta\theta}(r,\theta_0)\mathrm{d}r\,. \tag{24}$$

Here, $r_1 = r_1(\theta_0)$, $r_2 = r_2(\theta_0)$ and r_N is the value of r at the neutral point N.

d) The resultant force X_1 acting on S_1 (named the back force) and the resultant force X_2 acting on S_2 (named the drawing force) are both given.

e) The speed of the strip equals the speed of the rolls at the neutral point

$$v_r(r,\theta_0)\big|_{r=r_N} = -\,\omega R\,, \tag{25}$$

where ω is the roll angular velocity and R is the radius of the roll.

3. APPROXIMATION OF THE SOLUTION

We introduce the dimensionless variables (denoted by the "o"index)

$$x_i = R_2 x_i^o,\ v_i = \mathrm{V}_2 v_i^o,\ p = \frac{\eta \mathrm{V}_2}{R_2} p^o,\ \mathbf{T} = \frac{\eta \mathrm{V}_2}{R_2}\mathbf{T}^o,\ \mathbf{D} = \frac{\mathrm{V}_2}{R_2}\mathbf{D}^o,\ \psi(x_1,x_2) = R_2 \mathrm{V}_2 \psi^o(x_1,x_2)\,, \tag{26}$$

where R_2 is a characteristic length and V_2 is a characteristic velocity (see Fig. 1).

By using (26) in the system of equations (1)-(4), the non-dimensional Bingham and Reynolds numbers put into evidence

$$\mathrm{Bg} = \frac{kR_2}{\eta \mathrm{V}_2},\qquad \mathrm{Re} = \frac{\rho \mathrm{V}_2 R_2}{\eta}\,. \tag{27}$$

In what follows, we assume $\mathrm{Bg} < 1$ and $\mathrm{Re} < 1$.

Following the mathematical methods from [5] and [7], in this study the unknown functions $\psi^o(x_1,x_2)$ and $p^o(x_1,x_2)$ are expanded in power series with respect to the "small" parameters - Bingham and Reynolds numbers. Thus,

$$\begin{aligned}
\psi(x_1,x_2) &= R_2\mathrm{V}_2\left[\psi_0^o(x_1,x_2) + \mathrm{Bg}\,\psi_1^o(x_1,x_2) + \mathrm{Re}\,\psi_2^o(x_1,x_2) + \mathcal{O}(\mathrm{Bg}^2,\mathrm{BgRe},\mathrm{Re}^2)\right],\\
p(x_1,x_2) &= \frac{\eta \mathrm{V}_2}{R_2}\left[p_0^o(x_1,x_2) + \mathrm{Bg}\,p_1^o(x_1,x_2) + \mathrm{Re}\,p_2^o(x_1,x_2) + \mathcal{O}(\mathrm{Bg}^2,\mathrm{BgRe},\mathrm{Re}^2)\right].
\end{aligned} \tag{28}$$

We substitute (28) in (14) and we equal the terms of the same degree in Bingham and Reynolds numbers, we obtain

$$\frac{\partial}{\partial \overline{z}}(p_0^o + iR_2^2\Delta\psi_0^o) = 0\,, \tag{29}$$

$$\frac{\partial}{\partial \overline{z}}(p_1^o + iR_2^2\Delta\psi_1^o) = -i\frac{\partial}{\partial z}\left(\frac{\dfrac{\partial^2\psi_0^o}{\partial \overline{z}^2}}{\left|\dfrac{\partial^2\psi_0^o}{\partial \overline{z}^2}\right|}\right), \tag{30}$$

$$\frac{\partial}{\partial \overline{z}}(p_2^o + iR_2^2\Delta\psi_2^o) = -2R_2^2\left[\frac{\partial}{\partial \overline{z}}\left(\frac{\partial\psi_0^o}{\partial z}\frac{\partial\psi_0^o}{\partial \overline{z}}\right) - \frac{\partial}{\partial z}\left(\frac{\partial\psi_0^o}{\partial \overline{z}}\right)^2\right]. \tag{31}$$

From (29)–(31) we find that the biharmonic operator governs the flat rolling problem, i.e.

$$\Delta\Delta\,\psi_0^o(x_1,x_2) = 0\,, \tag{32}$$

$$\Delta\Delta\psi_1^o(x_1,x_2) = -\frac{2}{R_2^2}\left[\frac{\partial^2}{\partial z^2}\left(\frac{\frac{\partial^2\psi_0^o}{\partial\bar{z}^2}}{\left|\frac{\partial^2\psi_0^o}{\partial\bar{z}^2}\right|}\right) + \frac{\partial^2}{\partial\bar{z}^2}\left(\frac{\frac{\partial^2\psi_0^o}{\partial z^2}}{\left|\frac{\partial^2\psi_0^o}{\partial z^2}\right|}\right)\right], \tag{33}$$

$$\Delta\Delta\psi_2^o(x_1,x_2) = -4i\left[\frac{\partial^2}{\partial z^2}\left(\frac{\partial\psi_0^o}{\partial\bar{z}}\right)^2 - \frac{\partial^2}{\partial\bar{z}^2}\left(\frac{\partial\psi_0^o}{\partial z}\right)^2\right]. \tag{34}$$

Integrating analytically the equations (32)–(34) with the boundaries conditions (21)–(25), we determine the Stokes function $\psi^o = \psi^o(r,\theta)$. Further on, the pressure function $p^o = p^o(r,\theta)$ is computed from the system of equations (29)–(31) by using the expression found for $\psi^o(r,\theta)$. A detailed solution is given in [11].

4. THE GEOMETRY OF THE DEFORMATION ZONE

In the analytical model of the flat rolling problem presented in [11], the discontinuity surfaces S_1 (at entrance) and S_2 (at exit) are not a priori given as in [9], [10]. The discontinuity surfaces S_1 and S_2 are simultaneously determined with the solution by using the dynamic compatibility condition (6), which expresses the continuity of the normal velocity v_n. We find their equations as below

$$S_1: \quad \mathrm{V}_1 r_1(\theta)\sin\theta + \psi(r_1(\theta),\theta) = 0, \tag{35}$$

$$S_2: \quad \mathrm{V}_2 r_2(\theta)\sin\theta + \psi(r_2(\theta),\theta) = 0. \tag{36}$$

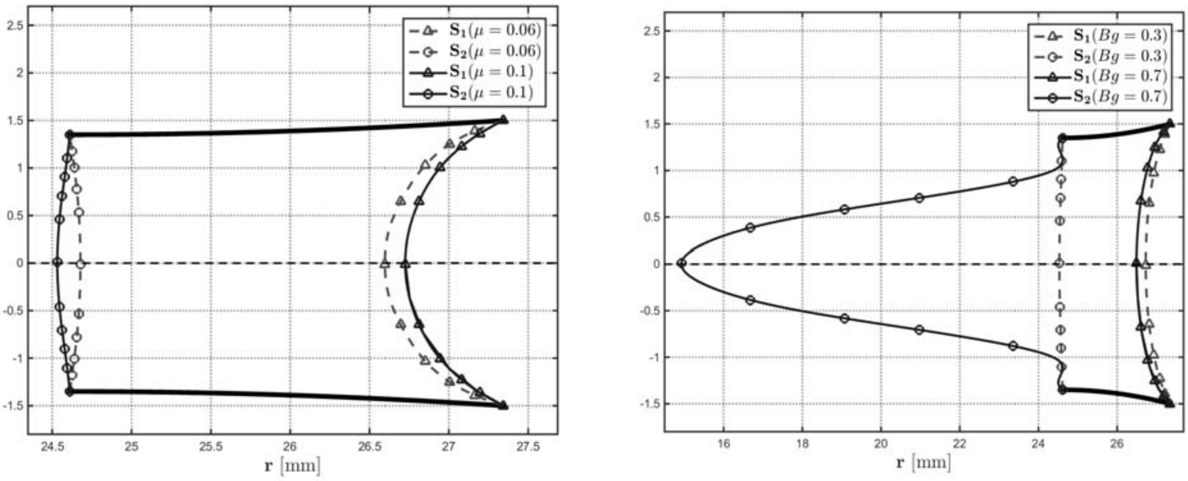
In what follows, the shape of the discontinuity surfaces is determined within the framework of the first two approximations which results from (29) and (30). In order to illustrate the theoretical results, some numerical examples are considered by using a MATLAB programme. In the flat rolling problem, the following parameters are given: the semi-thickness of the strip at entrance $R_1 = 1.5$ mm, the radius of the roll $R = (R_1 - R_2)/(2\sin^2\theta_0)$, the thickness reduction $r[\%] = 100(1 - R_2/R_1)$, the working speed V_2 at exit and the values of the material constants k and η.

In Figs. 2 and 3, we plot the discontinuity surfaces S_1 and S_2. The discontinuity surface S_1 (at entrance) is marked by triangle, while the discontinuity surface S_2 (at exit) is marked by circle. The convexity and the concavity properties of the graphs for S_1 and S_2 are represented with respect to the positive orientation of the Ox_1 axis. We analyse the influence of the friction coefficient μ and of the work velocity V_2 (equivalently, of the Bg number according to (27)) on the curvature and on the shape of the discontinuity surfaces S_1 and S_2. Two cases for the thickness reduction of the strip are considered, namely for 10% and 20%.

a) The geometry variation of the deformation zone for a $10\,\%$ *reduction.* Let the roll radius be $R = 25$ mm.

Figs. 2a, b illustrate that S_1 (at entry) remains a convex curve, but changes the curvature depending on parameters μ and V_2. Thus, the rigid part of the strip extends inside the roll gap area.

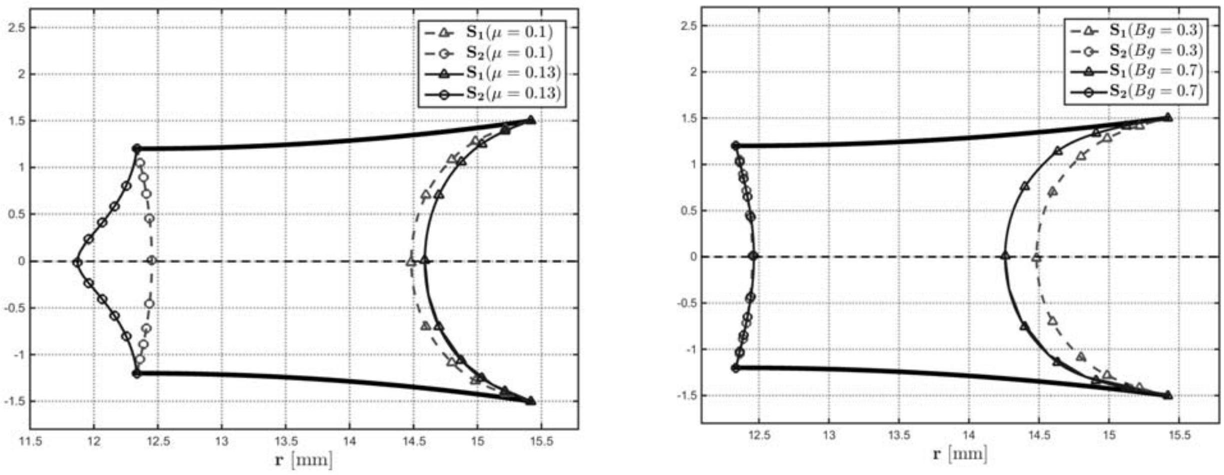
The shape of the discontinuity surface S_2 (at exit) changes the convexity. For instance, if $\mu = 0.06$ the curve is concave, while for $\mu = 0.1$ the curve is convex (see Fig. 2a). Unlike S_1, the shape of the discontinuity surface S_2 is strongly influenced by the working speed values, as can be noticed in Fig. 2b, for $\mu = 0.1$. We also remark from Fig. 2b that S_2 is a concave curve near the arc of contact and it changes to a convex curve which it bulges more towards to the origin of the Cartesian coordinate system Ox_1x_2.



(a) $Bg = 0.3$ (b) $\mu = 0.1$

Fig. 2 – The deformation zone for 10% reduction when (a) μ is varying and V_2 is fixed; (b) V_2 is varying and μ is fixed.

b) The geometry variation of the deformation zone for a 20 % reduction. Let the roll radius be $R = 16$ mm.



(a) Bg = 0.3 (b) $\mu = 0.1$

Fig. 3 – The deformation zone for 20% reduction when (a) μ is varying and V_2 is fixed; (b) V_2 is varying and μ is fixed.

Again, S_1 (at entrance) is a convex curve. Its curvature is influenced by the different values of the friction coefficient μ (see Fig. 3a), or by the different values of the working speed V_2 (see Fig. 3b).

In Fig. 3a, we illustrate the influence of the friction coefficient μ. The shape of the discontinuity surface S_2 (at exit) is concave when $\mu = 0.1$ and it is concave-convex when $\mu = 0.13$. In Fig. 3b, for two different working speed values (via Bg number), we notice that S_2 is a concave curve.

5. CONCLUSIONS

From the numerical results presented in this paper, we emphasize that the geometry of the deformation zone is strongly influenced by the rolling conditions, such as the thickness reduction of the strip, the working speed and the value of the friction coefficient. In the model described here, we determine the equations of the discontinuity surfaces, boundaring the deformation zone, together with the solution of the problem, unlike some other mathematical models of rolling problem (models based on the slab method analysis, the upper-bound method or finite element method).

Our analysis contributes to explaining the apparently evidence that the geometry of the deformation zone is variable. This remark awaits experimental verifications.

The theoretical results of this study allow to perform an accurate analysis of the viscoplastic deformation process in the flat rolling problem. A further analysis to predict the inertial effect in the rolling process will be considered in a future work.

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