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BRIGHT AND DARK SOLITONS IN OPTICAL MATERIALS WITH POLYNOMIAL LAW NONLINEARITY

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Abstract. We find families of analytical solutions for a recently proposed nonlinear partial differential equation describing pulse propagation in optical materials with polynomial law nonlinearity. By employing the Lie symmetry method, new explicit bright and dark soliton solutions are given. Furthermore, new doubly periodic solutions in terms of Jacobi elliptic functions are also obtained. The reported solutions have the advantage of being expressed in explicit forms in contrast with other works where they are presented in implicit forms.

***Key words*:** soliton solutions, bright solitons, dark solitons, nonlinear optical materials, Lie symmetry method; Jacobi elliptic functions.

1. INTRODUCTION

The study of solitons that propagate stably in optical fibers and other optical materials is a very fascinating area of research in nonlinear optics and photonics. Optical solitons are the ideal bits of information and form the basis of data transmission across transcontinental and transoceanic distances [1–20]. The theoretical and experimental study of solitons in a plethora of optical materials is a promising lead in this direction.

In this paper, we study the propagation of Raman-type solitons through an optical material that is governed by the generalized nonlinear Schrödinger's equation given in Ref. [3]:

$$\begin{aligned} &\mathrm{i}q_t + aq_{xx} + (c_1|q|^2 + c_2|q|^4 + c_3|q|^6)q = \mathrm{i}\alpha q_x + \mathrm{i}\lambda(|q|^2 q)_x + \mathrm{i}\nu(|q|^2)_x q + \\ &+\theta_1(|q|^2 q)_{xx} + \theta_2|q|^2 q_{xx} + \theta_3 q^2 q^*_{xx}\ , \end{aligned} \tag{1}$$

where $q(x,t)$ represents the complex-valued wave function. The independent variables x and t represent spatial and temporal variables, respectively. The constant a is the group velocity dispersion. The coefficients c_j and θ_j for $j = 1, 2, 3$ are constants. And finally α represents the coefficient of inter-modal dispersion.

Bright implicit soliton solutions of Eq. (1) are obtained in Ref. [3] by using a direct integration method. Also, Eq. (1) has been investigated in Ref. [4] by using the dynamical system approach. The authors have obtained many exact solutions in parametric forms. These solutions were obtained when $\beta = \dfrac{6\theta_1}{3\theta_1 + \theta_2 + \theta_3} = 1, 2, -2, -3$. In this paper we obtain new explicit doubly periodic, bright, and dark soliton solutions of Eq. (1).

The travelling wave hypothesis given in Refs. [5–7] can be applied to carry out the analysis as follows:

$$q(x,t) = y(z)\mathrm{e}^{i\phi}\ , \tag{2}$$

where, $y(z)$ is the shape of the wave profile, z is defined as $z = x - vt$, and v is the speed of the wave. Here $\phi(x,t)$ is the phase component defined as $\phi = -\kappa x + \omega t + \theta$, where ω is the soliton frequency, κ represents the wave number, and θ is the phase constant.

Substituting Eq. (2) into Eq. (1), and then equating real and imaginary parts to zero, we obtain two equations. More specifically, the real part gives

$$[-c_1 - \kappa(-\lambda + \kappa\theta_1 + \kappa\theta_2 + \kappa\theta_3)]y^3 - c_2 y^5 - c_3 y^7 + y(\alpha\kappa + a\kappa^2 + \omega + 6\theta_1 y'^2) \\ -ay'' + (3\theta_1 + \theta_2 + \theta_3) y^2 y'' = 0 , \tag{3}$$

whereas the imaginary part gives

$$v = -2a\kappa - \alpha , \tag{4}$$

$$3\lambda + 2v = 2\kappa(3\theta_1 + \theta_2 - \theta_3) . \tag{5}$$

The following Sections will be organized as follows: In Section 2, Eq. (3) will be reduced to a first order ordinary differential equation (ODE) using the Lie point symmetry method. Explicit forms of some new bright and dark soliton solutions of the generalized nonlinear Schrödinger's equation Eq. (1) will be obtained in Section 3.

2. LIE POINT SYMMETRY METHOD

In this Section, we investigate Eq. (3) using the Lie point symmetry analysis [8-14]. The autonomous ODE (3) admits the Lie point symmetry generator [10]:

$$\Gamma = \frac{\partial}{\partial z} . \tag{6}$$

The canonical coordinates $(r, s(r))$ associated to the infinitesimal generator (6) are given by

$$r = y , \; s(r) = z , \tag{7}$$

which are prolonged to

$$\frac{ds}{dr} = \frac{1}{y'} , \; \frac{d^2 s}{dr^2} = -\frac{y''}{y'^3} . \tag{8}$$

Substituting Eq. (7) and Eq. (8) into Eq. (3) we get

$$\frac{\mathrm{d}^2 s}{dr^2} = \frac{r(\mathrm{d}s/\mathrm{d}r)}{a - r^2(3\theta_1 + \theta_2 + \theta_3)}[(-\alpha\kappa - a\kappa^2 - r^2\kappa\lambda - \omega + r^2 c_1 + r^4 c_2 + r^6 c_3 + r^2\kappa^2\theta_2 + r^2\kappa^2\theta_3) \\ \times(\mathrm{d}s/\mathrm{d}r)^2 + \theta_1(-6 + r^2\kappa^2(\mathrm{d}s/\mathrm{d}r)^2)]. \tag{9}$$

Let

$$g = \frac{\mathrm{d}s}{\mathrm{d}r} . \tag{10}$$

Hence, Eq. (9) becomes

$$\frac{\mathrm{d}g}{\mathrm{d}r} = \frac{rg}{a - r^2(3\theta_1 + \theta_2 + \theta_3)}[(-\alpha\kappa - a\kappa^2 - r^2\kappa\lambda - \omega + \\ + r^2(c_1 + r^2 c_2 + r^4 c_3) + r^2\kappa^2(\theta_1 + \theta_2 + \theta_3))g^2 - 6\theta_1]. \tag{11}$$

Equation (11) is the Bernoulli equation that can be solved easily to obtain

$$\frac{1}{g^2}=\frac{a^3c_3-(5\theta_1+\theta_2+\theta_3)\begin{bmatrix}-a^2c_2+(6\theta_1+\theta_2+\theta_3)(a\kappa\lambda-ac_1+(9\alpha\kappa+8a\kappa^2+9\omega)\theta_1\\+(\alpha\kappa+\omega)(\theta_2+\theta_3))\end{bmatrix}}{6\theta_1(5\theta_1+\theta_2+\theta_3)(6\theta_1+\theta_2+\theta_3)(9\theta_1+\theta_2+\theta_3)}+$$
$$+\frac{a^2c_3+(5\theta_1+\theta_2+\theta_3)[ac_2+(6\theta_1+\theta_2+\theta_3)(c_1+\kappa(-\lambda+\kappa(\theta_1+\theta_2+\theta_3)))]}{(5\theta_1+\theta_2+\theta_3)(6\theta_1+\theta_2+\theta_3)(9\theta_1+\theta_2+\theta_3)}r^2+ \tag{12}$$
$$+\frac{ac_3+c_2(5\theta_1+\theta_2+\theta_3)}{2(5\theta_1+\theta_2+\theta_3)(6\theta_1+\theta_2+\theta_3)}r^4+\frac{c_3}{3(5\theta_1+\theta_2+\theta_3)}r^6+c_4[-a+r^2(3\theta_1+\theta_2+\theta_3)]^{-6\theta_1/(3\theta_1+\theta_2+\theta_3)}.$$

Using Eq. (7), Eq. (8), and Eq. (10), then Eq. (12) becomes

$$y'^2=\frac{a^3c_3-(5\theta_1+\theta_2+\theta_3)\begin{bmatrix}-a^2c_2+(6\theta_1+\theta_2+\theta_3)(a\kappa\lambda-ac_1+(9\alpha\kappa+8a\kappa^2+9\omega)\theta_1\\+(\alpha\kappa+\omega)(\theta_2+\theta_3))\end{bmatrix}}{6\theta_1(5\theta_1+\theta_2+\theta_3)(6\theta_1+\theta_2+\theta_3)(9\theta_1+\theta_2+\theta_3)}+$$
$$+\frac{a^2c_3+(5\theta_1+\theta_2+\theta_3)[ac_2+(6\theta_1+\theta_2+\theta_3)(c_1+\kappa(-\lambda+\kappa(\theta_1+\theta_2+\theta_3)))]}{(5\theta_1+\theta_2+\theta_3)(6\theta_1+\theta_2+\theta_3)(9\theta_1+\theta_2+\theta_3)}y^2+ \tag{13}$$
$$+\frac{ac_3+c_2(5\theta_1+\theta_2+\theta_3)}{2(5\theta_1+\theta_2+\theta_3)(6\theta_1+\theta_2+\theta_3)}y^4+\frac{c_3}{3(5\theta_1+\theta_2+\theta_3)}y^6+c_4[-a+y^2(3\theta_1+\theta_2+\theta_3)]^{-6\theta_1/(3\theta_1+\theta_2+\theta_3)}.$$

Without loss of generality we can put $c_4=0$ as it is a constant of integration.

Equation (13) has many solutions but we are interested in soliton solutions as will appear in the next Section.

3. FAMILIES OF SOLITON SOLUTIONS

Let

$$y^2=u+b. \tag{14}$$

Substituting Eq. (14) into Eq. (13), we get

$$u'^2=A+Bu+Cu^2+Du^3+Eu^4, \tag{15}$$

where

$$A=\frac{2b\begin{bmatrix}c_3(a^3+108b^3\theta_1^3+3b^2\theta_1^2(9a+10b\theta_2+10b\theta_3)+b\theta_1(6a^2+2b^2\theta_2^2+3ab\theta_3+2b^2\theta_3^2\\+b\theta_2(3a+4b\theta_3)))+(5\theta_1+\theta_2+\theta_3)(c_2(a^2+27b^2\theta_1^2+3b\theta_1(2a+b\theta_2+b\theta_3))\\-(6\theta_1+\theta_2+\theta_3)(a\kappa\lambda-6b\kappa^2\theta_1^2-c_1(a+6b\theta_1)+\alpha\kappa\theta_2+\omega\theta_2+\alpha\kappa\theta_3+\omega\theta_3)\\+\theta_1(9\alpha\kappa+8a\kappa^2+6b\kappa\lambda+9\omega-6b\kappa^2\theta_2-6b\kappa^2\theta_3)))\end{bmatrix}}{3\theta_1(5\theta_1+\theta_2+\theta_3)(6\theta_1+\theta_2+\theta_3)(9\theta_1+\theta_2+\theta_3)}, \tag{16}$$

$$B=\frac{2\begin{bmatrix}c_3(a^3+432b^3\theta_1^3+3b^2\theta_1^2(27a+40b\theta_2+40b\theta_3)+b\theta_1(12a^2+8b^2\theta_2^2+9ab\theta_3\\+8b^2\theta_3^2+b\theta_2(9a+16b\theta_3)))+(5\theta_1+\theta_2+\theta_3)(c_2(a^2+81b^2\theta_1^2+3b\theta_1(4a+3b\theta_2+3b\theta_3))\\-(6\theta_1+\theta_2+\theta_3)(a\kappa\lambda-12b\kappa^2\theta_1^2-c_1(a+12b\theta_1)+\alpha\kappa\theta_2+\omega\theta_2+\alpha\kappa\theta_3+\omega\theta_3\\+\theta_1(9\alpha\kappa+8a\kappa^2+12b\kappa\lambda+9\omega-12b\kappa^2\theta_2-12b\kappa^2\theta_3)))\end{bmatrix}}{3\theta_1(5\theta_1+\theta_2+\theta_3)(6\theta_1+\theta_2+\theta_3)(9\theta_1+\theta_2+\theta_3)} \tag{17}$$

$$C=\frac{2\left[\begin{array}{l}c_3(2a^2+216b^2\theta_1^2+4b^2\theta_2^2+3ab\theta_3+4b^2\theta_3^2+b\theta_2(3a+8b\theta_3)+3b\theta_1(9a+20b\theta_2+20b\theta_3))\\+(5\theta_1+\theta_2+\theta_3)(c_2(2a+27b\theta_1+3b\theta_2+3b\theta_3)\\+2(6\theta_1+\theta_2+\theta_3)(c_1+\kappa(-\lambda+\kappa\theta_1+\kappa\theta_2+\kappa\theta_3)))\end{array}\right]}{(5\theta_1+\theta_2+\theta_3)(6\theta_1+\theta_2+\theta_3)(9\theta_1+\theta_2+\theta_3)} \tag{18}$$

$$D=\frac{2[3c_2(5\theta_1+\theta_2+\theta_3)+c_3(3a+48b\theta_1+8b\theta_2+8b\theta_3)]}{3(5\theta_1+\theta_2+\theta_3)(6\theta_1+\theta_2+\theta_3)}, \tag{19}$$

$$E=\frac{4c_3}{3(5\theta_1+\theta_2+\theta_3)}. \tag{20}$$

The explicit forms of bright and dark soliton solutions of Eq. (15) depend on its parameters. This issue will be discussed in the following two cases.

Case 1. When $b=0$, Eq. (15) has the following form

$$\begin{aligned}u'^2=&\frac{2a^3c_3-2(5\theta_1+\theta_2+\theta_3)\left[\begin{array}{l}-a^2c_2+(6\theta_1+\theta_2+\theta_3)(a\kappa\lambda-ac_1+(9\alpha\kappa+8a\kappa^2+9\omega)\theta_1\\+(\alpha\kappa+\omega)(\theta_2+\theta_3))\end{array}\right]}{3\theta_1(5\theta_1+\theta_2+\theta_3)(6\theta_1+\theta_2+\theta_3)(9\theta_1+\theta_2+\theta_3)}u+\\&+\frac{4[a^2c_3+(5\theta_1+\theta_2+\theta_3)(ac_2+(6\theta_1+\theta_2+\theta_3)(c_1+\kappa(-\lambda+\kappa(\theta_1+\theta_2+\theta_3)))]}{(5\theta_1+\theta_2+\theta_3)(6\theta_1+\theta_2+\theta_3)(9\theta_1+\theta_2+\theta_3)}u^2+\\&+\frac{2[ac_3+c_2(5\theta_1+\theta_2+\theta_3)]}{(5\theta_1+\theta_2+\theta_3)(6\theta_1+\theta_2+\theta_3)}u^3+\frac{4c_3}{3(5\theta_1+\theta_2+\theta_3)}u^4.\end{aligned} \tag{21}$$

Equation (21) has many solutions mentioned in Appendix A in Ref. [5], we choose from them the following explicit solution

$$u=\frac{1}{1+cn^2(z,m)}, \tag{22}$$

where $cn(z,m)$ is the Jacobi elliptic cosine function with modulus m. Solution (22) is constrained by

$$c_1=-6a+16am^2+\kappa\lambda+(-9+45m^2-\kappa^2)\theta_1+(-1+5m^2-\kappa^2)\theta_2-\theta_3+5m^2\theta_3-\kappa^2\theta_3, \tag{23}$$

$$c_2=-2[-3a+6am^2+6(-3+8m^2)\theta_1+(-3+8m^2)\theta_2-3\theta_3+8m^2\theta_3], \tag{24}$$

$$c_3=6(-1+2m^2)(5\theta_1+\theta_2+\theta_3), \tag{25}$$

$$\omega=-\alpha\kappa+a(-1+5m^2-\kappa^2)+6m^2\theta_1. \tag{26}$$

Substituting Eq. (22) into Eq. (14), we get

$$y=\frac{1}{\sqrt{1+cn^2(z,m)}}. \tag{27}$$

When $m=1$, the solution (27) is constrained by

$$c_1=10a+\kappa\lambda-(-36+\kappa^2)\theta_1-(-4+\kappa^2)\theta_2+4\theta_3-\kappa^2\theta_3, \tag{28}$$

$$c_2 = -2(3a + 30\theta_1 + 5\theta_2 + 5\theta_3), \tag{29}$$

$$c_3 = 6(5\theta_1 + \theta_2 + \theta_3), \tag{30}$$

$$\omega = -\alpha\kappa - a(-4 + \kappa^2) + 6\theta_1, \tag{31}$$

and the solution (27) degenerates to the following dark soliton solution

$$y = \frac{1}{\sqrt{1 + \operatorname{sech}^2 z}}. \tag{32}$$

The dark soliton solution (32) is shown in Fig. 1.

Case 2. When $b \neq 0$, Eq. (15) has many solutions mentioned in Table A.1 in Ref. [6] , we choose from them the following solution

$$u = cn(z, m) + dn(z, m), \tag{33}$$

where $cn(z, m)$ and $dn(z, m)$ are Jacobi elliptic functions with modulus m . Solution (33) is constrained by

$$c_1 = -\frac{2a(1+m) - 4\kappa\lambda + (9 + 27m + 9m^2 + 4\kappa^2)\theta_1}{4} - \frac{(1 + 3m + m^2 + 4\kappa^2)\theta_2 + (1 + 3m + m^2 + 4\kappa^2)\theta_3}{4}, \tag{34}$$

$$c_2 = \frac{3a + 48(1+m)\theta_1 + 8(1+m)\theta_2 + 8(1+m)\theta_3}{16}, \tag{35}$$

$$c_3 = -\frac{3(5\theta_1 + \theta_2 + \theta_3)}{16}, \tag{36}$$

$$\omega = \frac{-4\alpha\kappa - a(1 + 3m + m^2 + 4\kappa^2) - 12m(1+m)\theta_1}{4}, \tag{37}$$

$$b = 1 + m. \tag{38}$$

Substituting Eq. (33) and Eqs. (34)-(38) into Eq. (14) we get

$$y = \sqrt{cn(z, m) + dn(z, m) + 1 + m}. \tag{39}$$

When $m = 1$, the solution (33) is constrained by

$$c_1 = \frac{-4a + 4\kappa\lambda - (45 + 4\kappa^2)\theta_1 - (5 + 4\kappa^2)\theta_2 - 5\theta_3 - 4\kappa^2\theta_3}{4}, \tag{40}$$

$$c_2 = \frac{3a}{16} + 6\theta_1 + \theta_2 + \theta_3, \tag{41}$$

$$c_3 = -\frac{3(5\theta_1 + \theta_2 + \theta_3)}{16}, \tag{42}$$

$$\omega = -\alpha\kappa - \frac{a(5 + 4\kappa^2)}{4} - 6\theta_1, \tag{43}$$

$$b = 2 . \tag{44}$$

and solution (39) degenerates to the following bright soliton solution

$$y = \sqrt{2\,\mathrm{sech}\, z + 2} . \tag{45}$$

The bright soliton solution (45) is shown in Fig. 2.

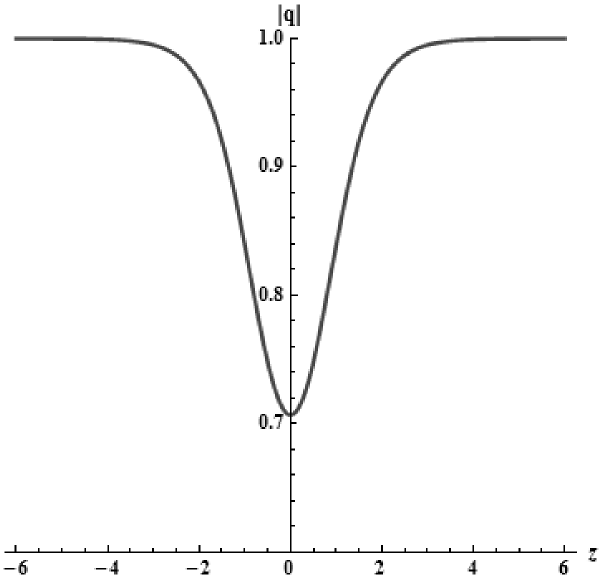

Fig. 1 – The dark soliton solution (32).

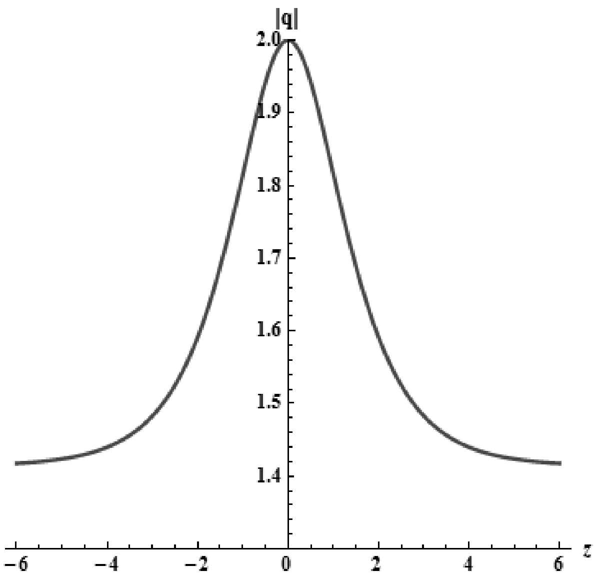

Fig. 2 – The bright soliton solution (45).

4. CONCLUSIONS

New exact solutions of the generalized nonlinear Schrödinger's equation (1) are reported in explicit form. In particular, explicit forms of both dark and bright soliton solutions of equation (1) and the associated nonlinear differential equation (3) are obtained using the Lie symmetry method. To the best of authors' knowledge, this is the first time to express explicitly these solutions for this physical model. Also, some new doubly periodic solutions of equations (1) and (3) are explicitly obtained in the form of Jacobi elliptic functions.

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