THE PUBLISHING HOUSE
OF THE ROMANIAN ACADEMY

PROCEEDINGS OF THE ROMANIAN ACADEMY, Series A,
Volume 19, Number 3/2018, pp. 463–471

TECHNICAL SCIENCES

COMPUTING THE GROUND FAULT CURRENT DISTRIBUTION ON ELECTRICAL POWER TRANSMISSION NETWORKS

Maria VINTAN

"Lucian Blaga" University of Sibiu
E-mail: maria.vintan@ulbsibiu.ro

Abstract. This paper evaluates the ground fault current distribution in an effectively grounded power network. The methods described here provide useful results for ground fault at any tower along the single and double circuit 3-phase transmission lines, with one ground wire, non-uniform span lengths and non-uniform tower resistances. The ground fault current distribution in an effectively grounded power network is affected by various factors. The effects of some of these factors on the ground fault current distribution are carefully analyzed. There are presented some useful qualitative and quantitative results obtained through a dedicated developed MATLAB 7.0 program.

Key words: power system faults, transmission lines, ground fault current.

1. INTRODUCTION

When a ground fault occurs on an overhead transmission line in a power network with grounded neutral, the fault current returns to the grounded neutral through the ground wires, towers and ground return paths. The estimation of the ground fault current distribution is an important step to design a safe substation grounding grid and the associated line's grounding systems and – especially in a single circuit transmission line case – had been undertaken by many researches, and numerous analytical methods have been published [2, 3, 5–9]. In our previous works there were presented analytical methods in order to determine the ground fault current distribution in effectively grounded power networks, for a ground fault located anywhere along the transmission line [10–12]. In all these cases it was assumed uniform spans lengths and towers resistances. But usually these parameters are not the same on the entirely transmission line. Sebo [8] developed an approach which could be used to compute the ground fault current distribution in ground return circuits for a single circuit transmission line, considering non-uniform span lengths and non-uniform tower resistances. In this paper, the method introduced by Sebo [8] has been generalized in order to consider the case of a double circuit transmission line with one ground wire, considering non-uniform spans lengths and non-uniform towers resistances. Also a complex MATLAB computer program based on this extended method has been developed. The effects of towers footing resistances, type and material of the overhead ground wires, number of spans of power lines, soil resistivity on the ground fault current distribution is analyzed through this software. The calculation method is based on the following assumptions: the impedances are lumped parameters in each span of the transmission line; the line's capacitances, the contact resistance between the tower and the ground wire, the contact resistance between the tower and the faulted phase, are all neglected; the network is considered linear in the sinusoidal steady-state.

2. FAULTS ON A SINGLE CIRCUIT 3-PHASE TRANSMISSION LINES WITH ONE GROUND WIRE

In this section it is considered a network containing a source station *S* that supplies a distribution station *D* through an overhead transmission line. It is considered a single circuit transmission line with one ground wire connected to the ground at every tower of the line, each transmission tower having its own grounding electrode or grid. When the fault appears, part of the ground fault current will get to the ground through the faulted tower, and the rest of the fault current will get diverted to the ground wire and other towers. Fig. 1 presents the connection of the ground wire connected to earth through transmission towers and the ground fault current distribution, when a single line-to-ground fault appears at any tower and the fault is supplied from one side only. Also it is considered that the span lengths $l_{(k)}$ and tower resistances $Z_{t(k)}$ are non-uniform (k is the number of considered span).

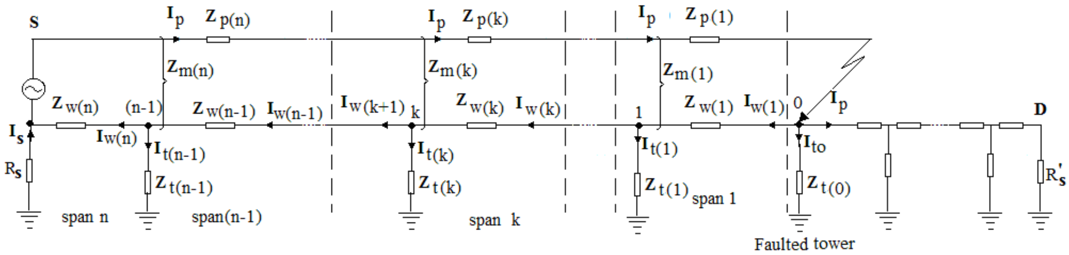

Fig. 1 – Ground fault current distribution on an single circuit transmission line.

The self-impedance of the ground wire connected between two grounded towers, called the self-impedance per span, was noted with $Z_{w(k)}$. The self-impedance of the faulted phase conductor per span was noted with $Z_p(k)$; $Z_m(k)$ represents the mutual-impedance between the ground wire and the faulted phase conductor, per span. The source and the distribution stations grounding systems resistances are R_s and respectively R'_s. When the fault is supplied from one side only, the equivalent circuit presented in Fig. 2 could be constructed, where Z_e represents the resultant impedance of the ladder network extended beyond the fault, containing the ground wire sections and tower footing resistances.

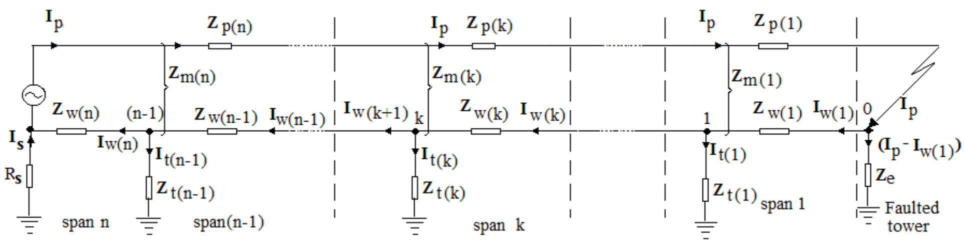

Fig. 2 – Ground fault current distribution equivalent circuit.

Considering span k presented in Fig. 3 between two consecutive towers, the following expressions, which relate the left-side quantities $U_{p(k+1)}$, $U_{w(k+1)}$, $I_{w(k+1)}$ and $I_{p(k+1)}$ of the span with its right-side quantities $U_{p(k)}$, $U_{w(k)}$, $I_{w(k)}$ and $I_{p(k)}$, can be written [8]:

$$\begin{cases} I_{w(k+1)} = I_{w(k)} - I_{t(k)} = (1+\dfrac{Z_{w(k)}}{Z_{t(k)}})I_{wk)} - \dfrac{Z_{m(k)}}{Z_{t(k)}} \cdot I_p - \dfrac{1}{Z_{t(k)}} U_{w(k)} \\ U_{p(k+1)} = U_{p(k)} + Z_{p(k)} \cdot I_p - Z_{m(k)} I_{w(k)} \\ U_{w(k+1)} = U_{w(k)} + Z_{m(k)} \cdot I_p - Z_{w(k)} I_{w(k)} \\ I_{p(k+1)} = I_{p(k)} = I_p \end{cases} \tag{1}$$

Expressions (1) could be written in a matrix form, as follows:

$$[M_{(k+1)}]=[E_{(k)}]\cdot[N_{(k)}] \tag{2}$$

where:

$$M_{(k+1)}=\begin{bmatrix}U_{p(k+1)}\\ I_p\\ U_{w(k+1)}\\ I_{w(k+1)}\end{bmatrix},\ E_{(k)}=\begin{bmatrix}1 & Z_{p(k)} & 0 & -Z_{m(k)}\\ 0 & 1 & 0 & 0\\ 0 & Z_{m(k)} & 1 & -Z_{m(k)}\\ 0 & -\dfrac{Z_{m(k)}}{Z_{t(k)}} & -\dfrac{1}{Z_{t(k)}} & (1+\dfrac{Z_{w(k)}}{Z_{t(k)}})\end{bmatrix},\ N_{(k)}=\begin{bmatrix}U_{p(k)}\\ I_p\\ U_{w(k)}\\ I_{w(k)}\end{bmatrix} \tag{3}$$

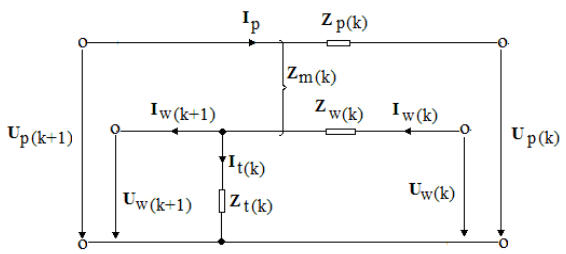

Fig. 3 – Span k between two towers.

In the same way, the right-side quantities of the span k could be expressed as a function of the left-side quantities of the same span:

$$[N_{(k)}]=[E_{(k)}^{-1}]\cdot[M_{(k+1)}] \tag{4}$$

where $[E_{(k)}^{-1}]$ is inverse matrix of $[E_{(k)}]$. Recurrently applying expression (3) for all the transmission line spans, it will be obtained:

$$\begin{gathered}[M_{(2)}]=[E_{(1)}]\cdot[N_{(1)}];\quad [M_{(3)}]=[E_{(2)}]\cdot[N_{(2)}]=[E_{(2)}]\cdot[E_{(1)}]\cdot[N_{(1)}];\ ....\\ [M_{(n+1)}]=[E_{(n)}]\cdot[E_{(n-1)}]\cdot...\cdot[E_{(1)}]\cdot[N_{(1)}]\overset{NOTATION}{=}[E^{(n)}]\cdot[N_{(1)}].\end{gathered} \tag{5}$$

Next, the following boundary conditions are necessary. At the faulted tower, on the right side of the first span, the faulted phase conductor and the ground wire are connected by the phase-to-ground fault, thus $U_{p(1)}=U_{w(1)}=(I_p-I_{w(1)})\cdot Z_e$. At the left terminal $I_s=I_p-I_{w(n)}$. Therefore, the column vector $[N_{(1)}]$ and the column vector $[M_{(n)}]$ are obtained. The expression (2) is applied to the span (n-1) which contains the last tower of the transmission line (see Fig. 2) and the following matrix equation can be written:

$$[M_{(n-1)}]=[E^{(n-1)}]\cdot[N_{(1)}];\qquad [M_{(n-1)}]=\begin{bmatrix}U_{p(n)}\\ I_p\\ (I_{w(n-1)}-I_{w(n)})\cdot Z_{t(n-1)}\\ I_{w(n)}\end{bmatrix} \tag{6}$$

$[E^{(n-1)}]$ will be computed as in expression (5). Additionally, in order to gain the necessary number of equations, for the last span (span n) the next expressions could be written:

$$R_s\cdot I_s-I_{w(n)}\cdot Z_{w(n)}+I_p\cdot Z_{m(n)}+U_{w(n)}=0 \tag{7}$$

$$I_p = I_{w(n)} + I_s; \quad U_{w(n)} = I_{t(n-1)} \cdot Z_{t(n-1)}; \quad I_{t(n-1)} = I_{w(n-1)} - I_{w(n)}. \tag{8}$$

Taking into account relations (8), expression (7) becomes:

$$I_p \cdot (R_s + Z_{m(n)}) - I_{w(n)} \cdot (Z_{t(n-1)} + R_s + Z_{w(n)}) + I_{w(n-1)} \cdot Z_{t(n-1)} = 0. \tag{9}$$

Taking into account the boundary conditions for the case of the fault fed from one side, all the unknown quantities $U_{p(n)}$, I_p, $I_{w(1)}$, $I_{w(n-1)}$ and $I_{w(n)}$ can be computed from expressions (6) and (9). By choosing I_p as the given reference value, all the currents are obtained as per-unit complex values referred to the pure reference quantity of I_p [8]. In order to obtain all the ground wires currents, in every span, it is enough to observe that:

$$[M_{(n-2)}] = [E^{-1}] \cdot [M_{(n-1)}]. \tag{10}$$

Because $[M_{(n-1)}]$ is known from expressions (6), by proceeding toward the fault, all ground wires currents could now be determined.

3. FAULTS ON A DOUBLE CIRCUIT 3-PHASE TRANSMISSION LINE WITH ONE GROUND WIRE

In this section we generalize the method presented in the previous section for the case of a double circuit transmission line. As a convenient example it used the same network as above, but this time it is considered a double circuit transmission line with one ground wire, as it is presented in Fig. 4. With I'_{p2} and I''_{p2} were noted fault current components through the faulted line on the left, respectively on the right side of the fault; I_{p1} represents the current in the un-faulted line, and I_p is the total ground fault current.

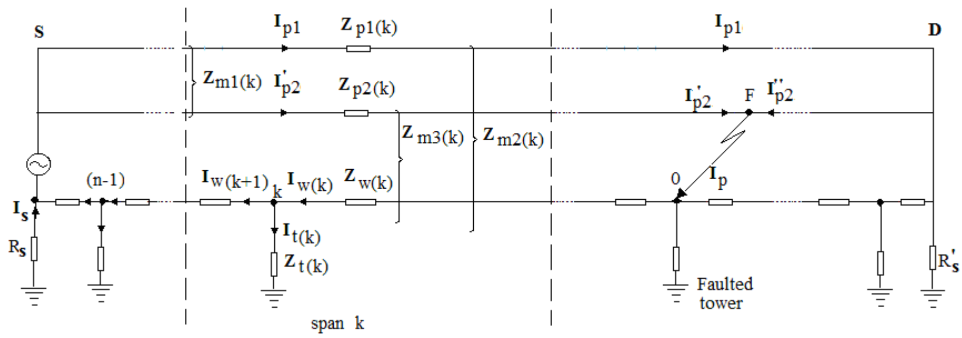

Fig. 4 – Ground fault current distribution on a double circuit transmission line.

$Z_{m1(k)}$ in Fig. 4 represents the mutual-impedance between the faulted phase conductor and the un-faulted phase conductor per span; $Z_{m2(k)}$ represents the mutual-impedance between the ground wire and the un-faulted phase conductor per span; $Z_{m3(k)}$ represents the mutual-impedance between the ground wire and the faulted phase conductor per span. Generally, if two parallel circuits are bused together at both ends of the line, both circuits will carry fault currents when a single line-to-ground fault appears on one of them. Due to the magnetic coupling between the ground wire and the phase conductors in the parallel circuits, there is an influence of the fault current flowing in these circuits on the magnitude of the ground return currents. In principle, any one of the external parallel conductors that carry the fault current can be replaced by two circuits, both grounded at the fault location [3]. The neutrals in distribution station are usually insulated. Zero sequence impedance z_S^0 of the system at the station *S*, respectively zero sequence impedance z_D^0 of the system at the station *D* fulfils this condition: $z_D^0 \gg z_S^0$ [6]. Due to this fact, the currents $I_{p2}^{"}$ and I_{p1}, in Fig. 4, are equal. As a consequence, in Fig. 5 is presented the equivalent circuit. Z_0 represents the zero sequence impedance of the transmission line from the fault location to the station *D* and Z_e represents the resultant impedance of the ladder network extended beyond the fault, containing the ground wire sections and tower footing resistances.

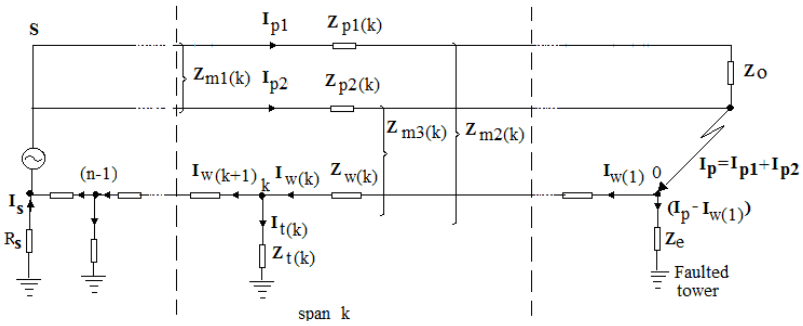

Fig. 5 – Equivalent circuit of a ground fault current distribution on a double circuit transmission line.

Considering span *k* between two towers (Fig. 5), as it was presented above, the following expressions which relate the left-side quantities of the span *k* with the right-side quantities can be written as:

$$\begin{cases} I_{w(k+1)} = (1+\dfrac{Z_{w(k)}}{Z_{t(k)}})I_{wk)} - \dfrac{Z_{m2(k)}}{Z_{t(k)}}\cdot I_{p1} - \dfrac{Z_{m3(k)}}{Z_{t(k)}}\cdot I_{p2} - \dfrac{1}{Z_{t(k)}}U_{w(k)} \\ U_{p1(k+1)} = U_{p1(k)} + Z_{p1(k)}\cdot I_{p1} + Z_{m1(k)}\cdot I_{p2} - Z_{m2(k)}I_{w(k)} \\ U_{p2(k+1)} = U_{p2(k)} + Z_{p2(k)}\cdot I_{p2} + Z_{m1(k)}\cdot I_{p1} - Z_{m3(k)}I_{w(k)} \\ U_{w(k+1)} = U_{w(k)} + Z_{m2(k)}\cdot I_{p1} + Z_{m3(k)}\cdot I_{p2} - Z_{w(k)}I_{w(k)} \\ I_{p1(k+1)} = I_{p1(k)} = I_{p1} \\ I_{p2(k+1)} = I_{p2(k)} = I_{p2} \end{cases} \tag{11}$$

As above, expressions (11) could be written in a matrix form:

$$[M_{(k+1)}] = [E_{(k)}]\cdot[N_{(k)}], \tag{12}$$

where:

$$M_{(k+1)} = \begin{bmatrix} U_{p1(k+1)} \\ U_{p2(k+1)} \\ I_{p1} \\ I_{p2} \\ U_{w(k+1)} \\ I_{w(k+1)} \end{bmatrix}, \quad E_{(k)} = \begin{bmatrix} 1 & 0 & Z_{p1(k)} & Z_{m1(k)} & 0 & -Z_{m2(k)} \\ 0 & 1 & Z_{m1(k)} & Z_{p2(k)} & 0 & -Z_{m3(k)} \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & Z_{m2(k)} & Z_{m3(k)} & 1 & -Z_{w(k)} \\ 0 & 0 & -\dfrac{Z_{m2(k)}}{Z_{t(k)}} & -\dfrac{Z_{m3(k)}}{Z_{t(k)}} & -\dfrac{1}{Z_{t(k)}} & (1+\dfrac{Z_{w(k)}}{Z_{t(k)}}) \end{bmatrix}, \quad N_{(k)} = \begin{bmatrix} U_{p1(k)} \\ U_{p2(k)} \\ I_{p1} \\ I_{p2} \\ U_{w(k)} \\ I_{w(k)} \end{bmatrix} \tag{13}$$

In order to determine the unknown quantities, the boundary conditions are necessary. Taking into account that at the faulted tower $U_{w1} = U_{p1} = (I_{p2} + I_{p1} - I_{w(1)}) \cdot Z_e$, the column vector $[N_{(1)}]$ for the fault tower and the column vector $[M_{(n-1)}]$ for the left terminal could be easily written. The expression (12) applied to the span (*n*–1), which contains the last tower of the transmission line, will result in the matrix equation given by (6). This time matrix $[E^{(n-1)}]$ will be compute considering expression from (17). Following the same method as above, all ground wires currents could now be determined.

4. VALIDATION CASES AND RESULTS

In order to illustrate and validate the theoretical approach outlined in the sections above, it is considered that the line which connects those two stations is a 110 kV double circuit transmission line with aluminum-steel (ACSR) 240/40 mm^2 phase conductors and one aluminum-steel 160/95 mm^2 ground wire (Fig. 6) [4]. Ground wire impedance per one span $Z_{w(k)}$ and the mutual impedances per one span are calculated for different values of the soil resistivity ρ with formulas based on Carson's theory of the ground return path [1]. Ground wire is in symmetrical position relative to the phase conductors of the two transmission line circuits, which means that these impedance per one span has the same value: $Z_{m2(k)} = Z_{m3(k)}$. The fault was assumed to occur on the phase which is the furthest from the ground wire, because the lowest coupling between the phase and the ground wire will produce the highest tower voltage. The total fault current was chosen as the reference value given, thus all the currents are obtained as per-unit complex values referred to the pure reference quantity of the total fault current.

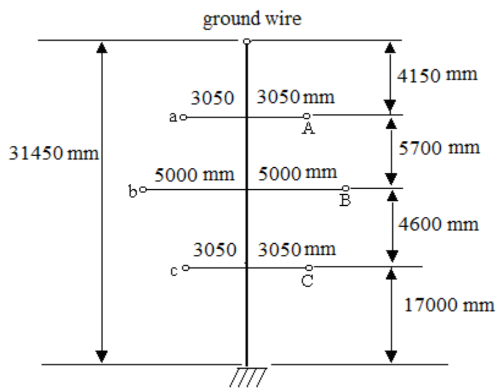

Fig. 6 – Disposition of the transmission line conductors.

Figure 7 presents the zero sequence impedance of the transmission line, considering the mutual coupling between the two circuits as a function of the soil resistivity. In the absence of mutual coupling the fault current will flow to the ground through a smaller number of towers than in the mutual coupling presence. As a consequence, the voltage rise of the faulted tower and also its neighbours' voltage rises will be higher in an artificial manner.

Figure 8 presents the zero sequence impedance of the transmission line as a function of the soil resistivity, for the case of an ACSR 160/95 mm^2 ground wire, respectively the case of a 95 mm^2 steel ground wire.

Figures 9 and 10 show the currents flowing in the ground wire in the case of a fault that appears at the 10-th tower of the line counted from the source station, as a function of the spans between the faulted tower and the source station for $R_s = 0.1\Omega$, respectively for $R_s = 1\Omega$, considering different values for the tower impedances. It was assumed that the line has a total number of 30 towers and tower impedances are uniform

($Z_t = 10\,\Omega$) and respectively non-uniform. In the last case (non-uniform impedances) it was assumed that the first 7 towers (counted from the faulted tower to the source station) have the same impedance $Z_t = 5\,\Omega$, and the last 3 towers (before the source station) have $Z_t = 10\,\Omega$. The resultant impedance Z_e of the ladder network extended beyond the fault is computed considering that the remaining 20 towers between the faulted tower and the distribution station were split in two equal categories. In the first one all of the towers have $Z_t = 5\,\Omega$, and in the second one all of them have $Z_t = 10\,\Omega$. It can be said that in the case of a relatively short transmission line, all towers will carry the fault current to the ground. In the case of relative long lines, the towers near the source station start collecting currents from ground as they are part of the station grounding systems.

Figure 11 shows the currents flowing in the ground wire in the case of a fault that appears at the 5-th tower of the line counted from the source station, for different values of the soil resistivity, considering the tower impedances uniform: $Z_t = 5\,\Omega$.

Figure 12 shows the currents flowing through the transmission line towers as a function of the tower impedances, in the case of a fault that appears at the 10-th tower of the line counted from the source station, considering a double circuit transmission line. It was assumed that the line has a total number of 30 towers and tower impedances are uniform ($Z_t = 10\,\Omega$ and $30\,\Omega$) and respectively non-uniform. In the last case (non-uniform impedances) it was assumed that first 7 towers have the same impedance $Z_t = 5\,\Omega$, and the last 3 tower have $Z_t = 10\Omega$. It was considered that $R_s = 0.1\Omega$.

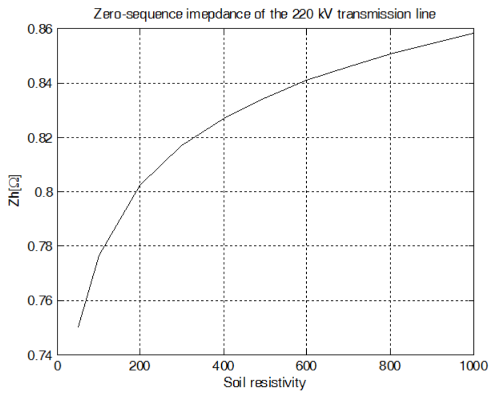

Fig. 7 – Zero sequence impedance of the transmission line, considering the mutual coupling between circuits.

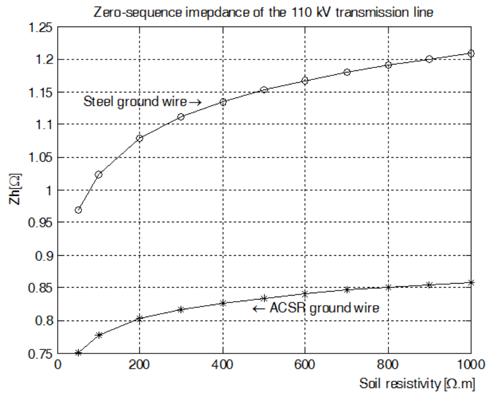

Fig. 8 – Zero sequence impedance of the 110 kV transmission line as a function of the soil resistivity.

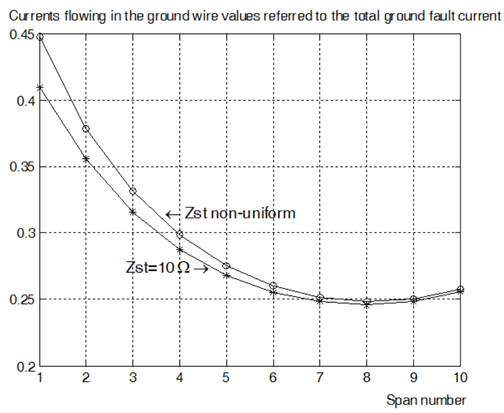

Fig. 9 – Currents flowing in ground wire, for $R_s = 0.1\,\Omega$.

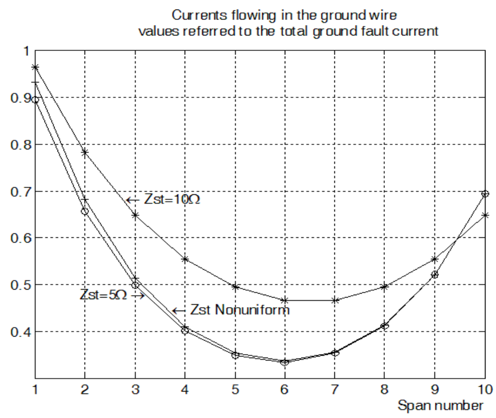

Fig. 10 – Currents flowing in ground wire, for $R_s = 1\,\Omega$.

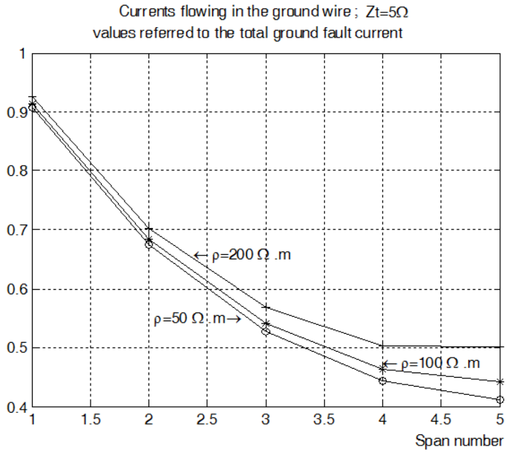

Fig. 11 – Currents flowing in the ground wire; $Z_t = 5\,\Omega$.

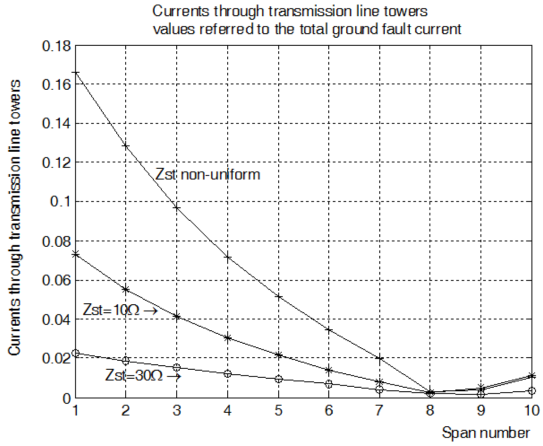

Fig. 12 – Currents through transmission line towers.

All the presented quantitative results are based on the theoretical approaches developed during the previous sections. In order to do this there were developed some complex numerical intensive programs written in Matlab 7.0 software frame, completely implementing the previous presented theoretical methods.

5. CONCLUSIONS

In order to evaluate the ground fault current distribution in substations, overhead ground wires and towers, a mathematical model inspired by Sebo's work [8] was described. This model is valid for a single 3-phase transmission line with one ground wire having non-uniform span lengths and non-uniform tower resistances, too. After this presentation we generalized Sebo's model for the case of a double circuit transmission line. Also a complex MATLAB computer program based on this extended method has been developed. The models were simulated on different realistic validation cases, generating useful results well correlated with other related works [5, 6]. The extended model has the advantages, in comparison with other available approaches, of being less complex and thus easily implemented and simulated. A laborious parametric analysis was done in order to study the effects of towers footing resistances, configuration and parameters of overhead ground wires and power conductors, number of spans of power lines, soil resistivity on the ground fault current distribution in substations, overhead ground wires and towers. The currents which return through ground wires were computed and examined for various realistic validation cases. From the above figures could be seen the strong influence of the soil resistivity on the ground fault currents distribution. When the soil resistivity has high values and the ground wire impedance has low values, the fraction of the fault current which returns to the source station trough the ground wire will also be high. The mutual impedance between the ground conductor and phase conductors reduces the total circuit impedance. Neglecting this mutual impedance the fault current would be significantly higher. Also, due to the magnetic coupling between the two circuits of the transmission line, there is an influence of the fault current flowing in these circuits on the magnitude of the ground return currents. The mutual coupling "keeps" the currents in ground wires and so they are returning to the source. Otherwise, these currents will play an important role in electromagnetic induction on neighboring circuits. The obtained results clearly show that ignoring the ground return currents may lead to grounding over-design.

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Received August 26, 2015