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CONSTRUCTAL DESIGN OF BRANCHED CONDUCTIVITY PATHWAYS INSERTED IN A TRAPEZOIDAL BODY: A NUMERICAL INVESTIGATION OF THE EFFECT OF BODY SHAPE ON OPTIMAL PATHWAY STRUCTURE

Tadeu Mendonca FAGUNDES[*], Neda YAGHOOBIAN[*], Luiz Alberto Oliveira ROCHA[**], Juan Carlos ORDONEZ[*]

[*] Florida State University, Department of Mechanical Engineering, Tallahassee, FL, 32310, United States
[**] Universidade Federal do Rio Grande do Sul,Department of Mechanical Engineering,
Rua Sarmento Leite, 425, Porto Alegre, RS, 90.050–170, Brazil

Corresponding author: Tadeu Mendonca FAGUNDES E-mail: tmf16d@my.fsu.edu

Abstract. This paper presents the application of constructal design to the geometry of a morphing branched conductivity pathway inserted in a trapezoidal body with a constant heat transfer rate at the base. The objective is to study the effect of conductivity ratio of the materials, and the strength of the convective cooling on the structure of the embedded pathway, whose geometrical features are deduced through constructal design. It is shown that the body global thermal resistance, represented by the maximum dimensionless temperature can be minimized by means of a constrained geometric optimization, in which the total area of the body remains constant. Five degrees of freedom were identified along the lines of constructal theory; three related to the pathway geometry and two related to the body geometry. The exploration of the search space was conducted via optimization by a genetic algorithm. The results indicate that when the conductivity pathway shape is free to morph, the thermal performance is improved according to the constructal principle of optimal distribution of imperfection. In addition, two different behaviours in the heat transfer process are identified: one for small values of heat transfer coefficient and other for high values. It is reported that the optimal pathway geometry changes under different conditions, with the combined system always aiming for the configuration that allows more ease for the currents within it.

Key words: Conductivity pathways, Constructal design, Genetic algorithm, Heat transfer, Trapezoidal body.

1. INTRODUCTION

The Constructal law dictates the universal phenomenon of design evolution for both animate and inanimate systems. Along with the first and second laws, the Constructal law elevates thermodynamics to a science of systems configuration [1–3] that finds application in the search for shapes and structures that facilitate flow [4–7], applicable to engineering systems.

Fins with relatively simple shapes (e.g., T-shape [8], Y-shape [9], and TY-shape [10]) have been widely studied in the context of Constructal theory. Moreover, recent studies show the importance of the basement body shape on the optimal fin configuration [11, 12]. There have also been significant studies on conduction pathways [13] with some notable being the X-shaped [14], phi and psi shaped [15] and V-shaped pathways [16]. In counterpart, for fluid flow distribution, similar geometries have been explored [17, 18].

In the present work, heat flow in a branched conductivity pathway inserted in a trapezoidal body is numerically investigated. The objective is to minimize the global thermal resistance of the system by allowing changes in the geometry of the inserted pathway in response to different conductivity ratios of the materials and the strength of the convective cooling. The mathematical and numerical models used in this study are presented in Sections 2 and 3, respectively, followed by results and conclusion in Sections 4 and 5.

2. MATHEMATICAL MODEL

Figure 1 illustrates the two-dimensional (2D) body under consideration. A branched high conductivity pathway (shaded) is inserted within a trapezoidal body of height H (m), and lower and upper base lengths of L and L' (m), respectively. The high conductivity pathway is symmetrically located in the trapezoidal body and it receives a constant heat rate q_1 (W) at the base of the pathway, which has the length of D_0 (m).

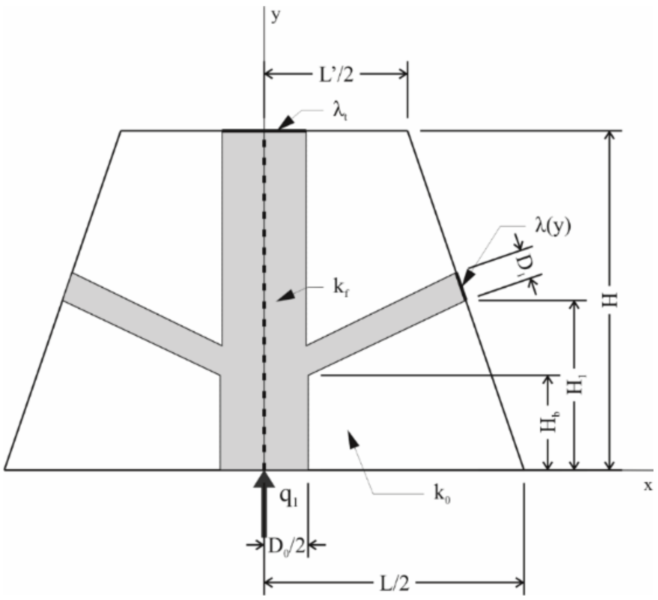

Fig. 1 – Sketch of the branched conductivity pathway inserted in a two-dimensional trapezoidal body.

Thermal conductivity of the pathway and the body are indicated by k_f and k_0 (W/(m K)), respectively. The high conductivity pathway is divided into three branches at height H_b (m). The extremities of these branches are subject to convective cooling at their endpoints, thus they work as heat sinks for the system. q_1 (W) and the convective coefficients h (W/(m^2 K)) at the end of the branches are known. The heat equation, Eq. (1), applies to both the pathway and the trapezoidal body:

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+q'''= =\rho c_p\frac{\partial T}{\partial t}, \tag{1}$$

where ρ (kg/m³) is the density, c_p (J/ (kg K)) is the specific heat, k (W/(m K)) is the thermal conductivity, q''' (W/m^3) represents the volumetric heat generation, T (K) is the temperature, and x (horizontal), y (vertical) and z represent the Cartesian coordinate of the system. It is considered that the problem is 2D, with the third dimension represented by a length W (m), being large compared to H, L, and L' ($\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)=0$). Under steady state, no internal heat generation, and uniform material properties, Eq. (1) becomes

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}=0. \tag{2}$$

The boundary condition at $-\frac{D_0}{2}\le x\le\frac{D_0}{2}$ and $y=0$ is $-k_f D_0 W\frac{\partial T}{\partial n}=q_1$, at the endpoints of the top and lateral branches are $-k_f\frac{\partial T}{\partial n}=h_T(T-T_{amb})$ and $-k_f\frac{\partial T}{\partial n}=h_L(T-T_{amb})$, respectively, and on the other surfaces is $\frac{\partial T}{\partial n}=0$ (adiabatic). T_{amb}, n, h_T and h_L represent the ambient temperature, the normal direction to the surface, and the top and lateral branch heat transfer coefficients, respectively. The objective of this analysis is to determine geometric parameters of the conductivity pathway (H_b, H_1, D_0 and D_1) for a given area of the pathway (A_p) and trapezoidal body (A), conductivity ratios (k_f/k_0), and convective heat transfer coefficients (h_T and h_L) that lead to minimum global thermal resistance. Based on the Constructal Design approach, the total area (trapezoidal body), and the branched pathway area (A_p) are the constraints. In addition, the area fraction is defined by $\phi = A_p/A$.

The geometrical parameters shown in Fig. 1 and the governing equations are represented in a dimensionless form using Eqs. (3), (4) and (5)

$$\tilde{a}=\frac{a}{A^{1/2}}, \tag{3}$$

$$\theta = \frac{T - T_0}{q_1 / (k_f W)}, \tag{4}$$

$$\tilde{k} = \frac{k_f}{k_0}, \tag{5}$$

where a represents any geometric parameter given in (m).

Thus, Eq. (2) can be rewritten in its dimensionless form as shown in Eq. (6) with its normalized boundary conditions as $-\frac{\partial \theta}{\partial \tilde{y}} = \frac{1}{\tilde{D}_0}$ at $-\frac{\tilde{D}_0}{2} \le \tilde{x} \le \frac{\tilde{D}_0}{2}$ and $\tilde{y} = 0$, $\frac{\partial \theta}{\partial \tilde{n}} = -\lambda_T \theta$ at the endpoint of the top branch, $\frac{\partial \theta}{\partial \tilde{n}} = -\lambda_L \theta$ at the endpoint of the lateral branches, and $\frac{\partial \theta}{\partial \tilde{n}} = 0$ on the other surfaces (adiabatic), where $\tilde{n}$ is the dimensionless direction normal to the boundary surface, and λ_N is defined by Eq. (7), similar to [2].

$$\frac{\partial^2 \theta}{\partial \tilde{x}^2} + \frac{\partial^2 \theta}{\partial \tilde{y}^2} = 0, \tag{6}$$

$$\lambda_N = \frac{h_N A^{1/2}}{k_f}. \tag{7}$$

Equation (6) with the provided boundary conditions can be solved numerically to obtain the temperature field.

In this work, the endpoints of the lateral branches are subject to convective cooling that is achieved by linking the lateral heat transfer coefficients to the top heat transfer coefficient through a linear approximation:

$$\lambda_L(y) = \lambda_T(y/H). \tag{8}$$

The maximum dimensionless temperature, θ_{max}, also represents the global thermal resistance of the configuration

$$\theta_{\max} = \frac{T_{\max} - T_0}{q_1 / k_f W}. \tag{9}$$

For the purposes of this work, five non-dimensional degrees of freedom were chosen: *H/L, L'/L,* H_b/H, H_1/H and D_1/D_0. For a given ϕ and these degrees of freedom the system geometry can be fully defined.

3. NUMERICAL PROCEDURES

The solution was obtained using a finite element approach with non-uniform, triangular elements in both x and y directions, implemented in MATLAB using the PDETool toolbox. This method has been verified in previous studies of cavities and fins [12, 16]. The appropriate mesh size was determined using successive refinements. In each refinement, the number of elements were increased four times until the criterion $|(\theta^j_{max} - \theta^{j+1}_{max})/\theta^j_{max}| < 1\times10^{-4}$ was satisfied. Here, θ^j_{max} and θ^{j+1}_{max} represent the maximum dimensionless temperatures calculated using the current and the next mesh sizes, respectively. The results presented in the following section were obtained using a mesh composed of approximately 28,000 triangular elements, in which the previous criterion was fulfilled. For the geometric optimization presented in this paper, a binary, single-objective, elitist genetic algorithm was employed, with a mutation rate of 10% and a crossover probability of 80%. The application of genetic algorithm in heat transfer problems has been extensively reviewed [19] and, for the sake of brevity, will not be detailed here.

4. RESULTS

Figure 2 illustrates the optimal geometric parameters of the conductive pathway and the minimized maximum dimensionless temperature $(\theta_{max})_m$ change with respect to the change in the conductivity ratio ($\tilde{k}$).

As expected, $(\theta_{max})_m$ decreases monotonically with the increase of $\tilde{k}$, since the higher conductivity material exhibits lower resistance to heat flow. Moreover, it is seen that the optimal lateral sink height $(H_1)_o$ of the pathway decreases as the optimal branching point $(H_b)_o$ decreases, and after the latter reach its minimum value, $(H_1)_o$ changes its behavior and increases. The subscript *'o'* denotes the optimal value for the specified parameters, while *'m'* denotes minimized and it is used on the objective function. Finally, as $\tilde{k}$ increases, the lateral branches increase their width (D_1) becoming more active in the heat removal, but the central branch width (D_0) becomes smaller. This is because the system adapts itself under different conditions, always seeking a way to give more and more access to its currents, in this case, the heat flow. These behaviors are further illustrated in Fig. 3.

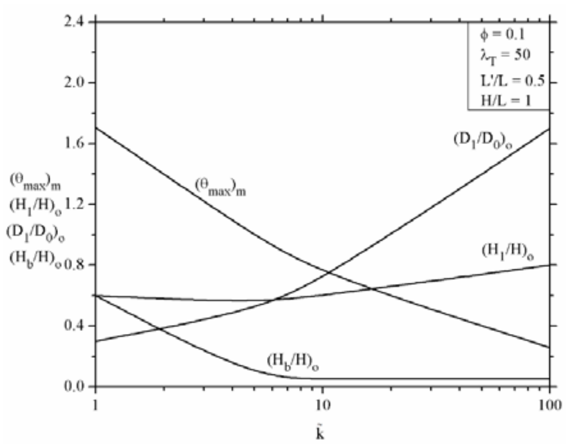

Fig. 2 – Influence of *k* over the optimal parameters of the conductive pathway.

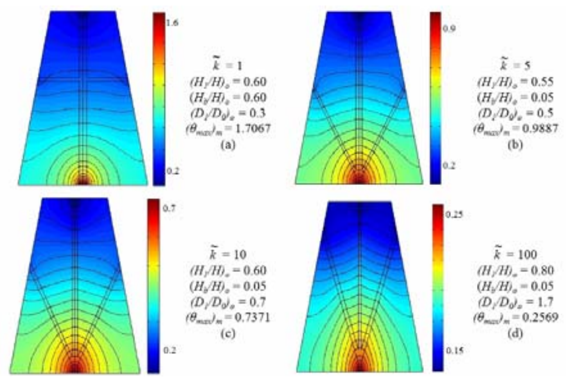

Fig. 3 – Optimal geometries for different values of *k* with □ = 1, $\lambda_T = 50$, $L'/L = 0.5$ and $H/L = 1.0$.

Besides exploring the effects of thermal conductivity ratio ($\tilde{k}$), effects of the convective heat transfer coefficients imposed at the end of the branches were investigated. Different values of the top branch dimensionless heat transfer coefficient, λ_T, were tested to understand the influence of this parameter over the optimal geometry of the conductivity pathway. Variability of $(\theta_{max})_m$ with respect to λ_T for different $\tilde{k}$ are shown in Fig. 4.

As expected, $(\theta_{max})_m$ of the system decreases as the top branch heat transfer coefficient increases. This happens because a higher heat transfer coefficient means a higher capacity of the sink to extract heat from the system. In addition, it was noticed that for values of λ_T less than 1, the decrease in $(\theta_{max})_m$ is small, and after this point, the temperature decreases significantly.

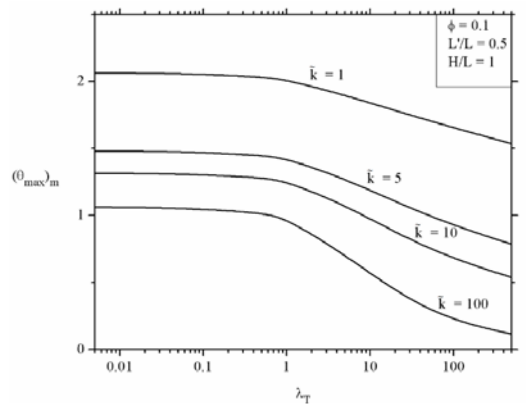

Fig. 4 – Influence of the dimensionless heat transfer coefficient (λ_T) over the minimized maximum dimensionless temperature $(\theta_{max})_m$ of the system for different values of *k*.

This indicates a change in regime on the system. Investigating further on this matter, the optimal configurations for different values of λ_T were analyzed.

The configurations are illustrated in Fig. 5. There are no drastic changes in the optimal configuration or temperature with the increase of λ_T. This reinforces that the change observed in $(\theta_{max})_m$ arises directly from the change in λ_T. Looking at the definition of λ_T, it could be interpreted as Biot number, and values greater than one indicate a predominance of the convection over the internal conduction. At that point, the sinks are more effective in removing heat from the base, as it can be observed in Fig. 5d by looking at the gradients near the end points of the pathway.

The influence of λ_T on the optimal values of D_1/D_0 for different values of $\tilde{k}$ was also analyzed. The results are shown in Fig. 6.

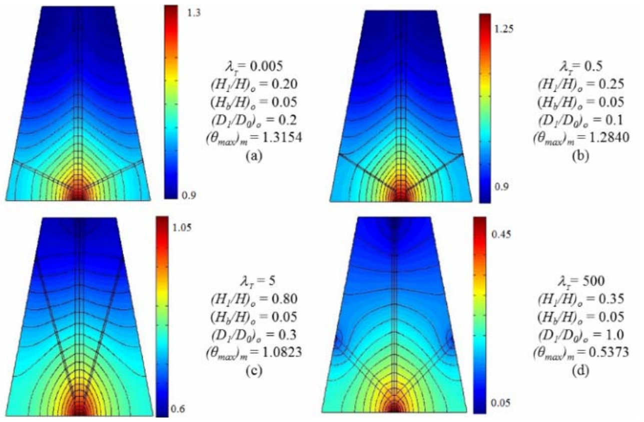

Fig. 5 – Optimal configurations of the system for different values of λ_T, with $\phi = 0.1$, $H/L = 1$, $L'/L = 0.5$ and $k = 10$.

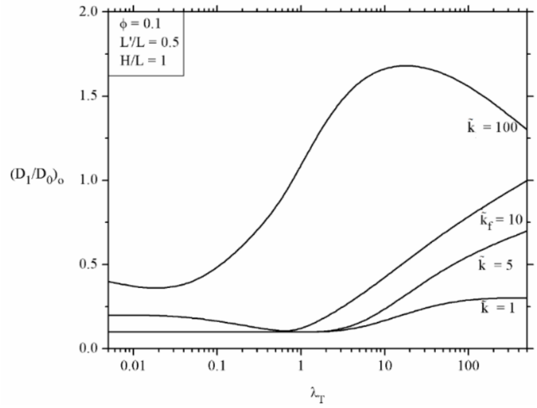

Fig. 6 – Influence of λT over the optimal lateral and central branches ratio $(D_1/D_0)_o$ for different values of k.

Fig. 6 illustrates that the optimal width (D_1) value of the lateral branch increases around the same λ_T, (≈ 1), in which the faster decrease of $(\theta_{max})_m$ starts. With a low value of λ_T, the system opts for a wider central pathway and lateral sinks as low as possible. Beyond $\lambda_T = 1$, this behavior changes, and the lateral sinks can remove heat effectively even on a relatively lower height, and then they become more important to the heat removal process, thus becoming wider and conducting a higher portion of heat throughout the system.

5. CONCLUSIONS

After exploring the configuration design space of a high conductivity pathway inserted within a trapezoidal body of lower conductivity it is observed that Constructal Design reveals configurations that allows more access to the flows within it. This phenomenon, predicted by the Constructal Law, is clearly seen in action in the system studied in this work. For this case, the balance between transporting the heat through the conductivity pathway or the background body led to structures that minimize the maximum dimensionless temperature. This minimization of the maximum dimensionless temperature is also the minimization of the thermal global resistance of the system.

By imposing a function for the lateral heat transfer coefficient dependent on the height, the system displays two distinct methods for dissipating the heat. For a small heat transfer coefficient, the system aims to dissipate the heat through the main channel of the pathway, while minimizing the area of the lateral branches and keeping them relatively low in the system, since they do not have a great influence in the heat transfer process. On the other hand, for a higher heat transfer coefficient, the system starts to increase the lateral branches and transport more heat through them. This is due to a balance involving the pathway conductivity, the heat transfer coefficient and the ratio between the lateral and main branches lengths.

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