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CONSTRUCTAL DESIGN AND NUMERICAL MODELING APPLIED TO STIFFENED STEEL PLATES SUBMITTED TO ELASTO-PLASTIC BUCKLING

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Abstract. Stiffened steel plates are components widely used in structural engineering, being especially indispensable in ship and aerospace structures. The stiffeners are beams fixed to the plates with the purpose to increase its mechanical strength. It is well known that if an axial compressive load is imposed to these components the undesired instability phenomenon of buckling can occur. For a specific load value, the limit stress is achieved and the plate suffers out-of-plane displacements indicating the buckling occurrence. Therefore, in the present work, allying the Constructal Design method, the exhaustive search technique and the computational modelling, the influence of rectangular stiffeners in the elasto-plastic plate buckling behaviour was analysed aiming its geometric optimization. To do so, a reference steel plate without stiffeners was adopted. Its total volume (V), length (a) and width (b) were preserved, but some portion of its material was transformed in stiffeners which leads a reduction in its thickness (t). The volume fraction (ϕ) parameter defines this steel portion by the ratio between the stiffeners volume (V_s) and V. In addition, as degree of freedoms the number of longitudinal and transversal stiffeners as well as the ratio h_s/t_s between height (h_s) and thickness (t_s) of the stiffeners were considered. The maximization buckling limit stress is adopted as objective function. The results indicate that significant improvements in the ultimate buckling stress value can be obtained when a stiffened plate is adopted in relation to a reference plate with the same in-plane dimensions and the same material volume. It was also possible to define the optimized geometric configuration for the stiffened plate that maximizes its ultimate buckling stress, hence conducting to a superior structural performance.

***Key words*:** Stiffened plates, Elasto-plastic buckling, Numerical simulation, Constructal design, Geometric optimization.

1. INTRODUCTION

Plates and panels are structural components widely employed in several engineering applications. One way to increase the mechanical strength of these elements is by insertion of stiffeners, that can be arranged longitudinally and/or transversely [1].

Among the different loads that can be applied to a plate structure, the compressive longitudinal load needs a special attention due to the possibility of the buckling phenomenon occurrence. Unlike columns the plates are capable to resist an increment of load after to suffer the elastic buckling [2]. Besides, the addition of stiffeners in a plate promotes an increase in its buckling limit stress with a small or even null increment in the structure weight. However, the presence of stiffeners also increases its geometric configuration complexity, being the computational modeling by means the finite element method (FEM) an effective approach for the analysis of these components.

2. BUCKLING PLATES

The critical stress that defines the elastic buckling in a thin uniaxial compressed plate is given by [3]

$$\sigma_{cr} = K_q \frac{\pi^2 E}{12\left(1-\upsilon^2\right)}\left(\frac{t}{b}\right)^2, \tag{1}$$

where t is the plate thickness, b is the plate width and $K_q = 4$ for a simply supported plate [2, 4].

The determination of elastic critical stress is important to understand the different buckling modes of thin plates [5]. However, it does not represent its real behavior because geometric and material nonlinearities must be taken into account. Therefore when the material yielding happens before the elastic critical stress is reached it is said that an inelastic buckling occurs; if it is achieved a stress level higher than critical stress the post-buckling stage is developed. Finally, the ultimate stress is defined by the maximum stress that the plate can resist before its collapse. In addition, these structural components can resist a significant additional compressive loading beyond the critical load allowing its maximum capacity be the sum of critical buckling load and the post-buckling load [2].

3. COMPUTATIONAL MODELING

It is well known that the finite element method (FEM) can be used to obtain approximate solutions for the mechanical behavior of plates with reasonable accuracy [6]. In the field of structural analysis it is usually adopted in its displacement formulation. For this, the structure continuum is divided into a number of small regions, the so-called finite elements that are assumed to be interconnected at a discrete number of nodal points located on their boundaries [7, 8]. More information about the FEM can be obtained in references [9, 10]. The ANSYS software is based on the FEM, being used for the numerical simulations of the present work by means the SHELL93 finite element.

3.1. Numerical analysis of elasto-plastic buckling

Because of the complexity of the stress-strain relation beyond the elastic buckling state [11] the determination of the buckling ultimate stress of a plate is a complex nonlinear analysis. Hence, numerical methods are widely recommended and employed for the analysis of the plates post buckling behavior.

To do so, in the present work a computational model was developed considering linear elastic perfectly plastic material behavior, i.e. with no strain hardening, being this assumption the most critical situation for the steel. Besides, as an initial condition for the nonlinear elasto-plastic buckling simulation, it is necessary that the plate has an imperfect geometric configuration. This initial imperfect geometry is obtained from the first elastic buckling mode with maximum lateral deflection defined as $b/2000$, being b the plate width [12].

A computational model based on the eigenvalue approach was employed to the first elastic buckling mode determination. More detailed information about the elastic buckling computational model, as well as about the elasto-plastic buckling computational model can be encountered in reference [13].

3.2. Verification and validation of computational model

A verification and validation of the elasto-plastic buckling computational model were performed considering a plate with longitudinal and transversal stiffeners, called SP1 in reference [14]. Fig. 1 shows the geometric configuration of this simply supported steel plate, being a = 1.16 m, b = 0.96 m, t_p = 0.01 m, c_1 = 0.28 m, c_2 = 0.32 m, a_1 = 0,1 m, b_1 = 0.06 m, h_s = 0.05 m and t_s = 0.005 m. Moreover the mechanical properties of the steel are σ_y = 218 MPa, E = 180 GPa and ν = 0.3.

Reference [14] presents experimental and numerical results for the ultimate buckling load of P_u = 983.00 kN and P_u = 1036.20 kN, respectively, while in the present work a value of P_u = 1075.55 kN was numerically obtained. An error of 9.41% and a difference of 3.79% were found when our numerical solution is compared respectively with experimental and numerical results of [14], validating and verifying the computational model used.

4. CONSTRUCTAL DESIGN METHOD

The Constructal Theory is based on a physics principle, which is the constructal law: “For a finite-size flow system to persist in time (to survive) its configuration must evolve freely in such a way that it provides an easier access to the currents that flow through it" [15,16]. The Constructal Law requests for configurations with successively smaller global flow resistances in time. Resistances (imperfection) cannot be eliminated. They can be matched neighbor to neighbor, and distributed so that their global effect is minimal, and the whole basin is the least imperfect that it can be [17].

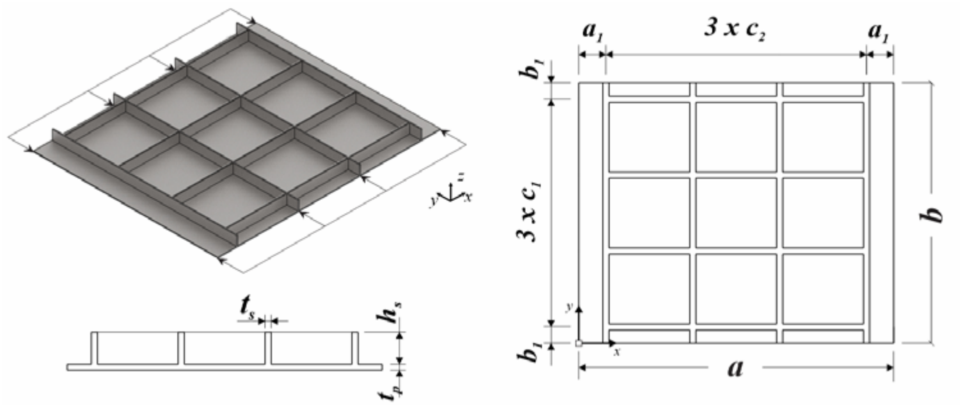

Fig. 1 – Steel plate with longitudinal and transversal stiffeners.

In turn, the Constructal Design method allows the use of the Constructal Law to improve engineering performances, seeking better strategies for generating the system geometry. Therefore, it guides the designer (in time) toward flow architectures that have greater global performance for the specific flow access conditions (fluid flow, heat flow, flow of stresses, etc.) pursuing the optimal distribution of imperfections [15, 16]. The proposal of the present work is to treat the mechanic of materials as the flow configurations are treated in fluid mechanics or heat transfer: mechanical structures are networks through which stresses flow from components to their neighbors [17].

It is well known in structural engineering that concentrations of maximum stresses are not good for mechanical performance. The best use of a mechanical resistant material is reached when the limit stresses are distributed uniformly through the available material, being this design principle in agreement with the principle of the optimal distribution of imperfections [13, 17].

Therefore, aiming to apply the Constructal Design method for the evaluation of the geometry influence in the elasto-plastic buckling of a stiffened plate, a reference plate without stiffeners (Fig. 2a) was adopted. The total material volume of the reference plate is a constraint, being kept constant in all studied cases. To transform part of the total volume in stiffeners, it was defined the volume fraction (ϕ) parameter:

$$\phi = \frac{V_s}{V} = \frac{N_{ls}\left(ah_st_s\right) + N_{ts}\left[\left(b - N_{ls}t_s\right)h_st_s\right]}{abt}, \tag{2}$$

where: V_s is the material volume of reference plate transformed in stiffeners, V is the total material volume, N_{ls} and N_{ts} are, respectively, the number of stiffeners in longitudinal and transversal directions (see Fig. 2b), h_s and t_s are, respectively, the height and thickness of the stiffeners.

The reference plate (Fig. 2a) with a = 2 000 mm, b = 1 000 mm, t = 14 mm and $V = 28 \times 106$ mm^3 was considered. Moreover volume fractions of ϕ = 0.1 and 0.4 were adopted for the stiffened plates (Fig. 2b), with combinations of N_{ls} = 2, 3, 4 and 5 and N_{ts} = 2, 3, 4 and 5 for several values of h_s/t_s. In addition, Fig. 2a presents a stiffened plate with N_{ls} = 2 and N_{ts} = 3, called P(2, 3). Steel AH-36 was adopted for these plates, having the follow mechanical properties: σ_y = 355 MPa, E = 210 Gpa, and ν = 0.3.

It is important to highlight that as the plate dimensions a and b are kept constant, the tp value is dependent of the ϕ parameter with the purpose of guaranteeing no variation in the total material volume.

5. RESULTS AND DISCUSSIONS

Considering a simply supported condition, the reference plate was numerically simulated and the elasto-plastic buckling ultimate stress obtained is σ_{uR} = 187.61 MPa. This value was adopted to normalize the buckling ultimate stress value of stiffened plates.

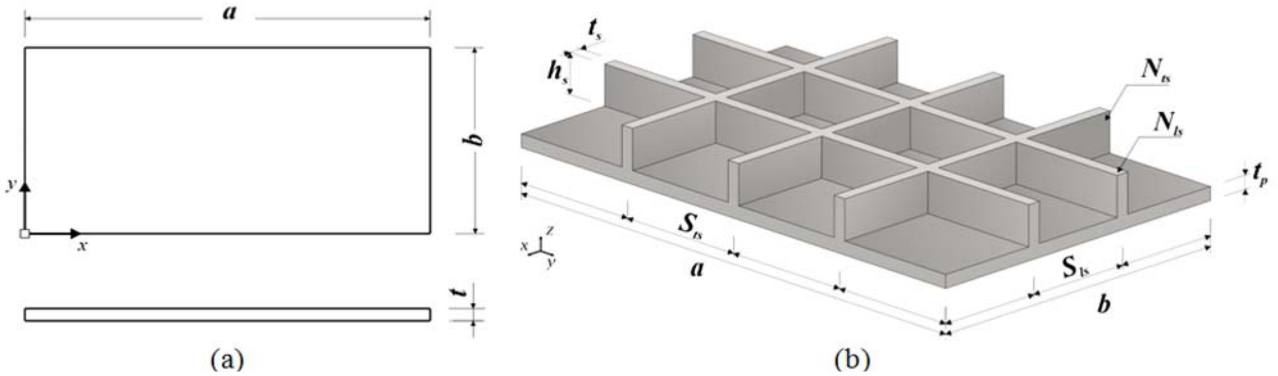

Fig. 2 – Illustration of: a) reference plate; b) a stiffened plate with N_{ls} = 2 and N_{ts} = 3, called P(2, 3).

For each studied ϕ value (ϕ = 0.1 and 0.4) and each stiffeners arrangement (P(2, 2), P(2, 3), P(2, 4), P(2, 5), P(3, 2), P(3, 3), P(3, 4), P(3, 5), P(4, 2), P(4, 3), P(4, 4), P(4, 5), P(5, 2), P(5, 3), P(5, 4) and P(5, 5)), the h_s/t_s variation allowed to identify an optimal plate geometry $(h_s/t_s)_o$ leading to a maximized normalized ultimate buckling stress $(\sigma_{uN})_m$. It is worth to emphasize that in each arrangement the h_s/t_s variation always conduct to the same mechanical behavior trend: from the lowest h_s/t_s value its increase promotes an augmentation of σ_{uN} until be reached $(h_s/t_s)_o$ and $(\sigma_{uN})_m$, thenceforth the σ_{uN} value decrease as h_s/t_s increase. In other words, the optimized geometric configuration was always obtained with an intermediate h_s/t_s ratio. This fact indicates that it is not possible to define the superior mechanical behavior without to perform a geometry evaluation. In this context, the geometric configuration variation proposed by the Constructal Design method allows to find the one that leads to the best performance, keeping constant the total material volume.

Then, for ϕ = 0.1 and 0.4, with the values of $(h_s/t_s)_o$ and $(\sigma_{uN})_m$ for each above mentioned arrangement it was possible to elaborate graphs relating $(\sigma_{uN})_m$ and $(h_s/t_s)_o$ as function of N_{ts}, respectively, in Figs. 3a and 3b.

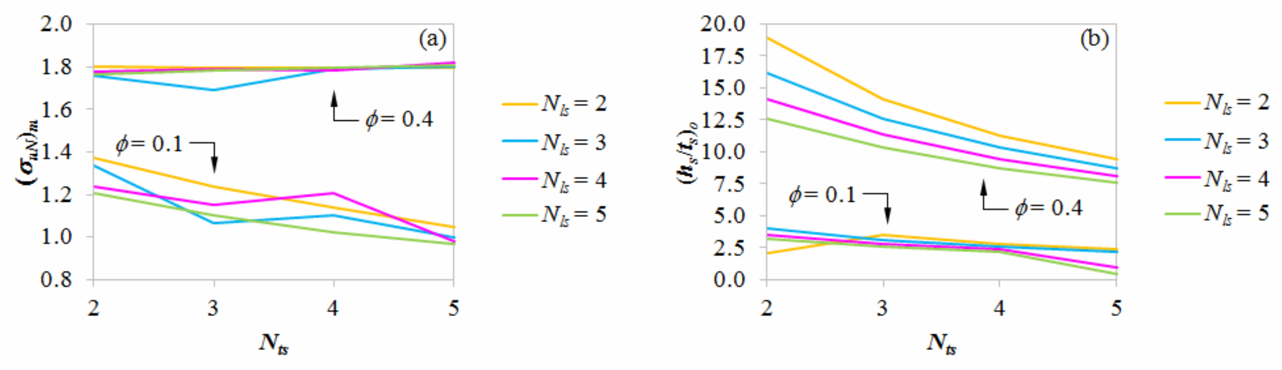

Fig. 3 – Influence of N_{ts}, for ϕ = 0.1 and ϕ = 0.4, over: a) $(\sigma_{uN})_m$; b) $(h_s/t_s)_o$.

It is possible to note in Fig. 3a for ϕ = 0.1 a reduction in $(\sigma_{uN})_m$ with the increase of N_{ts}. Considering that longitudinal and transversal stiffeners have the same ratio h_s/t_s, the increase of N_{ts} causes a reduction of material used in longitudinal stiffeners. As explained in [11], the longitudinal stiffeners are the main responsible to resist unixial buckling. So, when a little amount of material from reference plate is transformed in stiffeners, ϕ = 0.1 in in this case, as the N_{ts} increases it is expected a diminution in ultimate buckling stress. In addition, until N_{ts} = 4 the presence of stiffeners improved the mechanical capacity of plates in comparison with reference plate, i.e. for these cases it was obtained $(\sigma_{uN})_m > 1$. However, in Fig. 3a for ϕ = 0.4 there is a stabilization of $(\sigma_{uN})_m$ around 1.8 indicating that due the greatest material amount used as stiffeners a superior mechanical behavior can be achieved.

Regarding Fig. 3b, in a general way the increase of N_{ts} promotes a decrease in $(h_s/t_s)_0$ value, being this trend more evident for ϕ = 0.4.

After that, for each N_{ls} value it was defined an optimized value for N_{ts}, named $(N_{ts})_o$. Hence, it was also defined the ultimate buckling stress twice maximized, $(\sigma_{uN})_{mm}$, and h_s/t_s twice optimized, $(h_s/t_s)_{oo}$. These results are plotted in Fig. 4.

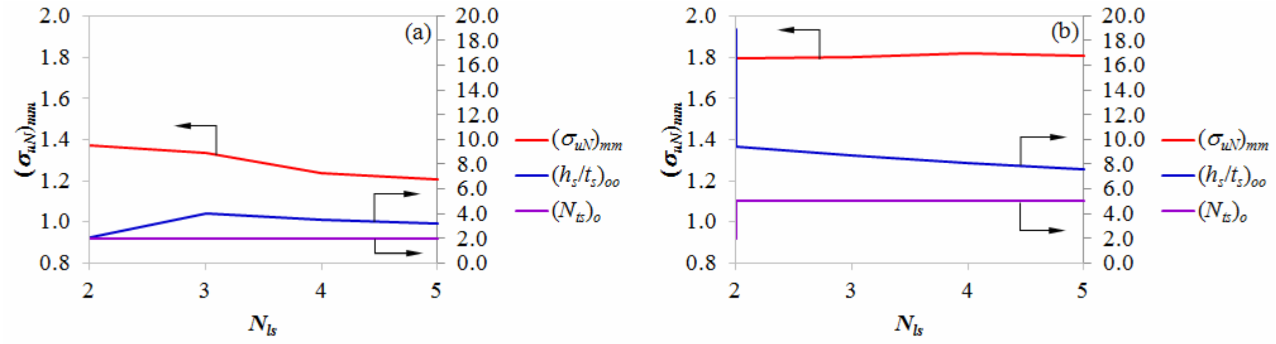

Fig. 4 – Variation of $(N_{ts})_o$, $(h_s/t_s)_{oo}$ and $(\sigma_{uN})_{mm}$ as a N_{ls} function: a) $\phi = 0.1$; b) $\phi = 0.4$.

Figures 4a and 4b indicate that $(N_{ts})_o = 2$ and $(N_{ts})_o = 5$ leads to superior structural performances independent of N_{ls}, respectively, for $\phi = 0.1$ and $\phi = 0.4$. However, in Fig. 4b, when $N_{ls} = 2$ the geometric configuration with $(N_{ts})_o = 2$ also leads to an optimized geometry. Besides, it is possible to note in Fig. 4a that $2.10 \geq (h_s/t_s)_{oo} \geq 4.04$ for $\phi = 0.1$, while in Fig. 4b one can observe that $7.59 \geq (h_s/t_s)_{oo} \geq 9.41$ for $\phi = 0.4$. Therefore, there are specific values for h_s/t_s around which the best geometry is defined. An exception occurs for plate P(2, 2), that has $(h_s/t_s)_{oo} = 18.92$ (Fig. 4b). Finally, regarding $(\sigma_{uN})_{mm}$, for $\phi = 0.1$ (Fig. 4a) the increase of N_{ls} causes a reduction in $(\sigma_{uN})_{mm}$, while for $\phi = 0.4$ (Fig. 4b) it is possible to note a stabilization of the $(\sigma_{uN})_{mm}$ value around 1.81, i.e. there is no significant influence of N_{ls} in ultimate buckling stress.

A last analysis was performed comparing the effect of volume fraction over the mechanical behavior of stiffened plates submitted to uniaxial buckling. To do so, the geometric configuration that maximizes the ultimate stress for each ϕ value was defined from Fig. 4, as can be seen in Tab. 1.

Table 1

Geometric configuration with $(N_{ls})_o$, $(N_{ts})_{oo}$, $(h_s/t_s)_{ooo}$ and $(\sigma_{uN})_{mmm}$

ϕ	Plate	$(N_{ls})_o$	$(N_{ts})_{oo}$	$(h_s/t_s)_{ooo}$	$(\sigma_{uN})_{mmm}$
0.1	P(2,2)	2	2	2.10	1.37
0.4	P(4,5)	4	5	8.12	1.82

The results of Tab. 1 indicate that between the two volume fractions value considered and among several geometries numerically simulated there is a geometry that conduct to the global best performance, being this the plate P(4, 5) with $\phi = 0.4$, $(h_s/t_s)_{ooo} = 8.12$ and $(\sigma_{uN})_{mmm} = 1.82$. It has an ultimate buckling stress 82% and 33% superior than the reference plate and the best geometry for $\phi = 0.1$ (P(2,2) with $(h_s/t_s)_{ooo} = 2.10$ and $(\sigma_{uN})_{mmm} = 1.37$. The von Mises stress distributions for stiffened plates of Tab. 1 are depicted in Fig. 5.

It is evident from Fig. 5 that the plate P(4,5) with $\phi = 0.4$ (Fig. 5b) can promote a better distribution of the limit stress (in red color) than the plate P(2, 2) with $\phi = 0.1$ (Fig. 5a). While the plate with the best global performance is almost all submitted to the limit stress (Fig. 5b), the other one has only few regions in this situation. This fact can be explained by the Constructal principle of optimal distribution of imperfection. Moreover, in Fig. 5 it is possible to prove that the transversal stiffeners are submitted to low stresses, as already mentioned.

6. CONCLUSIONS

In this work, numerical models for elastic and elasto-plastic buckling of plates allied to the Constructal Design and the Exhaustive Search were employed to perform a geometric optimization of stiffened plates.

A reference plate with no stiffeners was used. From it and taking into account the volume fraction (ϕ) parameter, plates with longitudinal and transversal stiffeners were defined but always keeping constant the total material volume. Two ϕ values were studied, having as degrees of freedom the ratio between the height and thickness of rectangular stiffeners (h_s/t_s) and the number of longitudinal (N_{ls}) and transversal (N_{ts}) stiffeners. The objective function was to maximize the ultimate buckling stress of stiffened plates.

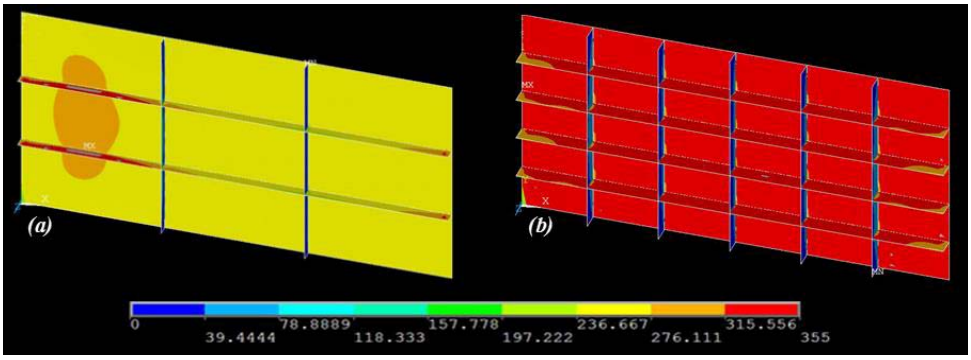

Fig. 5 – Von Mises stress distribution, in MPa: a) P(2, 2) with ϕ = 0.1; b) P(4,5) with ϕ = 0.4.

The results indicate that there is an optimized geometry that leads to a global superior performance among the analysed cases. This best geometric configuration (P(4, 5) with ϕ = 0.4, $(h_s/t_s)_{ooo}$ = 8.12, $(\sigma_{uN})_{mmm}$ = 1.82 and ϕ = 0.4) is 82% better than the reference plate and it is almost 367% better than the worst stiffened plate (P(2, 2) with h_s/t_s = 74.92, σ_{uN} = 0.39 and ϕ = 0.4). The result for the worst geometry shows that the transformation of part of the reference plate material into stiffeners, keeping constant its volume, not always improve its ultimate buckling stress. Therefore, the geometry evaluation in structural engineering is an important research subject and must be done in order to achieve superior mechanical behaviours and avoid improper geometries.

In addition, among the studied geometric configurations, the best shape was the one that better distributed the imperfections of the system, i.e. the one that have more regions submitted to the limit stress. This trend is in agreement with the constructal principle of optimal distribution of imperfection, proving the effectiveness of Constructal Design method.

In future works it is intended to analyze the influence of other ϕ values, type of stiffeners as well as to study the biaxial buckling phenomenon.

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