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CONSTRUCTAL OPTIMIZATIONS FOR LINE-TO-LINE FLUID NETWORKS IN A TRIANGULAR AREA BY RELEASING THE TUBE ANGLE CONSTRAINT

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Abstract. Based on constructal theory, the line-to-line fluid networks (LTLFNs) in a triangular area are investigated by releasing the tube angle constraint (TAC). The total pressure drop (TPD) of the LTLFN is taken as the optimization objective, and the total volume of the tubes and occupied area of the LTLFN are taken as the constraints. Constructal optimizations of the LTLFNs are implemented by optimizing the tube angles. The results show that the TPDs of the second, third and fourth order LTLFNs have their minimum values, and the corresponding optimal tube angles are different for different orders. Compared with the TPDs of the LTLFNs with the TAC, those of the second, third and fourth order LTLFNs by releasing TAC are decreased by 3.94%, 6.26% and 8.33%, respectively. One can see that the fluid flow performances of the LTLFNs are improved by releasing TAC. Moreover, when the total tube surface area is taken as the constraint, the optimal tube angles of the LTLFNs are different from those obtained with the total tube volume constraint.

Key words: Constructal theory, Total pressure drop, Line-to-line fluid network, Tube angle constraint, Generalized thermodynamic optimization.

1. INTRODUCTION

Fluid flow systems widely exist in the nature and engineering, such as blood vessels, bronchium, leaf veins, plant roots, water networks, etc. Suitable flow structures help to improve their flow performances, and many scholars have shown great interests in the structure optimizations of the fluid flow systems [1–5].

One of the popular structure optimization theories is constructal theory [6–21], which has been widely used in illustrating or solving natural, social and engineering problems. Tree-shaped flow structure (TSFS) [22–24] is one kind of superior flow systems, and many scholars [25–43] conducted constructal optimizations of the tree-shaped flow systems based on different optimization objectives. Bejan *et al*. [25] considered the T-shaped flow tubes in a rectangular area and Y-shaped flow tubes in rectangular and disc-shaped areas, and minimized the flow resistances of the tubes by varying the length ratios and tube diameter ratios, respectively. Wechsatol *et al*. [26] further analyzed the effects of junction pressure losses (JPLs) on the flow performances of T- and Y-shaped flow tubes, and found that the JPLs could be ignored when the svelteness number was larger than 104. Lorente *et al*. [27] optimized four kinds of TSFSs based on minimum path length objective, and provided a simple method to optimize the performances of the TSFSs. Lorente and Bejan [28] optimized a line-to-line fluid network (LTLFN) in porous medium subjected to tube angle constraint (TAC), and obtained the optimal pure flow performance of the network. Wechsatol *et al*. [29] built a detailed fluid flow model with TSFS in a disc, and minimized its total pressure drop (TPD) by varying the tube angles and central tube number, respectively. Gosselin *et al*. [30, 31] further conducted constructal designs of H- and Y-shaped flow networks based on minimum pumping power objective, and obtained new optimal constructs of the flow networks different from those based on TPD objective. Furthermore, loop structure [32], local junction loss [33], asymmetry network [34, 35] and tube surface area constraint (TSAC) [36, 37] were considered in the constructal designs. More practical results were obtained, and different requirements were satisfied in these researches. Azoumah *et al*. [38] and Bieupoude *et al*. [39] optimized T- and Y-shaped drink water networks based on TPD objective, and found that the flow performance of Y-shaped network was superior to that of T-shaped network. Moreover, constructal designs of distributor networks, comb-like networks and open flow networks were also conducted by Fan *et al*. [40, 41], Lee *et al*. [42] and Zhang *et al*. [43], respectively.

Based on the LTLFN model with TAS, a LTLFN model without TAS will be built in this paper. Occupied area of the LTLFN will be constrained, and the TPD of the LTLFN will be minimized. Optimal tube angles of the LTLFNs will be obtained subjected to the total tube volume and surface area constraints, respectively, and comparisons of the optimal constructs derived by different constraints will be implemented.

2. CONSTRUCTAL OPTIMIZATIONS FOR LINE-TO-LINE FLUID NETWORKS SUBJECTED TO TOTAL TUBE VOLUME CONSTRAINT

2.1. Second order of line-to-line fluid network

The model of a first order LTLFN in a triangular area is shown in Fig. 1 [28]. In the triangular area A ($=2d\times H$), the first order LTLFN is composed of one main tube (diameter D_1, length L_1) and two elemental tubes (diameter D_0, length L_0). Fully developed laminar flow (FDLF) is considered in the tubes of LTLFN. The stream (flow rate $\dot{m}_T$) enters the inlet of the LTLFN, and flows through the main and elemental tubes, respectively. Finally, it flows out of the LTLFN from the end of the elemental tube (flow rate $\dot{m}_0=\dot{m}_T/2$). The outlets of the elemental tubes uniformly locate at the edge of the triangular area, and the distance between the adjacent outlets is d . The angle of D_1 tube and left edge of the triangular area is α_1, and that of D_0 tube and left edge is α_0 .

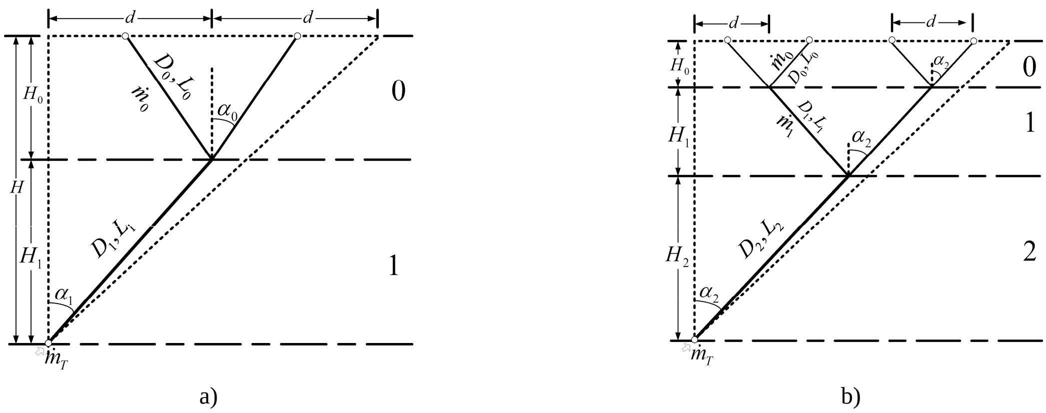

Fig. 1 – Line-to-line fluid networks [28]: a) first order b) second order.

Based on the model of the first order LTLFN, a second order LTLFN as shown in Fig. 1b was further built by Lorente and Bejan [28]. All the angles of the tubes and left edge are α_2 , which is a TAC of this model. If this TAC can be released, the performance of the LTLFN may be better. With this consideration, the model of a second order LTLFN without TAC is built in this paper. As shown in Fig. 2, the new model is composed of one main tube (diameter D_2, length L_2) and two first order LTLFNs. The stream (flow rate $\dot{m}_T$) enters the inlet of D_2 tube, and flows out of the LTLFN from the end of the elemental tube (flow rate $\dot{m}_0=\dot{m}_T/4$). The angle of D_2 tube and left edge of the triangular area is α_2 , and those for D_1 and D_0 tubes are α_1 and α_0, respectively. Different from the model in Fig. 1b, the tube angles α_0, α_1 and α_2 in Fig. 2 are different, which means that the TAC is released. How about the performance of this model? Constructal optimization of this model will be conducted to answer this question.

The total tube volume and occupied area of the second order LTLFN can be, respectively, given as

$$V = \pi(4D_0^{\ 2}L_0 + 2D_1^{\ 2}L_1 + D_2^{\ 2}L_2)/4, \tag{1}$$

$$A = 4d(H_0 + H_1 + H_2)/2. \tag{2}$$

When the tube diameter ratios of the LTLFN obey Murry law, the relationships of the diameters are:

$$D_2 = 2^{1/3}D_1 = 2^{2/3}D_0. \tag{3}$$

For the fixed total volume of the tubes, the diameter of the elemental tube can be obtained by substituting the diameter relationships into Eq. (1), i.e.,

$$D_0 = V^{1/2}\cdot[\pi(L_0 + 2^{-1/3}L_1 + 2^{-2/3}L_2)]^{-1/2}. \tag{4}$$

According to the structure of the second order LTLFN, the lengths and vertical distances of the tubes are, respectively, given as

$$L_0 = d/[2\sin(\alpha_0)],\ \ L_1 = d/\sin(\alpha_1),\ \ L_2 = 2d/\sin(\alpha_2), \tag{5}$$

$$H_0 = L_0\cos(\alpha_0),\ \ H_1 = L_1\cos(\alpha_1),\ \ H_2 = L_2\cos(\alpha_2). \tag{6}$$

For the fixed area A, substituting Eqs. (5) and (6) into Eq. (2) yields the distance between the adjacent outlets

$$d = A^{1/2}\cdot[\cot(\alpha_0) + 2\cot(\alpha_1) + 4\cot(\alpha_2)]^{-1/2}. \tag{7}$$

According to Refs. [29, 37], the TPD of the second order LTLFN for FDLF is

$$\Delta P_2 = 128\,\dot{m}_0 \nu L_0/(\pi D_0^{\ 4}) + 128\,\dot{m}_1 \nu L_1/(\pi D_1^{\ 4}) + 128\,\dot{m}_2 \nu L_2/(\pi D_2^{\ 4}). \tag{8}$$

From Eq. (8), the dimensionless total pressure drop (DTPD) can be expressed as

$$\Delta\tilde{P}_2 = \Delta P_2 V^2/(\pi\,\dot{m}_T \nu A^{3/2}) = 4(2\tilde{L}_0 + 2^{2/3}\tilde{L}_1 + 2^{1/3}\tilde{L}_2)^3, \tag{9}$$

where $(\tilde{L}_0, \tilde{L}_1, \tilde{L}_2) = (L_0, L_1, L_2)/A^{1/2}$. From Eq. (9), the DTPD $\Delta\tilde{P}_2$ of the second order LTLFN is a function of the tube angles α_0, α_1 and α_2, and constructal optimization of the second order LTLFN can be conducted by taking these parameters as optimization variables.

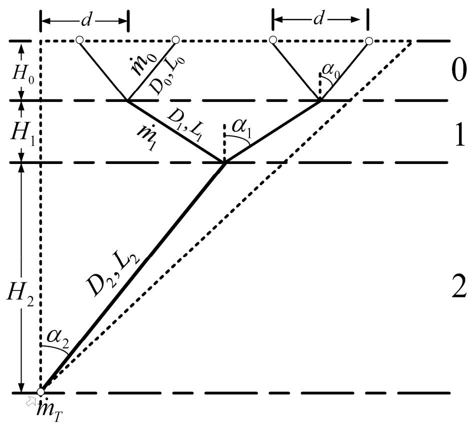

Fig. 2 – Second order line-to-line fluid network by releasing TAC.

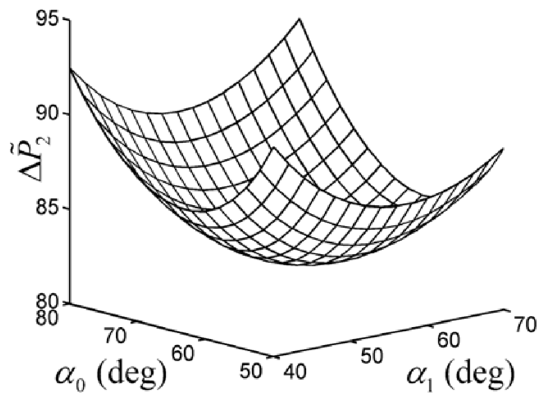

Fig. 3 – Characteristics of $\Delta\tilde{P}_2$ versus α_0 and α_1.

Figure 3 shows the three-dimensional relationship of the DTPD $\Delta\tilde{P}_2$ versus the tube angles α_0 and α_1 with $\alpha_2 = 40°$. From Fig. 3, there exist optimal tube angles ($\alpha_{0,opt}$ and $\alpha_{1,opt}$) which leads to double minimum value of the DTPD. Numerical calculation shows that the tube angle α_2 can be further optimized, the optimal tube angles of the second order LTLFN by releasing the TAC are $\alpha_{0,opt} = 60.16°$, $\alpha_{1,opt} = 51.18°$ and $\alpha_{2,opt} = 37.83°$. Compared with the DTPD of the LTLFN with the TAC, that of the LTLFN without the TAC is reduced by 3.94%. Therefore, the flow performance of the LTLFN is improved by releasing the TAC.

2.2. Third and fourth orders of line-to-line fluid networks

The model of a third order LTLFN without the TAC is shown in Fig. 4. It is composed of one main tube (diameter D_3, length L_3) and two second order LTLFNs. One can see that the models of the higher order LTLFNs can be further built by adopting this method. The stream (flow rate $\dot{m}_T$) enters the inlet of D_3 tube, then flows along D_i tubes (flow rate $\dot{m}_i = \dot{m}_T / 2^{3-i}$, $i = 3,2,1,0$), and finally flows out of triangular area from the outlet of D_0 tube. The angle of D_3 tube and triangular left edge is α_3.

The total tube volume and occupied area of the third order LTLFN can be, respectively, given as

$$V = \pi(8D_0{}^2L_0 + 4D_1{}^2L_1 + 2D_2{}^2L_2 + D_3{}^2L_3)/4, \tag{10}$$

$$A = 8d(H_0 + H_1 + H_2 + H_3)/2. \tag{11}$$

According to the structure of the LTLFN shown in Fig. 4, the length and vertical distance of each order tube are, respectively, expressed as

$$L_0 = d/[2\sin(\alpha_0)],\ L_1 = d/\sin(\alpha_1),\ L_2 = 2d/\sin(\alpha_2),\ L_3 = 4d/\sin(\alpha_3), \tag{12}$$

$$H_0 = L_0\cos(\alpha_0),\ H_1 = L_1\cos(\alpha_1),\ H_2 = L_2\cos(\alpha_2),\ H_3 = L_3\cos(\alpha_3). \tag{13}$$

Similar to the method adopted in section 2.1, the DTPD of the third order LTLFN can be obtained by combining Eqs. (10)–(13)

$$\Delta\tilde{P}_3 = \Delta P_3 V^2/(\pi\dot{m}_T \nu A^{3/2}) = 8(2\tilde{L}_0 + 2^{2/3}\tilde{L}_1 + 2^{1/3}\tilde{L}_2 + \tilde{L}_3)^3, \tag{14}$$

where the dimensionless tube lengths are defined as $(\tilde{L}_0, \tilde{L}_1, \tilde{L}_2, \tilde{L}_3) = (L_0, L_1, L_2, L_3)/A^{1/2}$. From Eq. (14), the DTPD $\Delta\tilde{P}_3$ of the third order LTLFN is a function of the tube angles α_0, α_1, α_2 and α_3, and constructal optimization of the third order LTLFN can be conducted by taking these parameters as optimization variables.

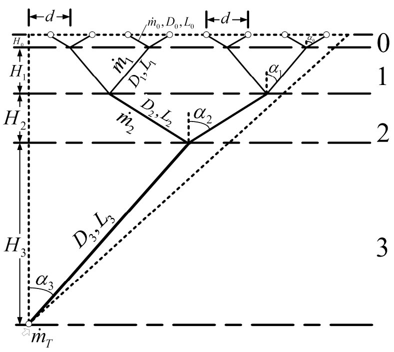

Fig. 4 – Third order line-to-line fluid network by releasing TAC.

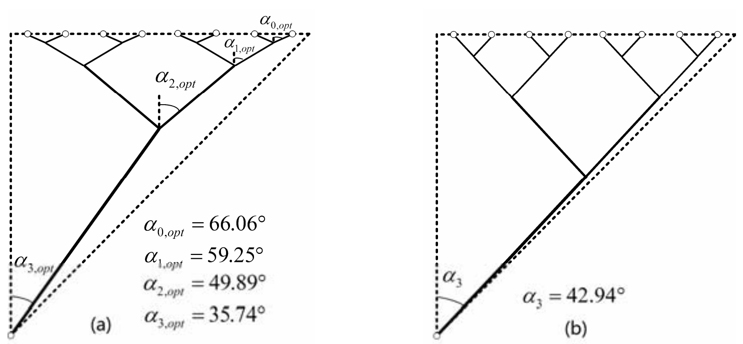

Fig. 5 – Comparison of the optimal constructs of the third order line-to-line fluid networks: (a) releasing TAC (b) preserving TAC.

Figure 5 shows the comparison of the third order LTLFN with and without the TACs. From Fig. 5, one can see that the optimal tube angles of the third order LTLFN without the TAC are $\alpha_{0,opt} = 66.06°$, $\alpha_{1,opt} = 59.25°$, $\alpha_{2,opt} = 49.89°$ and $\alpha_{3,opt} = 35.74°$, and the optimal length ratios of the tubes are $(L_3/L_2)_{opt} = 2.62$, $(L_2/L_1)_{opt} = 2.25$ and $(L_1/L_0)_{opt} = 2.13$, respectively. All the tube angles of the third order LTLFN with the TAC are $\alpha_0 = \alpha_1 = \alpha_2 = \alpha_3 = 42.94°$, and all length ratios of the tubes are 2 [28]. One can see that the optimal constructs of the third order LTLFN with and without the TACs are different. Numerical calculations show that compared with the DTPDs of the third and fourth order LTLFNs with the TAC, those of the LTLFNs without the TAC are reduced by 6.26% and 8.33%, respectively. Therefore, the flow performance of the LTLFN can be further improved by releasing the TAC and adopting higher order network simultaneously.

3. CONSTRUCTAL OPTIMIZATIONS FOR LINE-TO-LINE FLUID NETWORKS SUBJECTED TO TOTAL TUBE SURFACE AREA CONSTRAINT

The LTLFNs subjected to the tube volume constraint (TVC) are optimized in Section 2. The cost of a network is always associated with its total tube surface area [4, 25, 36, 37]. In the design of the network with finite cost, the total surface area is an important constraint in the optimizations. Due to this reason, constructal designs of the second and third orders of LTLFNs subjected to the TSAC will be conducted as examples.

The models of the second and third order LTLFNs without the TAC are shown in Figs. 2 and 4, respectively. The total tube surfaces of the second and third order LTLFNs can be, respectively, given as

$$A_T = \pi(4D_0L_0 + 2D_1L_1 + D_2L_2), \tag{15}$$

$$A_T = \pi(8D_0L_0 + 4D_1L_1 + 2D_2L_2 + D_3L_3). \tag{16}$$

For the total TSACs in Eqs. (15) and (16), constructal optimizations of the second and third order LTLFNs can be conducted by releasing the TAC similar to the method adopted in section 2.

Numerical calculations show that for the fixed total TSAC, the optimal tube angles of the second order LTLFN by releasing the TAC are $\alpha_{0,opt} = 68.43°$, $\alpha_{1,opt} = 56.14°$ and $\alpha_{2,opt} = 32.38°$; those of the third order LTLFN are $\alpha_{0,opt} = 75.39°$, $\alpha_{1,opt} = 67.52°$, $\alpha_{2,opt} = 54.58°$ and $\alpha_{3,opt} = 28.54°$; those of the fourth order LTLFN are $\alpha_{0,opt} = 80.19°$, $\alpha_{1,opt} = 75.03°$, $\alpha_{2,opt} = 66.95°$, $\alpha_{3,opt} = 53.59°$ and $\alpha_{4,opt} = 25.89°$, respectively. One can see that the optimal constructs of the second and higher order of LTLFNs with TSAC are different from those with TVC. Compared with the DTPDs of the second, third and fourth order LTLFNs with the TAC, those of the LTLFNs without the TAC are reduced by 20.05%, 31.36% and 40.98%, respectively. Therefore, releasing TAC exhibits obvious advantages in flow performance improvements of the second and higher order LTLFNs when total tube surface is taken as the constraint.

4. CONCLUSIONS

A LTLFN model in a triangular area without the TAC is built in this paper. The total volume of the tubes and occupied area of the LTLFN are taken as the constraints. Constructal optimizations of the LTLFNs are implemented by optimizing the tube angles, and the TPDs of the LTLFNs are minimized. The results show that when the total TVC is considered and the TAC is released, the optimal tube angles of the third order LTLFN are $\alpha_{0,opt} = 66.06°$, $\alpha_{1,opt} = 59.25°$, $\alpha_{2,opt} = 49.89°$ and $\alpha_{3,opt} = 35.74°$, and the corresponding optimal length ratios of the tubes are $(L_3 / L_2)_{opt} = 2.62$, $(L_2 / L_1)_{opt} = 2.25$ and $(L_1 / L_0)_{opt} = 2.13$, respectively. All the tube angles of the third order LTLFN with the TAC are $\alpha_0 = \alpha_1 = \alpha_2 = \alpha_3 = 42.94°$, and all length ratios of the tubes are 2. The optimal constructs of the third order LTLFN with and without the TACs are different. Compared with the DTPDs of the third and fourth order LTLFNs with the TAC, those of the LTLFNs without the TAC are reduced by 6.26% and 8.33%, respectively. Therefore, the flow performance of the LTLFN can be further improved by releasing the TAC and adopting higher order network simultaneously. Moreover, the optimal constructs of the LTLFNs subjected to TVC and TSAC are different, which can provide different guidelines for the designs of the fluid flow systems.

Actually, the LTLFN model built in this paper is an ideal one. The local pressure losses exist and turbulent flow may occur in the tubes. The mass flow rates in the tubes may not equal to each other, and different design requirements should be satisfied. Therefore, one can built more practical LTLFN models by considering local resistance losses, different flow regimes and nonuniform flow rate distributions, respectively, and further conduct constructal designs of the LTLFNs based on multi-objectives.

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