

## DESIGN AND PROPERTIES OF NEW CIC FIR FILTER FUNCTIONS

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In this paper, a new design of the CIC (Cascaded-Integrator-Comb) FIR (Finite Impulse Response) filter functions is presented by spreading the delays in the comb stages. In order to validate theoretical design, a few test examples are designed for different filter parameters. The superiority of the new CIC filter functions is established by comparing these novel CIC FIR architectures with existing classical CIC structures. The new filter functions maintain simplicity of FIR filters by avoiding multipliers, but show excellent performances versus classical CIC filter functions. They have the same level of constant group delay, as well as number of delay elements, but the new designed functions give higher insertion losses in stopband, as well as they have higher selectivity.

*Key words:* digital filters, CIC FIR filters, multiplier free design, linear phase, selective filters.

### 1. INTRODUCTION

The first traces of CIC (Cascaded-Integrator-Comb) filters date back to the 80's. E.B. Hogenauer [1] proposed a class of hardware-efficient linear phase finite impulse response (FIR) filters known as CIC filters or Hogenauer filters. The CIC filters are becoming very popular due to their properties such as multiplier free design and no memory is required for the storage of filter coefficients. These properties make them very efficient in terms of hardware implementation and computational complexity. A large number of methods for improving the efficiency of digital filters have been described in [2–4].

The one of CIC filter disadvantages is not flat passband, which is undesirable in many applications because the original signal can be destroyed. Also, the CIC filter ensuring high folding band attenuations has a high passband droop due to its sinc-like characteristic. Because of these disadvantages, it is of a great interest to improve magnitude response characteristic. Various improvements of classical CIC filters have been reported in the last two decades [5–18]. The improvement can be done by modifying the basic CIC structure [5–7], by connecting a compensation filter (CIC compensator) in the cascade with the original filter [8–15], as well as by designing novel classes of CIC FIR filter functions [16–18].

A CIC filter is cascade connection of simple integrator and comb filter stages. Design of a novel class of selective CIC filter functions based on the classical CIC filters, by spreading the delays in the CIC filter comb stages, is recently shown in [16–18]. In [16–17], novel CIC filter functions in the explicit compact form, as well as their frequency response characteristics and performance improvements over the classical CIC filters, are presented. The novel designed classes give higher insertion losses in the stopband region, and higher selectivity. The paper [18] provides graphs which can be used to design a novel class of selective CIC filters given specification which is suggested in [17]. They are very useful for the designers who will be able to do selection of the design parameters of the novel filter functions that they need for the particular design task.

In this paper, modified CIC FIR filter functions which preserve the CIC filter simplicity avoiding the multipliers, are designed. The novel filter functions are given in recursive and non-recursive forms, suitable for software and hardware realizations, respectively. The performance analysis in more detail through an example is done. It starts by showing locations of function zeros on the unit circle. Then, a detailed analysis of frequency response characteristics, as well as their comparison with the classical filter functions is presented in graphical and tabular forms. A comparative study of the performances is made with that of well-

known classical CIC filters. Comparisons are made under totally fair conditions: the same number of cascades and the same level of signal delay, which enters the filter. Also, some parameters of the novel class CIC filter functions (*i.e.* passband cut-off frequency and minimum attenuation in stopband) and their dependence on free parameters are given. The results illustrate the superiority of the suggested novel CIC filter functions and show that they can be a good alternative instead of classical CIC filters.

## 2. CLASSICAL CIC FILTER FUNCTIONS

The classical CIC FIR filter function of normalized amplitude response characteristic, represented in the  $z$ -domain, is defined as

$$H(N, K, z) = (H(N, z))^K = \left( \frac{1 - z^{-N}}{N \cdot (1 - z^{-1})} \right)^K = \left( \frac{1}{N} \sum_{r=0}^{N-1} z^{-r} \right)^K, \quad (1)$$

where  $N$  is the decimation factor, and  $K$  is the number of CIC sections [1]. The frequency response characteristic of CIC FIR filter function is evaluated by setting  $z = e^{j\omega}$ , where  $\omega = 2\pi \cdot f$  is angular frequency in radians per second. Using Euler's identity, it can be written as

$$H(N, K, z = e^{j\omega}) = e^{-jK(N-1)\omega/2} \cdot \left( \frac{\sin(N\omega/2)}{N \cdot \sin(\omega/2)} \right)^K. \quad (2)$$

The normalized amplitude response characteristic,  $A(N, K, \omega)$ , can be obtained from Eq. (2). The magnitude response characteristic,  $|H(N, K, e^{j\omega})|$ , is obtained as absolute value of normalized amplitude response characteristic. The linear phase response characteristic of the modified CIC FIR filter functions,  $\varphi(N, K, \omega)$ , is defined as the phase angle of the complex filter frequency response given in Eq. (2). A FIR filter has a linear phase and therefore a constant group delay. The constant group delay response characteristic of the modified CIC FIR filter functions is expressed as

$$\tau(N, K, \omega) = -d\varphi(N, K, \omega) / d\omega = (N-1) \cdot K / 2. \quad (3)$$

The CIC filter has a lowpass frequency characteristic and a linear phase characteristic. Also, it has a large droop in passband that depends on the decimation factor  $N$  and the section number  $K$ .

## 3. DESIGN OF NOVEL CIC FIR FILTER FUNCTIONS

### 3.1. NON-RECURSIVE AND RECURSIVE FORMS OF NOVEL CIC FILTER FUNCTIONS

A non-recursive form of the modified CIC FIR filter functions is

$$H(N, K, L, z) = \left( \frac{1}{N-1} \sum_{r=0}^{N-2} z^{-r} \right) \cdot \left( \frac{1}{N} \sum_{r=0}^{N-1} z^{-r} \right) \cdot \left( \frac{1}{N+1} \sum_{r=0}^N z^{-r} \right) \cdot \left[ \frac{1}{N-3} \cdot \frac{1}{N-2} \cdot \frac{1}{N-1} \cdot \frac{1}{N} \cdot \frac{1}{N+1} \cdot \frac{1}{N+2} \cdot \frac{1}{N+3} \right]^L \cdot \left[ \left( \sum_{r=0}^{N-4} z^{-r} \right) \cdot \left( \sum_{r=0}^{N-3} z^{-r} \right) \cdot \left( \sum_{r=0}^{N-2} z^{-r} \right) \cdot \left( \sum_{r=0}^{N-1} z^{-r} \right) \cdot \left( \sum_{r=0}^N z^{-r} \right) \cdot \left( \sum_{r=0}^{N+1} z^{-r} \right) \cdot \left( \sum_{r=0}^{N+2} z^{-r} \right) \right]^L, \quad (4)$$

where  $N$  and  $L$  are free integer parameters, and  $K = 7L + 3$ . This new filter class represents cascade connection of three non-identical CIC FIR filter sections  $H(N-1, z)$ ,  $H(N, z)$  and  $H(N+1, z)$ , as well as

seven cascade-connected non-identical CIC FIR filter sections ( $H(N-3, z)$ ,  $H(N-2, z)$ ,  $H(N-1, z)$ ,  $H(N, z)$ ,  $H(N+1, z)$ ,  $H(N+2, z)$  and  $H(N+3, z)$ ) which are repeated  $L$  times.

A recursive form of the modified CIC FIR filter functions is

$$H(N, K, L, z) = \frac{1-z^{-(N-1)}}{(N-1) \cdot (1-z^{-1})} \cdot \frac{1-z^{-N}}{N \cdot (1-z^{-1})} \cdot \frac{1-z^{-(N+1)}}{(N+1) \cdot (1-z^{-1})} \cdot \left( \frac{1-z^{-(N-3)}}{(N-3) \cdot (1-z^{-1})} \cdot \frac{1-z^{-(N-2)}}{(N-2) \cdot (1-z^{-1})} \right)^L \cdot \left( \frac{1-z^{-(N+1)}}{(N+1) \cdot (1-z^{-1})} \cdot \frac{1-z^{-(N+2)}}{(N+2) \cdot (1-z^{-1})} \cdot \frac{1-z^{-(N+3)}}{(N+3) \cdot (1-z^{-1})} \right)^L \quad (5)$$

and  $K = 7L + 3$ .

The proposed filter function has a normalized amplitude response characteristic.

### 3.2. FREQUENCY RESPONSE CHARACTERISTIC OF NOVEL CIC FILTER FUNCTIONS

Frequency response characteristic of designed FIR filter functions is obtained by evaluating the filter function in the  $z$ -plane at the sample points defined by setting  $z = e^{j\omega}$ , where  $\omega = 2\pi \cdot f$  is angular frequency in radians per second. Using Euler's identity, frequency response characteristic can be written as

$$H(N, K, L, z = e^{j\omega}) = e^{-jK(N-1)\omega/2} \cdot \frac{\sin((N-1)\omega/2)}{(N-1) \cdot \sin(\omega/2)} \cdot \frac{\sin(N\omega/2)}{N \cdot \sin(\omega/2)} \cdot \frac{\sin((N+1)\omega/2)}{(N+1) \cdot \sin(\omega/2)} \cdot \left( \frac{\sin((N-3)\omega/2)}{(N-3) \cdot \sin(\omega/2)} \cdot \frac{\sin((N-2)\omega/2)}{(N-2) \cdot \sin(\omega/2)} \cdot \frac{\sin((N-1)\omega/2)}{(N-1) \cdot \sin(\omega/2)} \cdot \frac{\sin(N\omega/2)}{N \cdot \sin(\omega/2)} \right)^L \cdot \left( \frac{\sin((N+1)\omega/2)}{(N+1) \cdot \sin(\omega/2)} \cdot \frac{\sin((N+2)\omega/2)}{(N+2) \cdot \sin(\omega/2)} \cdot \frac{\sin((N+3)\omega/2)}{(N+3) \cdot \sin(\omega/2)} \right)^L \quad (6)$$

where the parameter  $K = 7L + 3$ . The normalized amplitude response characteristic of the proposed filter functions,  $A(N, K, L, \omega)$ , is defined as the magnitude of the complex filter frequency response  $H(N, K, L, z = e^{j\omega})$ . The magnitude response characteristic  $|H(N, K, L, e^{j\omega})|$  is obtained as absolute value of normalized amplitude response characteristics.

The linear phase response characteristic of the proposed novel class of the modified CIC FIR filter functions is defined as the phase angle of the complex filter frequency response, and has the form

$$\varphi(N, K, L, \omega) = -(N-1) \cdot K \cdot \omega / 2 + 2 \cdot \nu \cdot \pi, \quad \nu = 0, 1, 2, \dots, \text{ and } K = 7L + 3. \quad (7)$$

A FIR filter has a linear phase and therefore a constant group delay. The constant group delay response characteristic of the proposed novel class of the modified CIC FIR filter functions is expressed as

$$\tau(N, K, L, \omega) = (N-1) \cdot K / 2, \text{ and } K = 7L + 3. \quad (8)$$

### 3.3. SELECTION OF THE DESIGN PARAMETERS

The choice of free integer parameters  $N$  and  $L$  is done in the same way as for CIC filters, there are the same restrictions on the group delay response. The parameter  $K$  can take different integer values,  $K = 7L + 3$ .

The attenuation in the stopband region is closely related to the parameter  $L$ . By increasing  $L$  for the constant value of  $N$ , the higher stopband attenuation is achieved.

The constant group delay  $\tau$  is equal for the classical CIC filters (Eq. (3)) and the novel modified CIC filter functions (Eq. (8)). The values of constant group delay for different values of parameters  $N$ ,  $L$  and  $K = 7L + 3$  are given in Table 1.

Table 1

Group delay  $\tau(\omega)$  a for  $N \in \{5, 6, \dots, 11\}$ ,  $L \in \{1, 2, 3\}$  and  $K = 7L + 3$

$N$		5	6	7	8	9	10	11
$\tau[s]$	$L = 1, K = 10$	20	25	30	35	40	45	50
	$L = 2, K = 17$	34	42.5	51	59.5	68	76.5	85
	$L = 3, K = 24$	48	60	72	84	96	108	120

#### 4. DESIGN EXAMPLE AND PROPERTIES OF CIC FIR FILTER FUNCTIONS

The locations of zeros in  $z$ -plane along with their multiplicities for the classical CIC and the proposed class of CIC filter functions are shown in Fig. 1. All zeros lied on the unit circle. They are shown for case of  $N = 9$ , and  $L = 2$ . The classical CIC filter function has  $N - 1$  different zeros,  $z_r = e^{j2\pi r/N}$ ,  $r = 1, 2, \dots, N - 1$ . The total number of zeros is  $(N - 1) \cdot K$ . Note that the classical CIC filters have all multiple zeros with maximum multiplicity equal to the number of cascades  $K$ , which is not the case in the proposed solutions. The zeros of the proposed filter classes are more evenly distributed with their multiplicities therefore reduced as can be seen in Figure 1b.

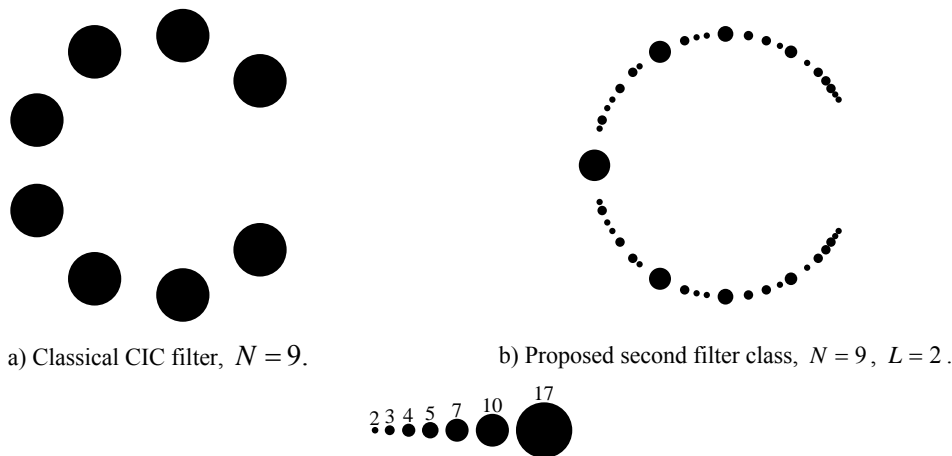


Fig. 1 – Locations and multiplicities of filter function zeros in  $z$ -plane for  $N = 9$ , and  $K = 17$  cascades.

The benefit of the novel CIC filter functions will be demonstrated by a few example functions. For that purpose, filter parameters are chosen as:  $N = 11$  and  $L \in \{1, 2, 3\}$  which gives  $K \in \{10, 17, 24\}$ .

In Tables 2 and 3, parameter values of both the classical CIC filter function  $H(N, K, z)$  and the novel CIC filter functions  $H(N, K, L, z)$  obtained for  $N = 11$ , and  $K \in \{10, 17, 24\}$ , are presented respectively. The given parameters are: passband and stopband cut-off frequencies,  $f_{cp}$  and  $f_{cs}$ , maximum attenuation in the passband,  $\alpha_{\max} = 0.28$  dB, and minimum attenuation in the stopband region,  $\alpha_{\min}$  [dB]. The classical CIC FIR filter functions and the designed CIC FIR filter functions have the same number  $K$  of cascaded sections with the difference that the CIC filters have an identical structure in all cascades, and the designed novel class has a cascade-connected CIC filter sections of different lengths. Also, they have the same level of constant group delay, as well as number of delay elements, but the novel designed filter functions give higher insertion losses in stopband, as well as it has higher selectivity. Achieved improvement of the stopband

attenuation is 32.71%, 27.18% and 28.62%, for values of  $L \in \{1, 2, 3\}$ , respectively. Note that the normalized stopband cut-off frequencies for novel filter functions are practically identical for different values of integer parameter  $L$ , but minimum attenuation in the stopband region increase rapidly by increasing its value.

Comparison of the normalized magnitude response characteristics in dB, for the classical CIC filters and the novel class of CIC FIR filter functions, is depicted in Fig. 2. Also, Fig. 2 gives zooms of the normalized magnitude response characteristics of the classical CIC filters and novel class of CIC filter functions in the transient and the stopband areas. In this figure benefits of attenuation of novel filter functions are depicted.

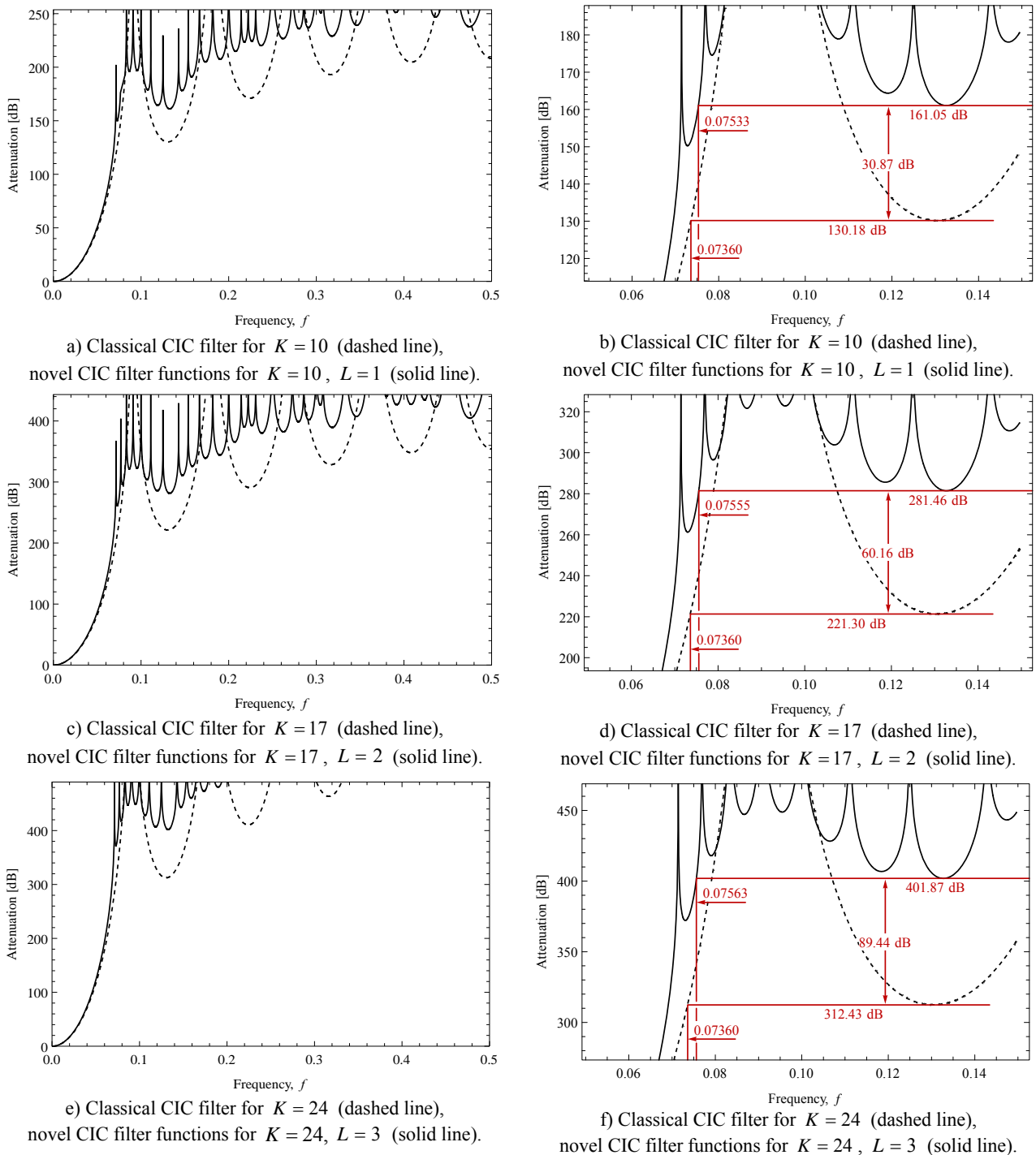


Fig. 2 – Comparison of normalized magnitude response characteristics in dB of classical CIC filter (dashed lines), and novel CIC FIR filter functions (solid lines), for  $N = 11$  (a, c, e – in whole frequency range and b, d, f – corresponding zooms).

Table 2

Cut-off frequencies in passband and stopband, constant group delay and stopband attenuation of classical CIC filter for  $K \in \{10, 17, 24\}$  and  $N = 11$

$N$	$K$	$f_{cp}$	$\alpha_{max}$ [dB]	$f_{cs}$	$\alpha_{min}$ [dB]
11	10	0.00404	0.28	0.07360	130.18
11	17	0.00310	0.28	0.07360	221.30
11	24	0.00261	0.28	0.07360	312.43

Table 3

Cut-off frequencies in passband and stopband, constant group delay and stopband attenuation of modified CIC filter functions for  $K \in \{10, 17, 24\}$  (obtained for  $L \in \{1, 2, 3\}$ ),  $N = 11$  and  $\alpha_{max} = 0.28$  dB

$N$	$L$	$K$	$f_{cp}$	$f_{cs}$	$\alpha_{min}$ [dB]
11	1	10	0.00399	0.07533	161.05
11	2	17	0.00306	0.07555	281.46
11	3	24	0.00257	0.07563	401.87

Figure 3 presents two-dimensional (2D) contour plots of normalized magnitude response characteristics of the classical CIC filters and the proposed novel CIC FIR filter functions. It shows overall and lower frequency part zoomed characteristics. As the value of the parameter  $N$  increases, as well as the normalized frequency increases, the benefits of the proposed filter class become less apparent, and the characteristics closely resemble those ones of the classical CIC filters. Therefore, it can be concluded that the proposed filter class is more efficient in lower part of frequency range and for smaller values of the parameter  $N$ .

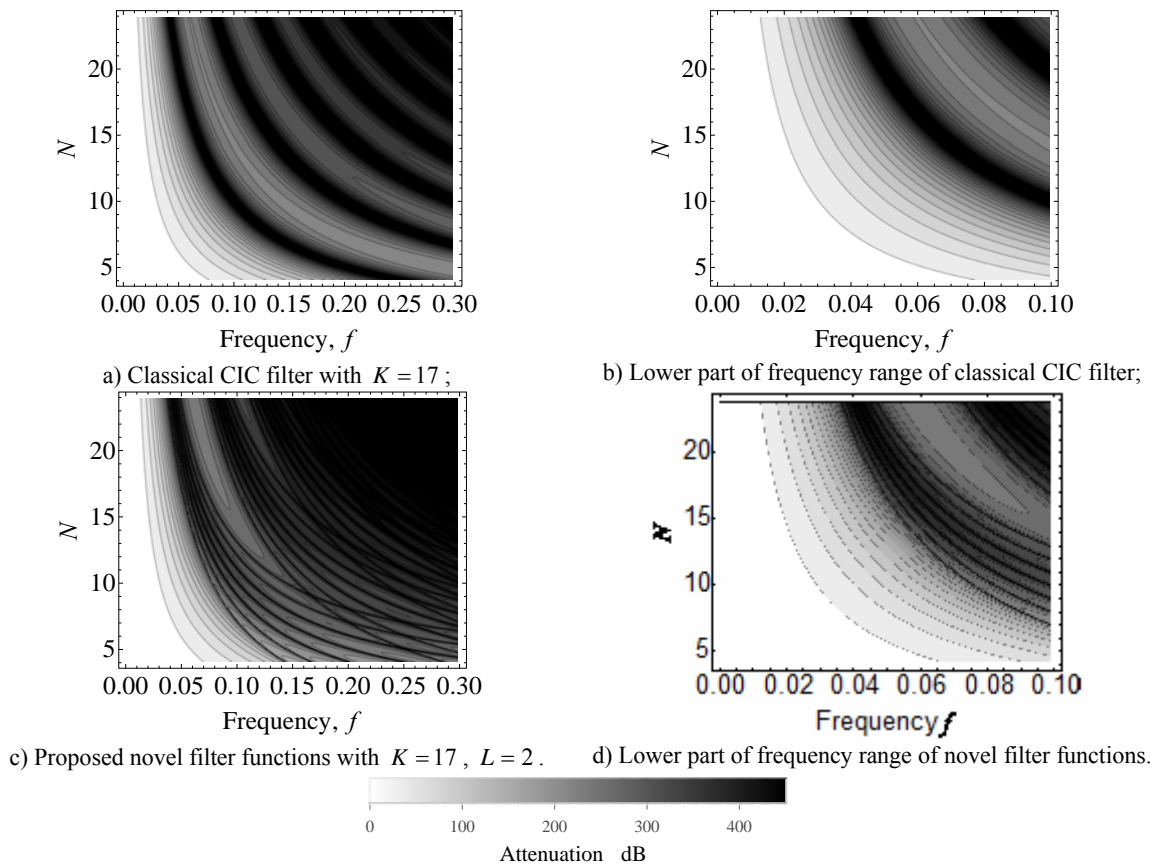


Fig. 3 – 2D contour plots of magnitude frequency response characteristics for classical and novel CIC FIR filter functions for  $N \in \{4 \div 24\}$  and  $K = 17$  .

In Fig. 4, three-dimensional (3D) plot of normalized magnitude response characteristic of novel CIC FIR filter functions is shown. There is shown normalized magnitude response in frequency domain as a function of parameter  $N \in \{4 \div 17\}$ , for case of  $L = 2$ . It is worth to noting that with the increase in the value of the parameter  $N$  the passband becomes narrower, as is expected. The number of transfer function zeros is increased and this is clearly visible in branching of high loss regions in magnitude response characteristics, especially for the smaller values of the parameter  $N$  and towards higher frequencies.

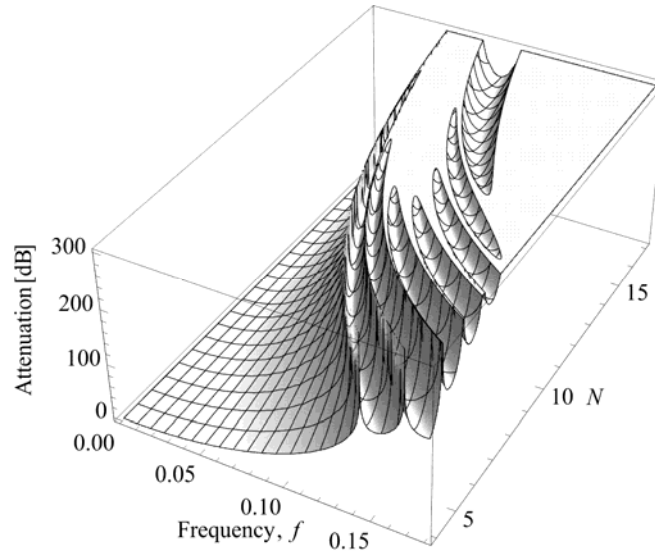


Fig. 4 – 3D plot of normalized attenuation response characteristic in dB of the novel CIC FIR filter functions for  $N \in \{4 \div 17\}$ , and  $K = 17$  obtained for  $L = 2$ .

## 5. CONCLUSION REMARKS

In this paper, an attempt has been made to introduce an innovative design of novel linear phase multiplierless finite duration impulse response (FIR) filter functions using several cascaded non-identical CIC FIR sections. In the last decades, CIC filters have been successfully used for sample rate conversion in modern communications systems [19–20]. Some modified filter structures for sigma-delta analog-to-digital applications are given in [21–24]. The novel filter classes suggested here can be used in these applications.

An important measure of the performance superiority of the proposed CIC FIR filter functions is to compare them to the characteristics of the classical CIC filters. The classical CIC filters and the designed novel CIC FIR filter functions have the same number of cascaded sections with the difference that the CIC filters have an identical structure in all cascades, and the designed novel functions have a cascade-connected CIC filter sections of different lengths. Also, they have the same level of constant group delay, as well as number of delay elements, but the novel designed CIC FIR filter functions give higher insertion losses in stopband region of interest, as well as they have higher selectivity.

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