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LOCALIZED OPTICAL STRUCTURES: AN OVERVIEW OF RECENT THEORETICAL AND EXPERIMENTAL DEVELOPMENTS

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We provide a brief overview of recent theoretical and experimental studies in the area of two- and three-dimensional localized optical structures, which were performed in a series of relevant physical settings. We aim to review recent works on formation and dynamics of localized structures in various Hamiltonian (dissipationless) and dissipative nonlinear optical systems.

Key words: localized optical structures, dissipative structures, spatial optical solitons, spatiotemporal optical solitons, light bullets.

1. INTRODUCTION

In the past years, the concept of *solitons* – more generally speaking, the notion of *localized structures* – has been generalized to systems that are not integrable in the strict mathematical sense [1, 2]. Thus, this concept has been extended to the most general class of nonlinear dissipative systems [3–20] with an infinite number of degree of freedom. It is worth noting that the Hamiltonian (conservative) nonlinear systems can be regarded as a subclass of dissipative ones, while the integrable (or completely integrable) can be considered as a subset of Hamiltonian ones; for an in-depth discussion of these key issues, see the relevant work of Akhmediev and Ankiewicz [6]. The localized optical structures form in a variety of conservative and dissipative nonlinear environments. One can distinguish between temporal, spatial, and spatiotemporal localized optical structures, see several comprehensive reviews in these research areas [21–28] and other relevant works [29–43]. It may be anticipated that the optical solitons can play a key role in all-optical processing of information and in future ultra high speed optical networks. It is worth noting that comprehensive studies of fundamental (vorticityless) and vortex solitons in nonlinear atomic media and in Bose-Einstein condensates have been also reported, see, e.g. the works [44–47] and references therein.

The *dissipative optical solitons* (sometimes called *autosolitons*) form in non-Hamiltonian nonlinear systems, i.e., in dissipative nonlinear optical media, and are characterized by a permanent energy exchange with the environment. Dissipative localized optical structures emerge as a result of a balance between dispersion/diffraction and nonlinearity and between gain and loss effects. Solitons in Hamiltonian systems form, in general, multi-parameter families of localized wave structures, while dissipative solitons form zero-parameter families and their key features are entirely fixed by the parameters of the underlying nonlinear optical system. Also, there are two main species of dissipative solitons, namely, *cavity solitons* and *propagating dissipative solitons*. The former are spatially localized transverse peaks in transmission or reflection in nonlinear Fabry-Perot cavities, while the latter do not require any feedback mechanism and are in fact forward propagating wave fields.

It is well known that the emergence of localized structures and localized patterns in dissipative media results from self-organization phenomena. The formation, i.e., the *morphogenesis* of dissipative localized structures in systems far from equilibrium has motivated a lot of activity since the pioneering papers of Turing [48] on the chemical basis of morphogenesis and Prigogine and Lefever [49] on symmetry breaking instabilities in dissipative systems. Such localized structures consist in regions in patterned state surrounded by a domain in the homogeneous steady state. The localized patterns arise in various fields such as optics and photonics, laser physics, fluid mechanics, plant ecology, chemistry, etc. A special issue on "Localized structures in dissipative media: from optics to plant ecology" has been recently published in *Philosophical Transaction of the Royal Society A*, see Ref. [50] for a comprehensive review of this topic and Refs. [51–55] for relevant recent works on key issues concerning localized structures such as their stability and dynamical properties.

This paper is organized as follows. In Section 2 we give a brief overview of recent theoretical and experimental advances in the area of localized structures in dissipative nonlinear media. In Section 3 we outline the unique properties of two- and three-dimensional dissipativeless localized structures, which form in a vast class of nonlinear optical media. Finally, in Section 4 we summarize our main conclusions.

2. LOCALIZED STRUCTURES IN DISSIPATIVE NONLINEAR OPTICAL MEDIA

The creation and stability of dissipative solitons (i.e., localized structures in dissipative optical media) rely on the simultaneous balance of diffraction/dispersion and self-focusing nonlinearity in the conservative part of the physical system, and the linear and nonlinear gain/loss terms in its dissipative part. In order to prevent collapse of the optical wavepacket in the (2+1)-dimensional model, the cubic self-focusing nonlinearity is supplemented by the quintic self-defocusing one. The generic physical model that adequately describes the formation of such localized structure is based on the complex Ginzburg-Landau (CGL) equation with the cubic-quintic gain and loss terms, combined with the background linear loss term, see, e.g. Refs. [56–73]. The generic CGL equations describe pattern-formation phenomena in diverse areas of physical sciences, such as superconductivity, reaction-diffusion systems, Bose-Einstein condensation of quasiparticles (exciton polaritons) in solid-state media, etc. In recent works, it has been found that inhomogeneous gain-loss landscapes can support a variety of stable dissipative localized structures in optical media, see e.g. Refs. [12, 61]. As a typical example, we consider the generic two-dimensional cubic-quintic CGL equation with a periodic lattice potential [56–58]:

$$iu_z + \frac{1}{2}(u_{xx} + u_{yy}) + |u|^2 u + \nu |u|^4 u = iN(u) - [R(x,y) - iL(x,y)]u, \tag{1}$$

where $u(x, y, z)$ is the wave amplitude, z is the propagation distance, and (x, y) are the transverse coordinates. In the case of a periodic axisymmetric lattice potential representing the cylindrical lattice, one has $R(x, y) = R(r)$ and $L(x, y) = L(r)$, where r is the radial coordinate. Further, the negative coefficient ν is the strength of the quintic self-defocusing term, and the combination $N(u)$ of the cubic-quintic nonlinear terms is given by $N(u) = -\alpha u + \varepsilon |u|^2 u + \mu |u|^4 u$, where $\alpha > 0$ is the linear-loss coefficient, $\mu < 0$ is the quintic-loss parameter, and the positive coefficient ε accounts for the cubic gain. In the case of the one-dimensional version of the two-dimensional cubic-quintic complex Ginzburg-Landau equation (1) the third term in the left-hand side of Eq. (1) is dropped, and the one-dimensional lattice potential is taken as $R(x, y) = R(x)$ and $L(x, y) = L(x)$.

Following Ref. [56], we consider the one-dimensional version of Eq. (1) and the loss-modulation function of the sinusoidal form, $L(x) = d \sin(x/5)$, while $R(x) = 0$. For certain amplitudes of the modulated loss, solitons spread in the course of the propagation, due to excess gain, when the linear loss coefficient α is smaller than a critical value. If the linear loss coefficient increases, the transverse gradient force produced by the inhomogeneous (periodic) loss induces a leftward drift of the soliton, see the left panel of the top of Fig. 1. We see that the soliton may even move with acceleration at the initial stage of propagation, due to the power loss suffered when the soliton passes through dissipative channels of the structure. When the linear-loss coefficient increases much more, the soliton performs a persistent swing, see the right panel on the top of Fig. 1. If the linear loss coefficient exceeds some threshold value (slightly depending of the value of the parameter d of the model), the solitons firstly drift to the right hand side over a short spatial interval, which is limited to a half period of the periodic loss-modulation function $L(x)$. The drift is followed by damped oscillations; see the left panel on the bottom of Fig. 1. Eventually, this swinging soliton transforms into an output stationary one, located to the right of the input position ($x = 0$), as seen in the left panel on the bottom of Fig. 1. When the parameter α exceeds a certain critical value, the soliton decays under the action of heavy losses, see the right panel on the bottom of Fig. 1; for detailed studies of these issues see Ref. [56].

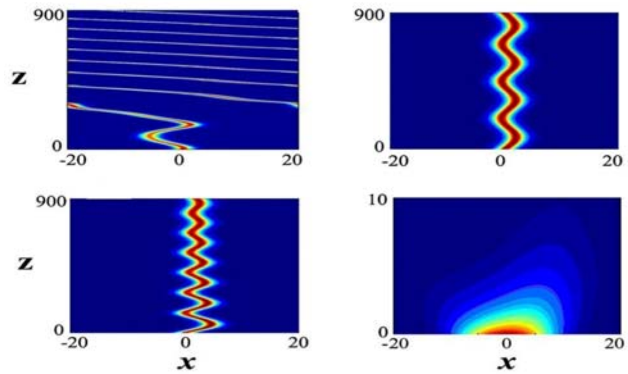

Fig. 1 – Typical propagation regimes in the case of inhomogeneous (sinusoidal-type) loss-modulation profile: the leftward drift of solitons (top, left); persistent swing scenario (top, right); f) damped oscillations of solitons (bottom, left); g) decay scenario (bottom, right) – adapted from Ref. [56].

Skarka *et al*. [62] have studied numerically a complex cubic-quintic Ginzburg-Landau equation with localized linear gain as a two-dimensional (2D) generic model for pattern formation proceeding via *spontaneous breaking of the axial symmetry.* Thus the generic model introduced in Ref. [62] was based on the (2+1)-dimensional complex cubic-quintic Ginzburg-Landau equation that governs the evolution of wave amplitude $u(x, y, z)$ in a medium with cubic-quintic nonlinearity:

$$iu_z + \frac{1}{2}\left(u_{xx} + u_{yy}\right) + \left(1 - i\varepsilon\right)|u|^2 u - \left(\nu - i\mu\right)|u|^4 u = ig(r)u. \qquad (2)$$

Here the positive coefficients ν, ε, and μ account, respectively, for the saturation of the cubic nonlinearity, cubic gain, and quintic loss. The "iceberg of the gain", $g(r) = \gamma - \Gamma r^2$ (r is the radial coordinate and γ, $\Gamma > 0$, where γ is the gain amplitude and Γ is the gain curvature) is protruding above the surface of the "loss sea". In a previous work by Skarka *et al*. [63], it was investigated a different physical setting with the "submerged iceberg", where the real-valued control parameter γ was negative. Starting from the steady-state solutions produced by the variational approximation developed by Skarka and Aleksić [64], extensive numerical simulations of the above Ginzburg-Landau equation generate a vast class of robust solitary-wave structures: (a) varieties of asymmetric rotating vortices carrying a topological charge, and (b) four- to ten-pointed revolving "stars" without intrinsic topological charge [62]. It was found that the four- and five-pointed "stars" feature a cyclic change of their structure in the course of the rotation, whereas six-, seven-, eight-, nine-, and ten-pointed "stars" steadily revolve, keeping constant the shapes and the angular momenta, with zero topological charge (vorticity), unlike vortices, angular momenta of which being proportional to the topological charge, see Ref. [62] for a detailed study of these issues. In Fig. 2 we illustrate the spontaneous formation of a four-pointed "star" with zero vorticity (S = 0). This four-pointed "star" features a periodic change of its structure during rotation; see Ref. [62] for a detailed discussion of this unique behavior of the four-pointed "star" pattern. Figure 3 shows the evolutions of the revolving eight- and ten-pointed "star-patterns" that keep undistorted their shapes; the star "patterns" are stable objects with nonzero angular momenta but with zero topological charges (vorticities).

Besse *et al*. [70–71] have analyzed pattern-formation scenarios in the 2D complex Ginzburg-Landau equation with the cubic-quintic nonlinearity and periodic external potential of the form $V(x, y) = V_0$ [cos(2x) + cos(2y)], where V_0 is the amplitude of the "cellular" potential. In Ref. [71] it was studied numerically the mobility of kicked soliton complexes, such as dipoles, quadrupoles, rhombic-shaped (onsite-centered) vortices, and square-shaped (offsite-centered) vortices. In Fig. 4 we see the result of the application of a kick to the square-shaped (offsite-centered) vortex in two different directions, i.e., for two distinct values of the kick angle θ. The initial kick breaks the symmetry between the top and the bottom rows of the soliton complex, generating an array of additional solitons in the up vertical direction (see the left panel in Fig. 4) or in the down vertical direction (see the right panel in Fig. 4). Thus, the possibility of controlling the direction of the emission of the soliton array by varying the direction of the initial kick is clearly illustrated in Fig. 4 [71].

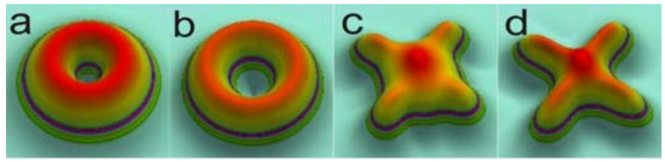

Fig. 2 – The spontaneous formation of a four-pointed "star" with zero vorticity ($S = 0$). The input ring-type structure: a) evolves into a vortex soliton; b) with vorticity (topological charge) $S = 1$. The spontaneous symmetry breaking of the vortex soliton produces an asymmetric "Celtic-cross"-type pattern; c) that subsequently transmutes into a symmetric four-pointed star-type pattern (adapted from Ref. [62]).

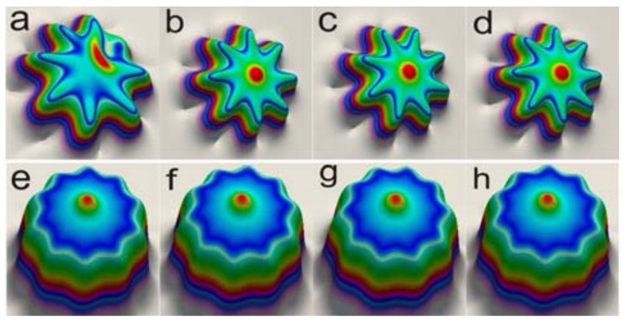

Fig. 3 – a) – d): The evolution of the revolving eight-pointed "star-pattern". In panel (a) it is seen the process of expelling the phase dislocation from the input vortical structure. The "octopus" soliton emerges (b), which rotates (c) and an identical configuration to that illustrated in pattern (b) is obtained eventually (d); e) – h): The typical rotation of a ten-pointed "star-pattern" (a "decapod" soliton) – adapted from Ref. [62].

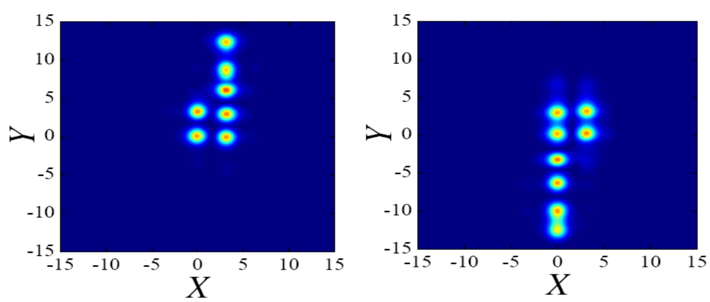

Fig. 4 – The pattern produced by the off-site-centered vortex kicked in two different directions. The kick angles are $\theta = 5\pi/8$ (left panel) and $\theta = 13\pi/8$ (right panel) – adapted from Ref. [71].

3. DISSIPATIVELESS LOCALIZED STRUCTURES IN NONLINEAR OPTICAL MEDIA

In this Section we briefly overview some recent experimental and theoretical works on multidimensional optical solitons that form in a variety of nonlinear optical media such as waveguide arrays, nonlocal optical media and optical lattices. In a previous theoretical study [74] it was predicted that hexagonal lattices of parallel linearly coupled waveguides, with the intrinsic cubic self-focusing nonlinearity, give rise to three species of stable semidiscrete soliton complexes with embedded vorticity (topological "charge") S: (a) triangular modes with vorticity $S = 1$, (b) hexagonal ones with vorticity $S = 2$, both modes being centered around an empty central core, and (c) compact triangles with vorticity $S = 1$, without the empty central core. We have also numerically simulated the collisions between stable triangular vortices, demonstrating the stoppage of the slowly moving vortex solitons, destabilizing rebounds, and quasielastic passage, depending on the collision velocity [74]. Eilenberger *et al.* [75] reported the first experimental observation of discrete vortex light bullets (spatiotemporal solitary waves with nonzero orbital angular momentum), which where first investigated theoretically in Ref. [74]. Conditions for their existence were analyzed and their rich properties and dynamics were investigated in detail. Such discrete vortex light bullets with vorticity number $S = 1$ were excited in fiber waveguide arrays (composed by 91-core silica arrays) with spatially shaped femtosecond pulses and were analyzed with a spatiotemporal cross correlator. These localized optical structures are robust to perturbations, in a limited range of energies. The vortex light bullets (vortex spatiotemporal optical solitons) are bound states of three temporally sinchronized pulses in a triangular configuration with phase shifts of $2\pi/3$ between them. They are semistable and decay into a set of three desynchronized light bullets after a certain propagation length, see Ref. [75] for a detailed study of these issues. Recently, Tran *et al*. [76] have studied theoretically the formation and dynamics of spatially broad light bullets generated in silica waveguide arrays. Such broad light bullets are metastable when high-order dispersion, coupling dispersion, and the Raman effect are included in the governing model. However, the narrow light bullets with energy located in three adjacent waveguides are extremely robust even in the presence of the Raman effect; see Ref. [76] for more details of this theoretical study.

It is well known that the creation of multidimensional solitons is a challenging problem in nonlinear optics and in the area of matter-waves (atomic Bose-Einstein condensates), see e.g. [24, 25, 44–47]. This challenge is due to the fact that the fundamental (vorticityless) solitons are prone to instabilities caused by the collapse phenomenon, while the vortex solitons are destroyed by the azimuthal instabilities that split them into a set of fundamental solitons. However, as reported in a recent work [77], models of optical self-trapping supported by a spatially growing strength of a repulsive cubic optical nonlinearity – the strength must grow from the center to periphery faster that r^3, where r is the radial coordinate in the three-dimensional space – gives rise to robust vortex tori, i.e., three-dimensional vortex solitons, with topological charges $S \geq 1$. The soliton family with vorticity $S = 1$ was found to be completely stable, while the one with topological charge $S = 2$ was found to have alternating regions of stability and instability. Moreover, it was also shown in Ref. [77] that application of a moderate torque to the vortex torus initiates a precession mode, with the torus' axle moving along a conical surface. In another recent work reported by the same group [78] it was shown that in a nonlinear medium with a repulsive nonlinearity that grows from the center to the periphery, a set of complex stationary and dynamical three-dimensional (3D) localized structures can be formed. Thus, peanut-shaped modulation profiles give rise to vertically symmetric and antisymmetric vortex solitons, and novel stationary hybrid vortex solitons, built of top and bottom vortices with opposite topological charges, as well as robust dynamical hybrids, which feature stable precession of a vortex on top of a zero-vorticity soliton. Stability regions for symmetric, antisymmetric, and hybrid solitons were found by extensive numerical simulations. These 3D hybrid vortex solitons might be realized in media with controllable cubic nonlinearities, e.g., in optics and in Bose-Einstein condensates [78].

The formation and stability of light bullets in spatially modulated Laguerre-Gauss optical lattices (in which both linear and nonlinear changes in the refractive index are spatially modulated) were investigated by numerical simulations [79]. It was demonstrated that the linear and nonlinear contributions considerably affect both the light bullet shape and its range of stability, while the nonlinear modulation depth affects the width of the stability domain. It was thus demonstrated that the properties of light bullets in Laguerre-Gauss optical lattices are much different from those in Bessel lattices, see Ref. [79] for more details. Also, the same group [80] reported the unique properties of 3D Hermite-Bessel solitons in strongly *nonlocal media* with variable potential coefficients. Self-similar Hermite-Bessel solitons and higher-order localized structures in the form of vortex solitons and multipole solitons were also investigated in Ref. [80]. Both 2D and 3D solitons in media with competing cubic-quintic nonlinearities and parity-time-symmetric complex-valued external potentials were investigated theoretically in Ref. [81]. Driben and Meier [82] have studied the dynamics of 3D Airy-vortex wave packets under the action of strong self-focusing Kerr nonlinearity. The emission of vortex light bullets with vorticities equal to those carried by the parental light structures was demonstrated [82]. Smetanina *et al*. [83] recorded the formation of light bullets during filamentation of femtosecond laser pulse in fused silica in the anomalous group velocity regime. This kind of light bullets have been observed, with duration of about two optical cycles in the near infrared (λ=1800 nm), see also Ref. [84] for a detailed theoretical and experimental study of these effects. The observation of *optical rogue waves* associated with the emission of high amplitude resonant radiation during the formation of 3D light bullets in sapphire nonlinear crystal with anomalous group velocity dispersion has been recently reported by Roger *et al*. [85]. The sapphire crystal was 8 mm long and the wavelength range for pump pulses was

1.7–2.2 μm [85]. Majus *et al*. [86] have revealed the nature of 3D spatiotemporal optical solitons (3D light bullets) in bulk Kerr media made of sapphire samples. Such light bullets were generated from the self-focusing of intense femtosecond pulses in bulk dielectric nonlinear media with anomalous group velocity dispersion. The self-focusing dynamics in sapphire crystals of 100 fs pulses at 1800 nm was captured in the full four-dimensional space by means of a 3D imaging technique, see Ref. [86] for more details of this study. It was demonstrated that the generated light bullet consists of a sharply localized high-intensity core that carries the self-compressed pulse (about 25% of the total energy) and a ring-shaped low-intensity periphery in the form of a Bessel-like profile [86].

4. CONCLUSIONS

In this work we have attempted to provide the interested reader with a general overview of the current state-of-the-art of the continuously growing research area of nonlinear optics of two- and three-dimensional localized structures in a variety of physical settings. We have briefly described both experimental and theoretical results that have recently been reported in the literature. The described results mainly refer to studies on fundamental and vortical localized structures in two and three dimensions. The huge experimental efforts have inspired and triggered a lot of recent theoretical investigations. We conclude with the hope that this brief overview on recent exciting theoretical and experimental developments in the field of nonlinear optics of two- and three-dimensional localized structures will inspire further studies.

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