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# MODELING THE PROCESSES OF DISTRIBUTION OF RESOURCE FLOWS

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In this paper we propose an expansion method for finding exact stable solutions for problems of distribution of stochastic streams of resources, in which the constraint matrix could be close to degenerate and the basic parameters of the problem could be uncertain. We propose a method of solving such problems in models complicated by randomness and possible (but not required) near singularity of the constraint matrix, which commonly arises in systems with parallel processes. The method finds solution of the problem by first solving a simpler problem with an "expanded" constraint set, and then performing a directed transition to the solution of the original problem. In completing these steps the method takes account of the stochastic nature of the problem.

*Key words*: parallel structure systems, modeling, random flows, optimization, extension method, small parameters.

## 1. INTRODUCTION

Allocation of physical resources or information flows in systems with parallel structures is a common practical problem. Finding an optimal solution to such problems often requires costly computational procedures and methods. These high computational costs arise because the parallel structures are typically, at least partially homogeneous, which leads to ill-conditioned constraint matrixes with near-singular determinants. As a result, optimization problems with such ill-conditioned constraint sets become highly unstable and hard to solve. There are methods that were developed to tackle these kinds of complications. For example, [1] studied a problem of finding stable solutions in systems with singularities in the constraint matrix by using "stabilizing functional". This idea has been also applied to problems with near singular constraint matrixes [2–5]. For instance, [2] proposes a method which first disregards small differences between nearly collinear constraints in order to extract a so called "characteristic system" of the problem, and then uses this system to assess the impact of small differences between constraints on optimal solution. The aforementioned computational methods have an important theoretical significance. However their applicability is conditional on a set of fairly stringent assumptions regarding the nature of singularity in the constraint set. Also these methods can only provide approximate solutions. An alternative solution method has been proposed in [6–8] by the first listed author of this paper and his colleagues. These studies develop an "Extension method," for solving problems of resource allocation in systems with parallel objects with possible (but not required) near singularity of the constraint matrix. The main idea of this method is to start from a solution of a simpler problem with an expanded (i.e. relaxed) constraint set, and then perform a directed transition to the optimal solution by re-introducing the original constraints, which happen to be binding at the solution of the relaxed problem. Such differentiation between binding and nonbinding constraints not only eliminates the sensitivity of the proposed method to near-singularity of the constraint set, but also allows obtaining exact solutions. The main contribution of this paper is a generalization of the Extension method to a new class of resource distribution problems with parallel resource flows (figure 1). More specifically, the authors generalize this method to problems admitting stochastic flows, non-linear objective functions, as well as small parametric differences (perturbations) among constraints implied by parallel objects. These results lay a theoretical foundation for the development of effective algorithms for solving similar problems in actual complex production processes with stochastic resource flows, as outlined in the concluding section.

## 2. DEFINING THE RESEARCH PROBLEM

Before we state the formal optimization problem in the next section, let us use this section to describe in more general terms the types of problems solvable by our method. Modern production processes in industry, agriculture and services involve complex stochastic flows of parts, raw materials, orders, etc. Often these flows are random not only in the timing of their arrival $t_j$, but also in many other parameters like, for example, volume $S_j$. These parameters could be discrete, continuous, and multi-dimensional. In this paper we consider modeling of resource flows in conjunction with the problem of their optimal allocation. To this end we formalize the problem as a problem of allocating random resource flows in systems with parallel structure (Fig. 1). As examples of systems with parallel structure we might consider multiple furnaces processing metal ore, or a large scale storage facility with multiple warehouses operating during a harvesting season.

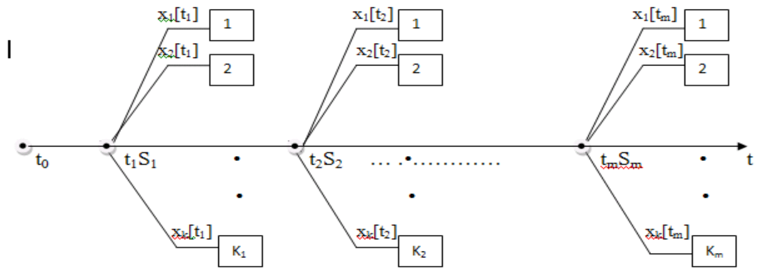


Fig. 1 – Allocating random resource flows in systems with parallel structure.

The aim of this study is to build the algorithms and a simulation model for modeling flow of resources and their allocation among parallel structures, which take in account various constraints and priorities imposed by economic and technological conditions, as well as small but economically important parametric differences among parallel objects. The existing models of technological processes and economic relations often formalize resource or order flows as non-stationary Poisson processes with a wide spectrum of random distributions characterizing various quantitative parameters of the flows (e.g. volumes). Numerous technical complications arise in modeling those resource allocation problems due to variable economic priorities or complex mathematical implementations of resource flows. General procedure for building resource flow and resource allocation models in systems with parallel objects involves the following steps:

- modeling processes of resource inflows;
- modeling the allocation of resources and solving the associated optimization problem at each moment of resource arrival;
- optimal analysis of the overall allocation efficiency and design of corrective algorithms.

The following sections will further specify these steps of the general procedure, paying special attention to the stochastic nature of resource flows and their arrival moments.

## 3. MODELING PROCESSES OF RESOURCE ARRIVALS

In modeling the elements of the chain $\{t_j\}$, let us consider a fairly general case when the set $T = \{t_j\}$ follows a stationary Palm's flow [9] with a given density function $\varphi(\tau)$ over intervals between its elements, starting from the second interval. To determine the moments $t_j$, let us use the standard formula

$$t_j = t_{j-1} + \tau_j,$$

where $j = 1,2,...,n$, $\tau_j$ are intervals between the elements of chain $T$.

To model Palm's flows it is insufficient to know $\varphi(\tau)$, because the density function over the first interval is different from $\varphi(\tau)$ [6], i.e. $\varphi_1(\tau) \neq \varphi(\tau)$. Thus, to find $\varphi_1(\tau)$ we need to use the following formula

$$\varphi_1(\tau_1) = \lambda \left( 1 - \int_0^{\tau_1} \varphi(\tau) d\tau \right), \tag{1}$$

where $\lambda$ is the intensity of the flow.

The values of the intervals $\tau_j$ between elements of the chain $T$ are determined with the help of inverse functions of random variables according to the principles stated in the following two cases [9]:

- Realized values of a continuously distributed random variable $\tau$ are determined from the formula

$$F(\tau) = \int \varphi(\tau)\, d\tau = u \tag{2}$$

or equivalently from

$$\tau = F^{-1}(u)$$

where $u$ is a random variable drawn from a uniform distribution $U(0,1)$ bounded between zero and one, and where $\varphi(\tau)$ is the density function.

- In case of a discrete distribution, the values $\tau_k$ are drawn from the table $\begin{pmatrix} \tau_1 & \tau_2 & \dots & \tau_m \\ p_1 & p_2 & \dots & p_m \end{pmatrix}$, and

realize with probability $p_k$ whenever the following condition is satisfied $u \in \Delta_k$, where $\Delta_k = p_k$.

If the density function $\varphi(\tau)$ represents one of the standard continuous distribution functions, then we can use the formulas provided in table 1 for modeling the random intervals $\tau_j$ between the elements of chain $T$ [6].

*Table 1*

The formulas for modeling standard continuous theoretical distributions

| Distribution | Normal | Uniform | Exponential | Linear | Gamma |
| --- | --- | --- | --- | --- | --- |
| The density function | $f(\tau) = \frac{1}{\sigma_\tau \sqrt{2\pi}} e^{-\frac{(\tau - m_\tau)^2}{2\sigma_\tau^2}}$, $-\infty < \tau < \infty$ | $f(\tau) = \frac{1}{b-a}$, $\tau \in [a,b]$ | $f(\tau) = \lambda e^{-\lambda\tau}$, $\tau \geq 0$ | $f(\tau) = \lambda(1 - \frac{\lambda}{2}\tau)$, $\tau \in [0, \frac{2}{\lambda}]$ | $f(\tau) = \frac{\alpha^k}{(k-1)!} t^{(k-1)} e^{-\alpha\tau}$, $\alpha > 0$, $k > 0$, $\tau \geq 0$ |
| Formula for modeling | $\tau = m_\tau + \sigma_\tau (\sum_{i=1}^{12} u_i - 6)$ | $\tau = a + u(b-a)$ | $\tau = -\frac{1}{\lambda} \ln u$ | $\tau = -\frac{2}{\lambda}(1 - \sqrt{u})$ | $\tau = -\frac{1}{\alpha} \ln(u_1 * u_2 * \dots * u_k)$ |

A more complete list of standard continuous theoretical distributions and their formulas is provided in [9].

The most effective algorithm for modeling non-stationary Poisson processes of resource arrival is based on the method of thinning intensity flows [10].

Let the process of resource inflow be represented by a pair $\{t_j, S_j\}$, describing the moment of resource arrival and its volume correspondingly. Assume that these flows follow a stochastic process with the intensity function given by $\lambda(t)$.

Depending on their distribution, the method of "inverse functions" as in formula (2) above, or a "John von Neumann's elimination" method described in [6] can be used for modeling volumes of resources.

We use the following steps to construct a non-stationary Poisson process for resource arrival moments from a uniform distribution:

*Step 1*. Set the initial parameters of the model: time interval ($T_0 - T_n$), number of subintervals ($n$), upper boundary of $j^{th}$ subinterval ($T[j]$), process intensity for each subinterval ($\lambda_j$), parameters of the uniform distribution, *a* and *b*.

*Step 2*. Enter and save the initial data in databases.

*Step 3*. Set $t = 0$, $\lambda_{\max} = \max\{\lambda[t]\}$, and $k = 1$.

*Step 4*. Generate $u_i \sim U(0,1)$ for $i = 1,2,3$.

*Step 5*. Compute Lewis-Shedler values of the arrival moments for the non-stationary Poisson process (thinning method) [10]

$$t_j = t_{j-1} - \frac{1}{\lambda_{\max}} * \ln u_1 .$$

*Step 6*. Iterate over all subintervals $(j)$.

*Step 7*. Check if $t \le (T(j) - T(j-1))$. If satisfied, set $\lambda(t) = \lambda(j)$, and skip directly to step 10.

*Step 8*. Update $j = j + 1$.

*Step 9*. If $j \le n$, go to step 7. If not, go to step 13.

*Step 10*. If $\{u_2 \le \lambda(t)/\lambda_{\max}$ and $t \le T_n\}$, set $t[k] = t$, $k = k + 1$ and go to step 12. If not, go to step 4.

*Step 11*. Compute a value of $S(t_j)$ from the inverse transformation $S(t_j) = a + u_3 * (b - a)$.

*Step 12*. Save simulated values of the arrival moment $t_j$ and of the volume $S(t_j)$ into results database. Transition to step 4.

*Step 13*. End of the loop.

*Step 14*. Print results.

## 4. RESOURCE ALLOCATION IN PARALLEL PROCESSES WITH PERTURBED PARAMETERS

The problem of resource allocation for any member of the set $T = \{t_j\}$ has the form

$$F(t_j) = \max_x f(x), \tag{3}$$

subject to constraints of:

– technological nature

$$\begin{aligned} g_1(x) &\le q_1(t_j), \\ &\dots \\ g_m(x) &\le q_m(t_j), \end{aligned} \tag{4}$$

– resource demand-supply balances

$$Ex = S(t_j), \tag{5}$$

– lower and upper limits on resource uses

$$V \le x \le W, \tag{6}$$

where $f(x)$ and $g_i(x)$ are assumed to be continuously differentiable real valued functions defined on a space of $n \times 1$ vectors $x$. The $1 \times n$ unit vector $E$ in the equation (5) sums the elements of vector $x$. Thus, equation (5) simply states that the elements of $x$ must sum to a real value $S(t_j)$, with the later representing the total amount of resource available. Finally, $n \times 1$ vectors $V$ and $W$ represent technologically imposed lower and upper bounds on the amounts of resources allocated to each element of vector $x$.

The functions $g_i(x)$ in the constraint (4) are the constraints imposed by the parallel objects. As such they can be very similar to each other and thus nearly collinear. To highlight this complication more prominently let us assume that all the constraints (4) are similar up to small parametric differences among them (i.e. perturbations). We can decompose these functions into the following general form:

$$g_1(x) = r_0(x) + \varepsilon r_1(x),$$
$$\dots$$
$$g_m(x) = r_0(x) + \varepsilon r_m(x),$$

where functions $r_0(x)$ and $r_1(x)....r_m(x)$ have the same functional forms as $g_i(x)$, with $r_0(x)$ capturing the common, or average parameterization, and $r_1(x)....r_m(x)$ capturing constraint-specific perturbations of parameters. A small positive scalar $\varepsilon$ is introduced in order to normalize the parameters of $r_1(x)....r_m(x)$ so that their values are close to those of $r_0(x)$.

In accordance with the extension method [6, 7] we introduce an auxiliary extended problem, obtained from the original problem by discarding the constraints of the form (4).

$$F(t_j) = \max_x f(x), \tag{7}$$

subject to constraints:

$$Ex = S(t_j), \tag{8}$$

$$V \le x \le W. \tag{9}$$

Suppose that the original problem (3–6) has a unique solution. Let us establish a connection between solution sets $X$ and $X^E$ for the original and extended problems respectively.

PROPOSITION 1. *The set of admissible solutions* $X$ *of the original problem* (3–6) *is always a subset of the set of solutions of the extended problem* (7–9), *i.e.* $X \subseteq X^E$.

*Proof.* Since $X^E$ was obtained by relaxing some of the constraints imposed on $X$, the proposition is obviously true.

PROPOSITION 2. *The optimal solution of the original problem coincides with the optimal solution of the extended problem if either*:

1) $X^E \subseteq X$, or
2) *the solution* $x^E$ *of the extended problem belongs to* $X$, *i.e.* $x^E \subseteq X$.

*Proof.* The objective functions of the extended and of the original problems are identical. Therefore, the equivalence of the admissible solution sets of these problems implies the equivalence of the problems themselves and, consequently, of their optimal solutions. Further, if $x^E \in X$, no constraint of the original problem, defined by the restriction (4), is binding.

In a more interesting case, if $x^E \notin X$, implement the following general algorithm of solving the problem of resource allocation among parallel objects.

I. Modeling elements of the chain $\{t_j\}$.

II. Solve the extended problem (7–9).

III. Verify if the obtained solution is admissible with respect to restrictions (4) of the original problem. If the decision is admissible, then it is optimal, otherwise go to step IV.

IV. Select the direction and step of a descent.

V. Transition to a new solution.

The new solution, obtained as a result of a descent would be optimal if the movement in this direction leads to the smallest change in the objective function value compared with other directions.

Let us now specify this algorithm in more details using concrete examples. We start with an example which has linear objective function and linear constraints [8].

$$F(t_j) = \max_x (cx), \quad (10)$$

$$A_0 + \varepsilon A_1 x \le q_1(t_j), \quad A_0 + \varepsilon A_2 x \le q_2(t_j), \quad (11)$$

$$Ex = S(t_j), \quad (12)$$

$$V \le x \le W, \quad (13)$$

where $c$ is a set of fixed weights of the objective function, and $A_0, A_1, A_2$ are $n \times n$ square matrixes, which represent linear analogs of the above mentioned constraint functions $r_0(x)$ and $r_1(x)....r_m(x)$, for the case of $m$=2. As is known [2], the likely near collinearity of the constraints (11) makes linear programming solution methods impractical, yielding imprecise and highly unstable solutions.

Based on the general algorithm outlined above, we can solve the original problem (10)-(13) by applying the extension method by realizing the following steps.

*Step 1*. Using the algorithm described in section 3 above, model the timing and the magnitudes of resource flows $S(t_j)$.

*Step 2*. Solve the extended problem (10), (12) and (13).

*Step 3*. Check if the obtained solution $x^E$ satisfies the constraints (11). If the solution is fully admissible, then it is optimal. Otherwise go to step 4.

*Step 4*. Analyze the coefficient matrixes of the objective and constraint functions, $c$ and $A_1, A_2$, in order to find the following differences:

$$\Delta c_{j,j+p} = c_j - c_{j+p}, \quad j = 1,2,...n, \quad p = 1,2,...n - j;$$
$$\Delta a^i_{j,j+p} = a^i_j - a^i_{j+p}, \quad j = 1,2,...n, \quad p = 1,2,...n - j, \quad i \in I_H,$$

where $I_H$ is a subset of constraints (11) which are binding at $x^E$. With those differences at hand, we can determine possible descent directions $N_B$:

$$N_B = N_B^{dir} \cup N_B^{rev},$$

where

$$N_B^{dir} = \left( (j,\ j+p) | \Delta c_{j,j+p} \ge 0 \ \text{ and } \ \Delta a^i_{j,j+p} > 0 \quad \forall i \in I_H \right);$$
$$N_B^{rev} = \left( (j+p,\ j) \,|\, \Delta c_{j,j+p} \le 0 \ \text{ and } \ \Delta a^i_{j,j+p} < 0 \quad \forall i \in I_H \right).$$

If $N_B = \varnothing$, then the system of original constraints is incompatible.

*Step 5*. Determine the optimal decent direction *(k, l)* from the condition

$$\max_{i \in I_H} \min_{(j,j+p) \in N_B} \left( \frac{\Delta c_{j,j+p}}{\Delta a^i_{j,j+p}} \left( q_i^E - q_i(t_j) \right) \right),$$

where $q_i^E$ represents the value of the $i^{th}$ constraint of (11) evaluated at $x^E$.

*Step 6*. Calculate the size of the decent in the selected direction

$$h_{kl} = \frac{\left( q_i^E - q_i(t_j) \right)}{\Delta a^i_{kl}}.$$

*Step 7*. Check if the step size is small enough to observe the lower and upper bound constraints (13)

$$h_{kl} \le \min\left\{x_k^p - V_k, W_l - x_l^p\right\}.$$

If the above inequality is satisfied, go to step 9. If $h_{kl} = 0$, eliminate this direction from consideration and return to step 5.

*Step 8*. Set

$$h_{kl} = \min\left\{x_k^p - V_k, W_l - x_l^p\right\}.$$

*Step 9*. Using formula

$$x_j = \begin{cases} x_j^p - h_{kl}, & \text{if } j = k; \\ x_j^p + h_{kl}, & \text{if } j = l; \\ x_j^p, & \text{if } j \ne k, l. \end{cases}$$

find new candidate solution of the problem (10–13) and go to step 3.

Further, we demonstrate how the extension method can be applied to nonlinear optimization problems using an example with a quadratic objective function.

$$F = \max(cx + x^T Dx), \tag{14}$$

subject to:

$$A_0 x + \varepsilon A_1 x \le q, \tag{15}$$

$$Ex = S. \tag{16}$$

where equation (15) possibly represents several inequality constraints. Once again it is decomposed in a way which highlights the typical problem arising when considering relatively homogeneous parallel objects. The constraint (15) might often have nearly collinear restrictions, leading to highly ill-conditioned constraint matrixes and unstable solutions.

Assume that the matrix $D$ is a negative definite matrix. The expanded problem is:

$$F^e = \max(cx + x^T Dx), \tag{17}$$

subject to:

$$Ex = S. \tag{18}$$

Since we eliminated problematic constraints (15), the expanded problem can be solved with the Lagrange multiplier method

$$\mathrm{L}(x, \lambda) = F(x) - \lambda(\sum x_i - S),$$

$$\frac{\partial \mathrm{L}}{\partial x_i} = \frac{\partial F(x)}{\partial x_i} - \lambda = 0, \tag{19}$$

$$\frac{\partial \mathrm{L}}{\partial \lambda} = \sum x_i - S = 0. \tag{20}$$

Solving the system of equations (19–20) we can find $x^E$ and $\lambda$. Now we expand the function $F(x)$ into a Taylor series of order two about the point $x^E$ [11]:

$$F(x) = F(x^{\mathrm{E}}) + \nabla F(x^{\mathrm{E}})h + 0.5h^T Hh. \tag{21}$$

where $H$ is Hessian matrix and $h = (x - x^E)$. Consider the second and the third terms of (21) separately

$$\nabla F(x^{\mathrm{E}})h = \left(\frac{\partial F}{\partial x_1}, \ldots, \frac{\partial F}{\partial x_k}, \ldots, \frac{\partial F}{\partial x_l}, \ldots, \frac{\partial F}{\partial x_n}\right) \times \begin{pmatrix} 0 \\ \vdots \\ -h_{kl} \\ \vdots \\ h_{kl} \\ \vdots \\ 0 \end{pmatrix}$$

$$= 0 + \ldots + \left(-\frac{\partial F(x^{\mathrm{E}})}{\partial x_k} + \frac{\partial F(x^{\mathrm{E}})}{\partial x_l}\right) \times h_{kl} + \ldots + 0.$$

Since (19) implies that

$$\frac{\partial F(x^{\mathrm{E}})}{\partial x_k} = \frac{\partial F(x^{\mathrm{E}})}{\partial x_l} = \lambda,$$

it follows that

$$\nabla F(x^{\mathrm{E}})h = 0.$$

Turning to the third term of (21)

$$0.5h^{\mathrm{T}}Hh = 0.5\left(0, \ldots, -h_{kl}, \ldots, h_{kl}, \ldots, 0\right) \times \begin{pmatrix} 2d_{11} & \ldots & d_{1k} & \ldots & d_{1l} & \ldots & d_{1n} \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ d_{k1} & \ldots & 2d_{kk} & \ldots & d_{kl} & \ldots & d_{kn} \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ d_{l1} & \ldots & d_{lk} & \ldots & 2d_{ll} & \ldots & d_{\ln} \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ d_{n1} & \ldots & d_{nk} & \ldots & d_{nl} & \ldots & d_{nn} \end{pmatrix} \times \begin{pmatrix} 0 \\ \vdots \\ -h_{kl} \\ \vdots \\ h_{kl} \\ \vdots \\ 0 \end{pmatrix} = \left(d_{kk} + d_{ll} - d_{kl}\right)h_{kl}^2.$$

Taking the above equalities in account we can re-write (21) as

$$F(x) = F(x^{\mathrm{E}}) + \left(d_{kk} + d_{ll} - d_{kl}\right)h_{kl}^2.$$

Due to *linearity* of constraints, we can substitute the value of $h_{kl}$ :

$$F(x) = F(x^E) - (d_{kl} - d_{kk} - d_{ll})\frac{\left(q_i^E - q_i(t_j)\right)^2}{(a_k^i - a_l^i)^2}.$$

Here $i$ is the index of the constraint (15), which is violated at $x^E$; $k$ is the index of the element of vector $x^E$ which is being decreased as a result of decent; $l$ is the index of the element of vector $x^E$ which is being increased as a result of decent.

PROPOSITION 3. *The point* $x = x^E + h$ *is a solution of the problem* (14–16*)* if *and only if, the parameters i, k, l in the step* $h_{kl} = \dfrac{q_i^E - \mathrm{q}_{\mathrm{i}}(t_j)}{a_k^i - a_l^i}$ *are determined from the following formula*

$$\beta = \max_{i \in I_H} \min_{(j,j+p) \in N_B} \left\{ \left(d_{kl} - d_{kk} - d_{ll}\right) \frac{\left(q_i^E - q_i(t_j)\right)^2}{\left(a_k^i - a_l^i\right)^2} \right\}.$$

The proof of this proposition is somewhat involved and for the sake of brevity is not presented here. It can be proven in the same way as the corresponding proposition for the problem (10–13), provided by one of the authors of this paper in [6].

Thus, the structure of the algorithm for solving the quadratic problem of resource allocation is similar to that one for the previous problem of resource allocation (10–13) with linear objective function.

*Step 1*. Solve the expanded problem (17–18).

*Step 2*. Check whether the obtained solution $x^E$ is in the feasible set of the original problem. If the solution satisfies all the constraints (15) then it is optimal. Otherwise go to the step 3.

*Step 3*. Calculate

$$\gamma_{kl} = d_{kl} - d_{kk} - d_{ll},\ k = 1,2,\ldots,n,\ \ l = k + m\ \ \forall m = 1,2,\ldots,n-k,$$

and determine a set of possible directions $N_B$ of a descent from the following

$$N_B = \{(k,l) \,|\, \gamma_{kl} > 0\}.$$

*Step 4*. Determine the optimal direction of a descent *(k, l)* from the following condition

$$\beta = \max_{i \in I_H} \min_{(k,l)\in N_B} \left( \gamma_{kl} \frac{\left(q_i^E - q_i(t_j)\right)^2}{\left(a_k^i - a_l^i\right)^2} \right).$$

*Step 5*. Calculate the value of a descent in the selected direction

$$h_{k^* l^*} = \frac{q_{i^*}^E - q_{i^*}(t_j)}{a_{k^*}^{i^*} - a_{l^*}^{i^*}}.$$

*Step 6*. Shift to the new solution $x = x^E + h$ and go back to the step 2.

## 5. CONCLUSION

The results of this study were used to develop a control system for one of the largest metal producers in the world, the Ust-Kamenogorsk lead-zinc plant, which is structured into an extensive network of sequential and parallel processes [6]. The designed system, which was developed on the basis of research presented in the current and previous papers [7, 8], controlled the sulfur acid production process, which had five different sequential production processes: 1) in dry electric filters, 2) in drying towers, 3) in wet electric filters, 4) in absorbers and 5) contact devices. Each process involved 4 to 10 parallel projects with very similar but non-identical characteristics, among which the sulfuric gas had to be optimally allocated. This technological process is often perturbed by various random disturbances. The main disturbances being due to frequent variations in the amounts of gas pumped from different metallurgical furnaces and due to outright maintenance shutdowns of either preventive nature or for repairing unexpected malfunctions. In order to prevent emergency discharges of these gases into the atmosphere, it was necessary to develop a control system capable of efficiently redirecting gases to the remaining parallel aggregates, with those tasks performed at randomly arriving time moments. Because of relative homogeneity of the parallel processes, the resulting constraint matrix of the formulated model had a high degree of multi-colinearity with high degree of sensitivity to small parameter variations. As is known from [12], such equation systems have near degenerate constraint matrixes, leading to solution instability and a low degree of precision of obtained solutions. Experimental application of the extension method for allocation of sulfuric gases among parallel furnaces, simulated with the help of Monte-Carlo methods, allowed obtaining a solution with the error margin of less than 0.5 percent. Standard mathematical programming optimization methods had error margins of 10 to 15 percent and were not approved by the plant management. Practical application of our method has shown that in contrast to other methods, the proposed procedure for solving optimal resource allocation problems allows finding precise and stable solutions even when the constraint matrix is near singular.

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