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ANALYTICAL SOLUTIONS OF THE MECHANICAL ANSWER OF THIN ORTHOTROPIC PLATES

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This article presents new aspects of the developing of some analytical solutions for static analysis of thin curved orthotropic plates, based on the Classical Laminated Plate Theory (CLPT). Various boundary conditions and a charge of uniform pressure are considered. The analytical solutions are obtained by Ritz method in conjunction with the weighted residue method for the coefficients calculation. The analytical solutions are further compared with the experimental results and the numerical solution considering similar boundary value problems as treated analytically. It is presented the experimental device and the experimental test results, as well. The thorough comparison between analytical solutions, numerical results and experimental data reveals a good agreement of the results.

Key words: thin plate, orthotropic elasticity, analytical solution, boundary conditions, experimental device, numerical models.

1. INTRODUCTION

The composites appeared as alternatives for scarce materials in the background of general raw material crisis. Afterwards, as a result of their promise to join the advantageous properties of each component into a new high performance material, the composites spread all over technical domains, especially in thin walled structures for aircraft industry.

Taking into account the serious weight of the structure mathematical model inside the final value of the safety coefficient, a correct estimation of its mechanical answer represents a fundamental step in aircraft design. And this condition was stated on the general requirement of the aircraft design regulations that impose a value of 1.5 for the safety coefficient. As the mechanical answer of the structures consists in the values that one or more field functions (generally displacement functions) have all over the domain that the structure fills, the proper form of the function will guarantee the accuracy of the results. The main constraints for the field function will refer to satisfy as exactly as possible:

- the differential equations of the mathematical model;
- the displacement boundary conditions;
- the force/moment boundary conditions.

According to these requirements, the possible cases for a chosen field function that approaches the solution can be in one of the following cases:

- approximates both the differential equations and the boundary conditions;
- fulfills the differential equations and approximates the boundary conditions;
- approximates the differential equations and fulfills the boundary conditions;
- fulfills both the differential equations and the boundary conditions.

Starting from these terms, the present article presents some analytical solutions that can offer a simple, efficient and quite precise way to estimate and quantify the behavior of thin composite plates both plane and curved, with large radius of curvature in different boundary condition cases.

Under some assumptions that facilitate the problem to be modeled as a differential equation, the proposed field form functions are susceptible to fulfill different types of boundary conditions and to approximate in good terms the general equation. The considered hypothesis and the area of applicability will be presented in detail in the next chapter.

Besides the main issue of the academic component to better understand the complex mechanical answer of composite plates, the analytical models and solutions remain a prospective useful tool to validate a numerical solution, they are less expensive in time and calculation terms than the numerical ones, they can emphasize how the solution depends on initial data, etc.

In order to quantify the degree of confidence of the analytical solutions proposed for different boundary conditions few critical analyzes were performed with respect to physical and numerical tests for similar cases.

2. MIDSURFACE PARTIAL DIFFERENTIAL EQUATION

2.1. Problem Formulation

It is considered a thin ortotropic elastic plate with a large radius of curvature sibjected to an uniform pressure. The thickness of the plate is h – a function of midsurface point. In general terms the plate geometry will be well defined if the median surface equation (a) as well as the variation of the thickness (b) are known.

$$z = z(x, y), \tag{a}$$

$$h = h(x, y). \tag{b}$$

The curves are given by relations [5, 7] $\frac{1}{R_x} = \frac{\partial^2 z}{\partial x^2}$, $\frac{1}{R_y} = \frac{\partial^2 z}{\partial y^2}$, $\frac{1}{R_{xy}} = \frac{\partial^2 z}{\partial x \partial y}$ and deformed surface equation of the plate is given by $z' = z + w_i + w$, where w is the normal displacement and w_i is the deviation from the ideal surface.

2.2 Assumptions

In order to determine the differential equation that modells the mechanical answer of the loaded composite plate, some assumptions were considered [1, 2, 4, 7, 8]. The main issues of the assumptions were to preserve the principal properties that have a major influence for the mechanical answer and to eliminate the minor aspects that would complicate the mathematical model.

1. The plate is considered to be linear elastic, thin, homogenous and anisotropic – the orthotropic model.
2. The curvature of the median surface is considered to be small in all points.
3. A normal line segment to the midsurface remains normal during the plate deformation, i.e., if Oz is the axis normal to the plate, $\gamma_{xz} = \gamma_{yz} = 0$ (Kirchhoff).
4. The length of a normal line segment to the midsurface remains constant during the plate deformation, i.e., if Oz is the axis normal to the plate, $\varepsilon_z = 0$ (Kirchhoff).
5. The displacement of the midsurface points is only along Oz axis (w), the displacements in tangent plane are being neglected ($u \approx v \approx 0$).
6. The rectilinear normal hipothesis (3) is ”softened” considering that

$$|\gamma_{yz}| << \left|\frac{\partial v}{\partial z}\right|, \quad |\gamma_{yz}| << \left|\frac{\partial w}{\partial y}\right|, \quad |\gamma_{zx}| << \left|\frac{\partial u}{\partial z}\right|, \quad |\gamma_{zx}| << \left|\frac{\partial w}{\partial x}\right|.$$

2.3. Composite Plate Equations

The elastic constitutive equation with respect to an arbitrary cartesian coordinate system can be written as:

$$\{T_\sigma\} = \left[\bar{Q}\right] \cdot \{T_\varepsilon\}, \tag{1}$$

$$\begin{gathered}\{T_\sigma\} = \begin{pmatrix}\sigma_{xx} & \sigma_{yy} & \sigma_{xy}\end{pmatrix}^{\mathrm{T}} \text{ – stress tensor; } \{T_\varepsilon\} = \begin{pmatrix}\varepsilon_{xx} & \varepsilon_{yy} & \varepsilon_{xy}\end{pmatrix}^{\mathrm{T}} \text{ – strain tensor;} \\ \left[\bar{Q}\right] = [R] \cdot [Q] \text{ – general compliance matrix; } [R] \text{ – rotation matrix}, \end{gathered} \tag{2}$$

$$Q_{11} = \frac{E_1}{1 - \nu_{12} \cdot \nu_{21}}, \quad Q_{22} = \frac{E_2}{1 - \nu_{12} \cdot \nu_{21}}, \quad Q_{12} = \frac{\nu_{21} \cdot E_1}{1 - \nu_{12} \cdot \nu_{21}} = \frac{\nu_{12} \cdot E_2}{1 - \nu_{12} \cdot \nu_{21}}, Q_{66} = G_{12},$$

where E_1, E_2 are elasticity moduli in the longitudinal and transversal directions, respectively, G_{12} is the in plane shear modulus, and ν_{12}, ν_{21} are the Poisson coefficients.

For thin plates with large radius and small deformations, it can be written:

$$\varepsilon_{ij} = -\frac{w}{R_{ij}} - z \cdot \frac{\partial^2 w}{\partial i \partial j}, \quad i, j = x, y. \tag{3}$$

The resulting internal efforts (forces and moments)

$$\{M\} = -\frac{h^3}{12} \cdot \left[\bar{Q}\right] \cdot \{\aleph\}, \quad \{N\} = -h \cdot \left[\bar{Q}\right] \cdot \{\Im\}, \tag{4}$$

where

$$\begin{gathered}\{M\} = \begin{pmatrix}M_{yy} & M_{xx} & M_{xy}\end{pmatrix}^{\mathrm{T}}, \quad \{N\} = \begin{pmatrix}N_{xx} & N_{yy} & N_{xy}\end{pmatrix}^{\mathrm{T}}, \\ \{\aleph\} = \begin{pmatrix}\dfrac{\partial^2 w}{\partial x^2} & \dfrac{\partial^2 w}{\partial y^2} & 2 \cdot \dfrac{\partial^2 w}{\partial x \partial y}\end{pmatrix}^{\mathrm{T}}, \quad \{\Im\} = \begin{pmatrix}\dfrac{w}{R_{xx}} & \dfrac{w}{R_{yy}} & 2 \cdot \dfrac{w}{R_{xy}}\end{pmatrix}^{\mathrm{T}}.\end{gathered} \tag{4'}$$

Simple calculations lead to the partial differential equation for *w* (the displacement on *Oz* direction) of the plate under a uniform pressure:

$$\begin{gathered}\bar{A}w + \frac{12}{h^2}\left[\left(\frac{\bar{Q}_{11}w}{R_x} + 2\frac{\bar{Q}_{16}w}{R_{xy}} + \frac{\bar{Q}_{12}w}{R_y}\right)\left(\frac{1}{R_x} + \frac{\partial^2(w_i + w)}{\partial x^2}\right) + \left(\frac{\bar{Q}_{12}w}{R_x} + 2\frac{\bar{Q}_{26}w}{R_{xy}} + \frac{\bar{Q}_{22}w}{R_y}\right)\left(\frac{1}{R_y} + \frac{\partial^2(w_i + w)}{\partial y^2}\right)\right] + \\ + \frac{24}{h^2}\left(\bar{Q}_{16}\frac{w}{R_x} + 2\bar{Q}_{66}\frac{w}{R_{xy}} + \bar{Q}_{26}\frac{w}{R_y}\right)\left(\frac{1}{R_{xy}} + \frac{\partial^2}{\partial x \partial y}(w_i + w)\right) - \frac{12p}{h^3} = 0,\end{gathered} \tag{5}$$

where $\bar{A} = \bar{Q}_{11}\dfrac{\partial^4}{\partial x^4} + \bar{Q}_{22}\dfrac{\partial^4}{\partial y^4} + 2\left(\bar{Q}_{12} + 2 \cdot \bar{Q}_{66}\right)\dfrac{\partial^4}{\partial x^2 \partial y^2} + 4\bar{Q}_{26}\dfrac{\partial^4}{\partial x \partial y^3} + 4\bar{Q}_{16}\dfrac{\partial^4}{\partial x^3 \partial y}$.

In the particular case of plane plate (infinite radii of curvature) equation (5) becomes considerably simpler [1, 6, 8, 10]:

$$\bar{A}w = \frac{12 \cdot p}{h^3}. \tag{6}$$

In particular case, when O*x*-axis is oriented along the fiber direction $\left(\theta = 0^\circ\right)$, equation (6) becomes [1, 6, 7, 9, 10]:

$$Aw = \frac{12 \cdot p}{h^3}, \tag{7}$$

where $A = Q_{11} \cdot \frac{\partial^4}{\partial x^4} + Q_{22} \cdot \frac{\partial^4}{\partial y^4} + 2 \cdot (Q_{12} + 2 \cdot G_{12}) \cdot \frac{\partial^4}{\partial x^2 \partial y^2}$.

Equations (6) and (7) can be considered as generalized Sophie-Germain equations for thin orthotropic plates with respect to arbitrary coordinate system and, respectively, with respect to natural anisotropic directions.

3. ANALYTICAL SOLUTION

3.1. Orthotropic rectangular plane plate with clamped edges

It is considered an orthotropic elastic thin rectangular plate subjected to an uniform pressure *p* on the bottom face of the plate, with constant thickness, *h* , *a* and *b* length and respectively width, under the boundary conditions of clamped edge for $x = 0$, $x = a$ and $y = 0$, $y = b$.

In case for angle $(\theta = 0^\circ)$**,** it must be solved the equation (7) with boundary conditions [1, 7, 10, 11]:

$$\begin{cases} w(0,y) = w(a,y) = 0, & \frac{\partial w}{\partial x}(0,y) = \frac{\partial w}{\partial x}(a,y) \\ w(x,0) = w(x,b) = 0, & \frac{\partial w}{\partial y}(x,0) = \frac{\partial w}{\partial y}(x,b) = 0 \end{cases}. \tag{8}$$

By using Ritz method, a potential solution may have the following form [3, 9, 10, 11]:

$$w(x,y) = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij} \cdot \left(\frac{x}{a}\right)^{i+1} \cdot \left(\frac{y}{b}\right)^{j+1} \cdot \left(1-\frac{x}{a}\right)^{i+1} \cdot \left(1-\frac{y}{b}\right)^{j+1}. \tag{9}$$

The coefficients $(c_{ij})_{i,j}$ will be determined by using the method of weighted residues.

According to this method, the following linear system is solved

$$\iint_D \left[A(w(x,y)) - \frac{12p}{h^3} \right] \cdot \Phi_{ij}(x,y)\,\mathrm{d}x\,\mathrm{d}y = 0, \quad i = \overline{1,n}, \quad j = \overline{1,m},$$

where $\Phi_{ij} = \left(\frac{x}{a}\right)^{i+1} \cdot \left(\frac{y}{b}\right)^{j+1} \cdot \left(1-\frac{x}{a}\right)^{i+1} \cdot \left(1-\frac{y}{b}\right)^{j+1}$, $i = \overline{1,n},\ j = \overline{1,m}$, are weighting functions (Galerkin method) and *D* is the domain of the composite plate.

Reddy in [7] proposes a solution for equation (7) with boundary conditions (8) as:

$$w(x,y) = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij} \cdot \left(\frac{x}{a}\right)^{i+1} \cdot \left(\frac{y}{b}\right)^{j+1} \cdot \left(1-\frac{x}{a}\right)^{2} \cdot \left(1-\frac{y}{b}\right)^{2}. \tag{10}$$

As an example, for a thin orthotropic rectangular plate with dimensions $300\ \text{mm} \times 200\ \text{mm} \times 1.5\ \text{mm}$, $E_1 = 22{,}051\ \text{MPa}$, $E_2 = 18{,}512\ \text{MPa}$, $G_{12} = 8{,}642\ \text{MPa}$, $\nu_{12} = 0.071$, subjected to a uniform pressure $p = 0.00419\ \text{MPa}$, by using the solution (9), the maximum value of the deflection $w\left(\frac{a}{2}, \frac{b}{2}\right) = 3.053\ \text{mm}$, was determined for $m = n = 3$, similar as the one obtained by using the solution (10).

It is important to note a good agreement between the two solutions (9) and (10), but a faster convergence is obtained by using the solution (10).

In case for angle $(\theta \neq 0^\circ)$, it must be solved the equation (6) with the conditions (8).

By using Ritz method, it is proposed a solution as follows:

$$w(x,y)=\sum_{i=1}^{m}\sum_{j=1}^{n}c_{ij}\cdot\left(\frac{x}{a}\right)^{i+1}\cdot\left(\frac{y}{b}\right)^{j+1}\cdot\left(1-\frac{x}{a}\right)^{i+1}\cdot\left(1-\frac{y}{b}\right)^{i+1}+k_1\cdot\text{tg}\,\theta\cdot f_1(x,y)+k_2\cdot\text{tg}\,\theta\cdot f_2(x,y). \quad (11)$$

The last two terms have been added to provide an asymmetric solution and to verify the imposed boundary conditions. In this case there were considered:

$$f_1(x,y)=\left(\frac{x}{a}\right)^2\left(1-\frac{x}{a}\right)^3\left(\frac{y}{b}\right)^2\left(1-\frac{y}{b}\right)^3,\ f_2(x,y)=\left(\frac{x}{a}\right)^3\left(1-\frac{x}{a}\right)^2\left(\frac{y}{b}\right)^3\left(1-\frac{y}{b}\right)^2.$$

It can be observed that for an angle $\theta=0^\circ$ the proposed solution becomes similar with the form (9) of the solution.

For a plate with $300\ \text{mm}\times 200\ \text{mm}\times 1.45\ \text{mm}$ $E_2=22{,}051\,\text{MPa}$, $E_1=18{,}512\,\text{MPa}$, $\nu_{12}=0.071$, $G_{12}=8{,}642\,\text{MPa}$, $\theta=20^\circ$, subjected to a uniform pressure, $p=0.00419\,\text{MPa}$ and considering the solution (11), the maximum obtained value of the deflection was $w_{\max}=3.0138$ mm.

In order to see the influence of the angle between the fiber and O*x* axis, an orthotropic plate was considered with the same characteristics previously presented, and with an angle $\theta=0^\circ$, and using the solution (9).

3.2. Orthotropic cylindrical plate with clamped edges

A cylindrical orthotropic plate, with median surface equation $z=z(x,y)$, is defined by

$$z(x,y)=-c+\sqrt{R^2-\left(y-\frac{b}{2}\right)^2}, \quad (12)$$

where $c=R-d$, $d=1$ mm, $a=300$ mm, $b=200$ mm, and large radius of curvature $R=\dfrac{\left(\frac{b}{2}\right)^2+d^2}{2\cdot d}$, plate with $E_1=22{,}051\,\text{MPa}$, $E_2=18{,}512\,\text{MPa}$, $G_{12}=8{,}642\,\text{MPa}$, $\nu_{12}=0.071$, subjected to uniform pressure $p=0.00419\,\text{MPa}$.

For this cylindrical plate, in equation (5), $R_x\to\infty$, $R_{xy}\to\infty$, so it results:

$$\begin{aligned}&-\frac{h^3}{12}\left[\bar{Q}_{11}\frac{\partial^4 w}{\partial x^4}+\bar{Q}_{22}\frac{\partial^4 w}{\partial y^4}+2\left(\bar{Q}_{12}+2\bar{Q}_{66}\right)\frac{\partial^4 w}{\partial x^2\partial y^2}\right]-\frac{h^3}{12}\left[4\bar{Q}_{16}\frac{\partial^4 w}{\partial x^3\partial y}+4\bar{Q}_{26}\frac{\partial^4 w}{\partial x\partial y^3}\right]-\\&-h\left(\bar{Q}_{12}\frac{w}{R_y}\right)\left(\frac{\partial^2}{\partial x^2}(w_i+w)\right)-h\left(\bar{Q}_{22}\frac{w}{R_y}\right)\left(\frac{1}{R_y}+\frac{\partial^2}{\partial y^2}(w_i+w)\right)-2h\left(\bar{Q}_{26}\frac{w}{R_y}\right)\left(\frac{\partial^2}{\partial x\partial y}(w_i+w)\right)+p=0,\end{aligned} \quad (13)$$

where $R_y=R.$

In case of a plate with clamped edges and the angle $\theta=0^\circ$, by using solution (9) and operator $A=Q_{11}\cdot\dfrac{\partial^4}{\partial x^4}+Q_{22}\cdot\dfrac{\partial^4}{\partial y^4}+2\cdot(Q_{12}+2\cdot G_{12})\cdot\dfrac{\partial^4}{\partial x^2\partial y^2}+\dfrac{12\cdot Q_{22}}{h^2\cdot R^2}-\dfrac{12\cdot p}{h^3}$ the maximum value of the deflection was obtained as $w\left(\dfrac{a}{2},\dfrac{b}{2}\right)=1.992$ mm.

4. NUMERICAL RESULTS

By using the FEM program ANSYS (four noded shell element SHELL63) the numerical solutions for similar boundary problems as described in paragraphs 2.4.1 and 2.4.2 were obtained. Three cases were considered:

Case 1: Rectangular orthotropic elastic thin plate with $a = 300$ mm, $b = 200$ mm and thickness $h = 1.45$ mm subjected to a uniform distributed pressure $p = 0.00419$ MPa on the bottom face with clamped edges with $\theta = 0^{o}$. The maximum value of the deflection was $w\left(\frac{a}{2}, \frac{b}{2}\right) = 3.055$ mm.

Case 2: Rectangular orthotropic elastic thin plate with $a = 300$ mm, $b = 200$ mm and thickness $h = 1.45$ mm subjected to a uniform distributed pressure $p = 0.00419$ MPa on the bottom face, with clamped edges, for $\theta = 20^{\circ}$. The maximum value of the deflection was $w_{max} \simeq 3.032$ mm.

Case 3: Cylindrical thin plate with large curvature radius $R = 4999$ mm, thickness $h = 1.45$ mm subjected to a uniform pressure $p = 0.00419$ MPa on the bottom face, having clamped edges. The maximum value of the deflection was $w\left(\frac{a}{2}, \frac{b}{2}\right) = 1.996$ mm. These values are in good agreement with the values obtained by analytical solution.

5. EXPERIMENTAL DEVICE

For experimental analysis there are considered plates manufactured by incorporating the 8 layers of fiberglass fabric, type EWR 300 g/mp in Polylite 440-M888 resin, and applying the Vacuum Assisted Resin Transfer Molding (VARTM) technology.

Taking into account the necessity of several sets of measurements of orthotropic plate deflection, under the imposed boundary conditions as previously mentioned, it was chosen to apply the uniform distributed pressure perpendicular on the bottom face of the plate by using a pressurized air pillow.

This technical solution allows repeated measurements and different changes of applied pressure in a convenient way.

The measurement of the air pressure in the chamber is performed by a manometer with liquid. To measure the deformed plate deflection, an inductive displacement transducer was used as presented in Fig. 1.

The entire experimental system contains the following modules:

- Pressurized air pillow module connected with the orthotropic plate;
- Displacement transducer module in the horizontal plane *x*O*y*;
- Deflection measuring hardware and software module;
- Positioning module consisting in two step by step electric engines for O*x* and O*y* axes respectively.

The air pressure pillow is a rectangular prism of plastic foil and rubber bottom, in which two valves are incorporated, one for air pumping and one for connection of the manometer (for pressure measuring).

The experimental program implies also the determining of the orthotropic elastic characteristics. This is performed by tensile and shearing tests on a INSTRON testing machine considering different flat specimens $1 \times 25 \times 300$ intrumented with strain gadges as presented in Fig. 2.

Fig. 1 – Displacement transducer.

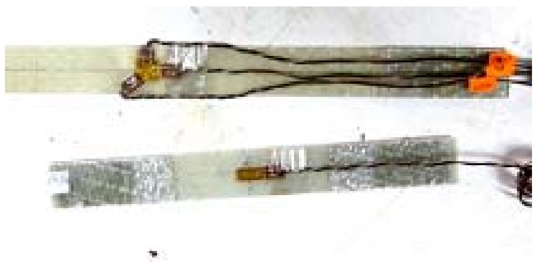

Fig. 2 – Flat specimens.

6. COMPARISON BETWEEN ANALYTICAL AND NUMERICAL SOLUTION AND EXPERIMENTAL DATA

To validate the proposed analytical solutions, there are presented some comparative results between experimental measurements, numerical and analytical results. For this purpose, some comparative diagrams are presented.

In Table 1 there are presented the comparative results between the experimental measurements, numerical and analytical results, for the first case of thin orthotropic plate, $300\ \text{mm} \times 200\ \text{mm} \times 1.45\ \text{mm}$, $\theta = 0^\circ$ subjected to the uniform pressure $p = 0.00419$ MPa .

Similarly, in Table 2 there are presented the comparative results for the orthotropic thin plate, $300\ \text{mm} \times 200\ \text{mm} \times 1.45\ \text{mm}$, $\theta = 20^\circ$, subjected to the uniform pressure $p = 0.00419$ MPa .

In table 3 it is presented the influence of curvature radius.

Table 1 ($p = 0.00419$ MPa , $\theta = 0^\circ$, for $y = 95$ mm)

Analytical (1)	0.274	1.518	2.564	3.023	2.878	2.119	1.529	0.869	0.277
Numerical (2)	0.286	1.539	2.571	3.021	2.872	2.121	1.539	0.884	0.286
Experimentally (3)	0.289	1.601	2.699	3.004	2.958	2.119	1.448	1.067	0.366
Difference (1)- (3) (%)	-5.474	-0.189	-0.135	0.019	-0.008	0	0.081	-0.19	-0.089

Table 2 ($p = 0.00419$ MPa , $\theta = 20^\circ$, for $y = 95$ mm)

Analytical (1)	0.283	0.883	1.539	2.117	2.556	2.842	2.982	2.84	2.112	0.880
Numerical (2)	0.289	0.89	1.542	2.117	2.56	2.854	2.999	2.852	2.114	0.886
Experimentally (3)	0.295	0.892	1.602	2.165	2.653	2.865	3.01	2.945	2.155	0.905
Difference (1) - (3) (%)	-4.24	-1.019	-4.093	-2.267	-3.794	-0.809	-0.938	-3.697	-2.035	-2.840

Table 3 ($p = 0.00419$ MPa , for $y = 95$ mm)

Analytical ($\theta = 0^\circ$)	0.274	0.862	1.518	2.106	2.564	2.87	3.023	2.878	1.529	0.869
Analytical ($\theta = 20^\circ$)	0.289	0.89	1.542	2.117	2.56	2.854	2.999	2.852	1.538	0.886
Difference in (mm)	-0.015	-0.028	-0.024	-0.011	0.004	0.016	0.024	0.026	-0.009	-0.017

Table 4 ($p = 0.00419$ MPa , $R = 4999$ mm , for $y = 95$ mm)

Rectangular	0.274	0.862	1.518	2.106	2.564	2.87	3.023	2.878	2.119	0.869
Cylindrical	0.206	0.63	1.077	1.452	1.724	1.893	1.973	1.893	1.452	0.63
Difference in (mm)	0.068	0.232	0.441	0.654	0.84	0.977	1.05	0.985	0.667	0.239

7. CONCLUSIONS

- The analitical solutions can offer accurate and rapid solutions with respect to the numerical ones enough close to the experimental tests.
- The additional terms of the solution can cover both different types of boundary conditions as well as different values of the anisotropy directions.
- The testing device can offer accurate loading system and continuous measurements along different curves chosen upon the free surface of the plate.
- As a result of imperfections of the plate clamping system, the result differences between the physical tests and numerical ones as well as the analytical solutions are significant (5.528%) in the boundary zones. These differences are significantly reduced far from the boundaries (0.223%).
- This article starts by using the curved plates theory, flat plate theory with analytical solutions considered in [10] is a particular case of this. It is propose a mathematical solution for cylindrical thin plate with large curvature radius, with clamped edges and it is presented the influence of curvature radius.

- The proposed analytical solution for orthotropic rectangular plate with fixed edges, when $(\theta \neq 0^\circ)$, has the same theory as presented in [10], except that it starts from the solution (11), not from (10) proposed by Reddy in [7].
- To perform experimental measurements was used the device in [10]. In this article, a comprehensive analysis on the influence of reinforcement fibers, (θ), only one big orthotropic plate was manufactured at STRAERO SA, having a thickness of 1.45 mm, which was cut in two rectangular plates of size (200/300), one with an angle $(\theta = 0^\circ)$, another with $(\theta = 20^\circ)$. The plates were subjected to the same uniform pressure.

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