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A STATISTICAL LINEARIZATION METHOD OF HYSTERETIC SYSTEMS BASED ON RAYLEIGH DISTRIBUTION

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A new statistical linearization method based on Rayleigh distribution is proposed in order to assess equivalent linear models for nonlinear systems with hysteretic characteristics. This method is based on the assumption that a set of hysteresis loops can be determined either from an analytical model or by laboratory experiments for imposed harmonic motion with different amplitudes, view as deviates from a Rayleigh distribution. A set of equivalent linear stiffness and damping coefficients dependent on the, are assessed as functions of imposed motion amplitudes by equating the dissipated energy and stored elastic energy per cycle corresponding to nonlinear and linear characteristics. The parameter defining the Rayleigh distribution is obtained by minimizing the error between r.m.s. outputs of equivalent linear system determined by two different statistical approaches. The main advantage of the proposed method is not only the fact that it can be easily applied for different mathematical models of hysteretic behavior, but also it can be used to assess statistic linear equivalent characteristics to experimental hysteretic loops with no available analytical models. A comparison between the proposed method and classical stochastic linearization method is presented in order to highlight the efficiency of this method.

Key words: hysteretic characteristics, Rayleigh distribution, stochastic equivalent linearization.

1. INTRODUCTION

An analytical description for the hysteretic behaviour of materials, structural elements or vibration isolators is given by the well known Bouc-Wen model [1, 2]. The following inverse problem often appears in practice: given a set of experimental input–output data, how to fit the Bouc-Wen model to experimental data. For identification of model parameters from experimental data of periodic vibration tests were developed analytical approaches [3, 4, 5] and different methods based on genetic algorithms for extended versions of Bouc-Wen model [6, 7]. The experimental data are usually obtained by imposing cyclic relative motions between the mounting ends on the testing rig of a material sample, structural element or vibration control device and by recording the evolution of the developed force versus the imposed displacement.

One of the most efficient techniques for approximating non-linear models within the operating domain is the equivalent linearization method, both in deterministic and stochastic approach. There are many studies about equivalent linearization of hysteretic characteristics, [8, 9, 10] that proved the applicability of this approach. Usually, the parameters of the equivalent linear stochastic system are obtained by minimization of the mean square error between linear and non-linear systems characteristics [8, 10]. One of its main limitations is that even in the case of a system with one degree of freedom system, it is not always very accurate, and the accuracy decreases as the magnitude of the nonlinear terms or the intensity of the random excitation increase [11].

In this paper is assumed that the imposed harmonic motions required for determining the hysteresis loops represent harmonic sample functions with statistically independent Rayleigh distributed amplitude and uniformly distributed phase. It is known that a random process with these characteristics is a Gaussian stationary process [12]. The frequency of these sample functions is chosen taking into account the fundamental natural frequency of the system with hysteretic characteristics or the dominant spectral components of the perturbation.

The proposed linearization algorithm consists of two steps for determining the mean square response of equivalent linear system as function of the single parameter that defines the Rayleigh probability density. The value of this parameter is obtained from the minimum condition of a relative error between the two mean squared responses. Without loss of generality, for the sake of clarity the proposed method is focused on a SDOF system with yielding hysteresis characteristic, typical for bracing anti-seismic devices.

2. MODEL FOR ANTI-SEISMIC SYSTEM

Let us consider the following nonlinear equation of motion of a SDOF system with bracing seismic protection devices BRAD (Buckling Restrained Axial Damper), manufactured by the Italian Company FIPP INDUSTRIALE (Fig. 1):

$$M\ddot{x} + c\dot{x} + kx + F\left(x\right) = -M\ddot{x}_0\left(t\right), \tag{1}$$

where $F\left(x\right) = \delta F_{\mathrm{d}}\left(x\right)$, $-F_{\mathrm{d,max}} \le F_{\mathrm{d}}\left(x\right) \le F_{\mathrm{d,max}}, -x_{\max} \le x \le x_{\max}$, is force-displacement characteristic of one hysteretic device and δ is a gain factor dependent on the number and mounting angles of devices, M is the sprung mass, the coefficients k and c define the structural stiffness and the damping properties, and $\ddot{x}_0\left(t\right)$ is a stationary Gaussian random process describing the base acceleration.

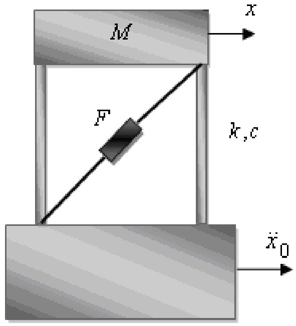

Fig. 1 – Schematic of mechanical system.

Using the following notations

$$\omega_1 = \sqrt{\frac{k}{M}}, \quad \zeta_1 = \frac{c}{2\sqrt{kM}} = \frac{c}{2M\omega_1}, \tag{2}$$

equation (1) becomes:

$$\ddot{x} + 2\zeta_1\omega_1\dot{x} + \omega_1^2 x + \frac{1}{M}F\left(x\right) = -\ddot{x}_0\left(t\right). \tag{3}$$

Introducing the dimensionless parameters:

$$\tau = \omega_1 t, \quad \xi\left(\tau\right) = \frac{x\left(\tau/\omega_1\right)}{x_{\max}}, \quad \xi'\left(\tau\right) = \frac{\dot{x}\left(\tau/\omega_1\right)}{\omega_1 x_{\max}}, \quad \xi''\left(\tau\right) = \frac{\ddot{x}\left(\tau/\omega_1\right)}{\omega_1^2 x_{\max}}, \tag{4}$$

the system (3) becomes

$$\xi'' + 2\zeta_1\xi' + \xi + \rho\Phi_{\mathrm{d}}\left(\xi\right) = \eta\left(\tau\right), \tag{5}$$

where

$$\rho = \delta\frac{F_{\mathrm{d,max}}}{\omega_1^2 x_{\max} M}, \quad \eta\left(\tau\right) = -\frac{\ddot{x}_g\left(t\right)}{\omega_1^2 x_{\max}}, \quad \Phi_{\mathrm{d}}\left(\xi\right) = \frac{F\left(x_{\max}\xi\right)}{F_{\mathrm{d,max}}} \tag{6}$$

and $\eta(\tau)$ is a stationary Gaussian random process with

$$E\left[\eta(\tau)\right]=0,\ \ R_\eta(\tau)=\mathrm{E}\left[\eta(\tau_1)\eta(\tau_1+\tau)\right],\ \ S_\eta(\nu)=\int_{-\infty}^{\infty}R_\eta(\tau)\exp(-\mathrm{j}\nu\tau)\mathrm{d}\tau\,. \tag{7}$$

A set of hysteresis loops determined for imposed cyclic displacements $\xi(\tau)=\xi_0\sin\tau$, where $0<\xi_0\le 1$, is considered for linearization of experimental hysteretic characteristics. The equivalent coefficients are:

$$\upsilon_e(\xi_0)=\sqrt{\frac{\Phi_\mathrm{d}(\xi_0)}{\xi_0}},\quad \zeta_\mathrm{e}(\xi_0)=\frac{E_\mathrm{d}(\xi_0)}{2\pi\upsilon_\mathrm{e}(\xi_0)\xi_0^2}\,. \tag{8}$$

The energy dissipated per cycle is equal with area captured by hysteresis loop:

$$E_\mathrm{d}(\xi_0)=\oint\Phi_\mathrm{d}(\xi_0\sin\tau)\mathrm{d}(\xi_0\sin\tau)=\xi_0\int_0^{2\pi}\Phi_\mathrm{d}(\xi_0\sin\tau)\cos\tau\mathrm{d}\tau\,. \tag{9}$$

3. GAUSSIAN STATIONARY RANDOM PROCESSES WITH HARMONIC SAMPLE FUNCTIONS

It is considered that the imposed displacements for determining the hysteretic loops represent samples of stationary random process:

$$\xi(\tau)=\xi_0\sin(\tau+\theta),\ \ 0<\xi_0\le 1\,, \tag{10}$$

where the random variables ξ_0 and θ are independent. The random variable θ is uniformly distributed on $(-\pi,\pi)$. The variable ξ_0 is Rayleigh distributed, i.e. its probability distribution and probability density functions are

$$F_{\xi_0}(\xi)=\mathrm{Prob}\{\xi_0<\xi\}=1-\exp\left(-\frac{\xi^2}{2\lambda^2}\right),\ f_{\xi_0}(\xi)=\frac{\xi}{\lambda^2}\exp\left(-\frac{\xi^2}{2\lambda^2}\right),\ \xi\ge 0,\ \text{otherwise zero.} \tag{11}$$

The mean, mean square and standard deviation of ξ_0 are given by

$$E[\xi_0]=\lambda\sqrt{\frac{\pi}{2}},\ \ E\left[\xi_0^2\right]=2\lambda^2,\ \ \sigma_{\xi_0}=\lambda\sqrt{\frac{4-\pi}{2}}\,. \tag{12}$$

In this case, $\xi(\tau)=\xi_0\sin(\tau+\theta)$ is a stationary Gaussian random process with zero mean, and standard deviation $\sigma_\xi=\lambda$. Imposing the condition that the probability of hysteretic device to work within its operating range is $\mathrm{Prob}\{\xi_0<1\}=0.99$, the first relation (11) yields the upper limit of Rayleigh distribution parameter:

$$\lambda\le 0.33\,. \tag{13}$$

4. DESCRIPTION OF LINEARIZATION METHOD BASED ON RAYLEIGH DISTRIBUTION

Step I. The equations of motion of the linear systems with parameters determined from experimental data at different amplitudes of imposed cyclic motion (10) are:

$$\xi''+2\left[\zeta_1+\delta\zeta_\mathrm{e}(\xi_0)\nu_\mathrm{e}(\xi_0)\right]\xi'+\left[1+\delta\nu_\mathrm{e}^2(\xi_0)\right]\xi=\eta(\tau)\,. \tag{14}$$

By using the notations

$$\nu_{1e}\left(\xi_0\right)=\sqrt{1+\delta\nu_e^2\left(\xi_0\right)},\ \zeta_{1e}\left(\xi_0\right)=\frac{\zeta_1+\delta\zeta_e\left(\xi_0\right)\nu_e\left(\xi_0\right)}{\sqrt{1+\delta\nu_e^2\left(\xi_0\right)}}, \tag{15}$$

the equations (14) can be written in standard form

$$\xi''+2\zeta_{1e}\left(\xi_0\right)\nu_{1e}\left(\xi_0\right)\xi'+\nu_{1e}^2\left(\xi_0\right)\xi=\eta\left(\tau\right). \tag{16}$$

In order to determine the transfer function $H\left(\nu,\xi_0\right)$ of system (16), it is considered the excitation $\eta\left(\tau\right)=\eta_0\exp(-j\nu\tau)$ and the response of system is expressed by $\xi\left(\tau\right)=H\left(\nu,\xi_0\right)\eta_0\exp(-j\nu\tau)$. Therefore, for each pair of linear equivalent coefficients $\nu_{1e}\left(\xi_0\right),\ \zeta_{1e}\left(\xi_0\right)$ determined from experimental hysteresis loops for different amplitudes of imposed cyclic one obtains the transfer functions

$$H\left(\nu,\xi_0\right)=\frac{1}{\nu_{1e}^2\left(\xi_0\right)-\nu^2+2j\zeta_{1e}\left(\xi_0\right)\nu_{1e}\left(\xi_0\right)\nu}. \tag{17}$$

Thus, for every ξ_0 the output mean square values of equivalent linear system (16) are given by

$$\sigma_{\xi}^2\left(\xi_0\right)=\int_{-\infty}^{\infty}\left|H\left(\nu,\xi_0\right)\right|^2 S_\eta\left(\nu\right)\mathrm{d}\nu,\quad \sigma_{\xi'}^2\left(\xi_0\right)=\int_{-\infty}^{\infty}\nu^2\left|H\left(\nu,\xi_0\right)\right|^2 S_\eta\left(\nu\right)\mathrm{d}\nu. \tag{18}$$

For different values of Rayleigh distribution parameter λ, the mean square output of equivalent linear system is determined as the statistical average of values (18):

$$\begin{aligned}\sigma_{1\xi}^2\left(\lambda\right)&=E\left[\sigma_{\xi}^2\left(\xi_0\right)\right]=\frac{1}{\lambda^2}\int_0^{\infty}\sigma_{\xi}^2\left(\xi_0\right)\xi_0\exp\left(-\frac{\xi_0^2}{2\lambda^2}\right)\mathrm{d}\xi_0,\\ \sigma_{1\xi'}^2\left(\lambda\right)&=E\left[\sigma_{\xi'}^2\left(\xi_0\right)\right]=\frac{1}{\lambda^2}\int_0^{\infty}\sigma_{\xi'}^2\left(\xi_0\right)\xi_0\exp\left(-\frac{\xi_0^2}{2\lambda^2}\right)\mathrm{d}\xi_0.\end{aligned} \tag{19}$$

Step II. The nonlinear system (5) with hysteretic characteristics is replaced by the following equivalent linear system:

$$\xi''+2\zeta_{1e}\left(\lambda\right)\nu_{1e}\left(\lambda\right)\xi'+\nu_{1e}^2\left(\lambda\right)\xi=\eta\left(\tau\right), \tag{20}$$

where the equivalent linear parameters $\upsilon_{le}\left(\lambda\right),\ \zeta_{le}\left(\lambda\right)$ are obtained by taking the statistical mean of $\upsilon_{le}\left(\xi_0\right),\ \zeta_{le}\left(\xi_0\right)$ with respect to Rayleigh distribution defined by parameter λ:

$$\nu_{1e}\left(\lambda\right)=E\left[\nu_{1e}\left(\xi_0\right)\right]=E\left[\sqrt{1+\delta\nu_e^2\left(\xi_0\right)}\right]=\frac{1}{\lambda}\int_0^{\infty}\xi_0\sqrt{1+\delta\nu_e^2\left(\xi_0\right)}\exp\left(-\frac{\xi_0^2}{2\lambda^2}\right)\mathrm{d}\xi_0. \tag{21}$$

In this case, displacement and velocity mean square values are:

$$\sigma_{2\xi}^2\left(\lambda\right)=\int_{-\infty}^{\infty}\left|H\left(\nu,\lambda\right)\right|^2 S_\eta\left(\nu\right)\mathrm{d}\nu,\ \sigma_{2\xi'}^2\left(\lambda\right)=\int_{-\infty}^{\infty}\nu^2\left|H\left(\nu,\lambda\right)\right|^2 S_\eta\left(\nu\right)\mathrm{d}\nu, \tag{22}$$

where the amplification factor is given by

$$\left|H\left(\nu,\lambda\right)\right|=\frac{1}{\sqrt{\left(\nu_{1e}^2\left(\lambda\right)-\nu^2\right)^2+4\zeta_{1e}^2\left(\lambda\right)\nu_{1e}^2\left(\lambda\right)\nu^2}}. \tag{23}$$

Finally, the parameter λ will be determined by minimizing the total relative error between the two mean square responses obtained in steps I, II with the condition that inequality is fulfilled:

$$\frac{\left|\sigma_{1\xi}(\lambda)-\sigma_{2\xi}(\lambda)\right|}{\sigma_{1\xi}(\lambda)}+\frac{\left|\sigma_{1\xi'}(\lambda)-\sigma_{2\xi'}(\lambda)\right|}{\sigma_{1\xi'}(\lambda)}=\min,\quad \lambda_{\min}\le 0.33\,. \tag{24}$$

Thus, for $\lambda=\lambda_{\min}$ the parameters of the equivalent linear system are

$$\nu_e=\nu_{1e}(\lambda_{\min}),\ \zeta_e=\zeta_{1e}(\lambda_{\min})\,. \tag{25}$$

The amplification factor and the mean square response of the final equivalent linear system are:

$$\left|H(\nu)\right|=\frac{1}{\sqrt{\left(\nu_e^2-\nu^2\right)^2+4\zeta_e^2\nu_e^2\nu^2}} \tag{26}$$

and

$$\sigma_{1e\xi}^2=\int_{-\infty}^{\infty}\left|H(\nu)\right|^2 S_\eta(\nu)\mathrm{d}\nu,\quad \sigma_{1e\xi'}^2=\int_{-\infty}^{\infty}\nu^2\left|H(\nu)\right|^2 S_\eta(\nu)\mathrm{d}\nu\ . \tag{27}$$

In the case of a Gaussian white noise excitation with properties

$$E\left[\eta(\tau)\right]=0,\quad R_\eta(\tau)=E\left[\eta(\tau_1)\eta(\tau_1+\tau)\right]=2\pi S_0\delta(\tau),\quad S_\eta(\nu)=S_0,\ \nu\in(-\infty,\infty), \tag{28}$$

the relations (29) and (22) become:

$$\begin{aligned}\sigma_{1\xi}^2(\lambda)&=E\left[\sigma_\xi^2(\xi_0)\right]=\frac{\pi S_0}{2}\int_0^{\infty}\frac{\xi_0}{\lambda^2\zeta_{1e}(\xi_0)\nu_{1e}^3(\xi_0)}\exp\left(-\frac{\xi_0^2}{2\lambda^2}\right)\mathrm{d}\xi_0,\\ \sigma_{1\xi'}^2(\lambda)&=E\left[\sigma_{\xi'}^2(\xi_0)\right]=\frac{\pi S_0}{2}\int_0^{\infty}\frac{\xi_0}{\lambda^2\zeta_{1e}(\xi_0)\nu_{1e}(\xi_0)}\exp\left(-\frac{\xi_0^2}{2\lambda^2}\right)\mathrm{d}\xi_0,\end{aligned} \tag{19'}$$

and

$$\begin{aligned}\sigma_{2\xi}^2(\lambda)&=\int_{-\infty}^{\infty}\left|H(\nu,\lambda)\right|^2 S_\eta(\nu)\mathrm{d}\nu=\frac{\pi S_0}{2\zeta_{1e}(\lambda)\nu_{1e}^3(\lambda)},\\ \sigma_{2\xi'}^2(\lambda)&=\int_{-\infty}^{\infty}\nu^2\left|H(\nu,\lambda)\right|^2 S_\eta(\nu)\mathrm{d}\nu=\frac{\pi S_0}{2\zeta_{1e}(\lambda)\nu_{1e}(\lambda)}\,.\end{aligned} \tag{22'}$$

5. APPLICATION OF THE LINIARIZATION METHOD

In this case study, the hysteresis characteristics are modeled by Bouc-Wen model

$$z'=h(\xi',z)=[A-|z|^n(\beta+\gamma\,\mathrm{sgn}(\xi' z))]\xi', \tag{29}$$

with parameters $A=1,\ \beta=0.1,\ \gamma=3,\ n=1$. Figure 2 shows the hysteresis loops generated by this model for a set of imposed motion amplitudes $\xi_0=0.1,0.2,...,1$. Their shape is typical for yielding behaviour of BRAD anti-seismic bracing devices. Therefore, this case study can be view as a virtual experiment, chosen in order to compare the results of the proposed method of statistical equivalent linearization based on Rayleigh distribution (REL) with those obtained by applying the classical statistical linearization method based on Gaussian distribution (GEL).

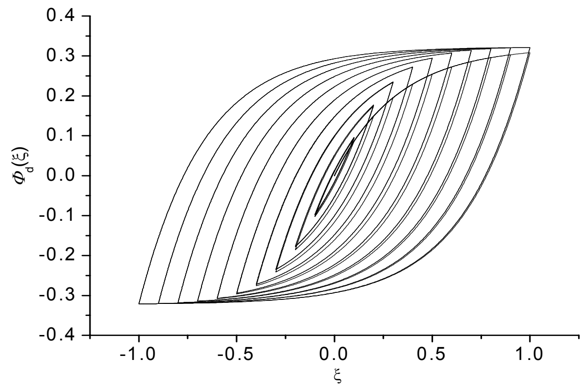

Fig. 2 – Hysteretic loops obtained by an virtual experiment.

For the considered values of the amplitude ξ_0 of imposed cyclic motion, the equivalent relative damping coefficients $\zeta_e(\xi_0)$ and the equivalent natural frequencies $\upsilon_e(\xi_0)$ are obtained by using the relations (8) and (9). The scatter plots of calculated values were approximated by the polynomial interpolations:

$$\begin{aligned} \zeta_e(\xi_0) &= -0.03 + 0.92\xi_0 - 1.04\xi_0^2 + 0.39\xi_0^3, \\ \nu_e(\xi_0) &= 1.07 - 0.56\xi_0 - 0.19\xi_0^2 + 0.25\xi_0^3. \end{aligned} \tag{30}$$

Figures 3 and 4 show the variation of calculated values and their polynomial fit.

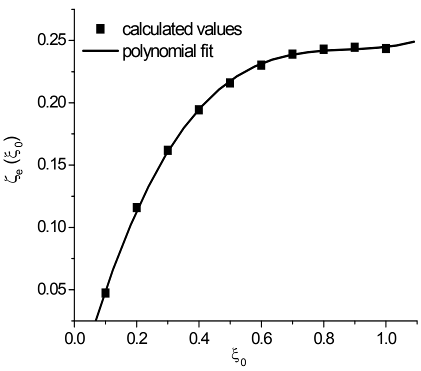

Fig. 3 – Equivalent relative damping coefficient *versus* amplitude of imposed cyclic motion.

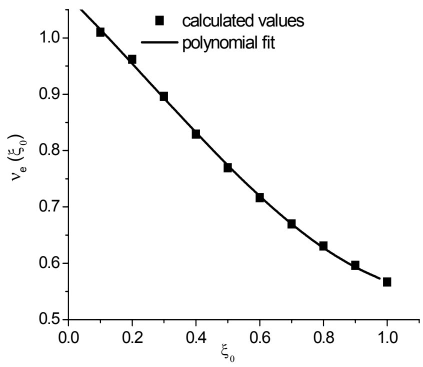

Fig. 4 – Equivalent natural frequency *versus* amplitude of imposed cyclic motion.

Applying the two steps of the proposed algorithm and using the values $\zeta_1 = 0.05$, $\delta=1$, $S_0 = 0.07$ yields

$$\lambda_{\min} = 0.31 < 0.33 \tag{31}$$

and

$$\nu_e(\lambda_{\min}) = 1.26,\ \zeta_e(\lambda_{\min}) = 0.14\,. \tag{32}$$

In order to verify the efficiency of the proposed method, the r.ms. response of the equivalent linear system

$$\xi'' + 2\zeta_e(\lambda_{\min})\nu_e(\lambda_{\min})\xi' + \nu_e^2(\lambda_{\min})\xi = \eta(\tau) \tag{33}$$

is compared with:

– the response of the equivalent linear system

$$\begin{cases} \xi'' + 2\zeta_1\xi' + \xi + z = \eta(\tau) \\ z' = az + b\xi' + c\xi \end{cases}, \tag{34}$$

where the coefficients $a=-1.76,\ b=0.5,\ c=0$ are calculated by minimization of the mean square error [10], [13]:

$$E\left[[h(\xi',z)-(az+b\xi'+c\xi)]^2\right]=\min\ ; \tag{35}$$

– the response obtained by numerical simulation of Bouc-Wen nonlinear system (36) excited by the same Gaussian white noise input with intensity $S_0=0.07$

$$\begin{cases}\xi''+2\zeta_1\xi'+\xi+z=\eta(\tau)\\ z'=[A-|z|^n(\beta+\gamma\,\mathrm{sgn}(\xi'z))]\xi'.\end{cases} \tag{36}$$

The results are presented in Table 1. The relative errors between the mean square response of Bouc-Wen nonlinear system and two equivalent linear systems are presented in Figs. 5–7 for different values of input intensity.

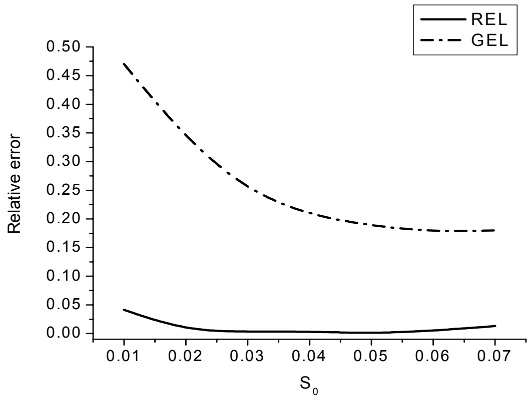

Fig. 5 – Relative errors of relative displacement standard deviations.

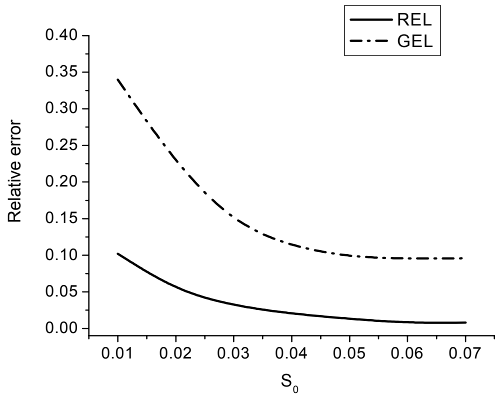

Fig. 6 – Relative errors of relative velocity standard deviations.

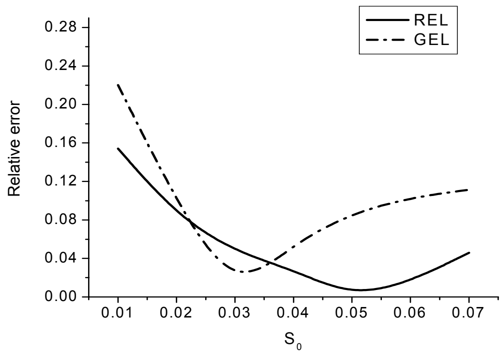

Fig. 7 – Relative errors of absolute acceleration standard deviations.

Table 1

Mean square response	Nonlinear Bouc-Wen system	Equivalent linear system GEL	Equivalent linear system REL
σ_{ξ}	0.30	0.354	0.296
$\sigma_{\xi'}$	0.375	0.411	0.372
$\sigma_{\xi_1''}$	0.476	0.423	0.498

As one can see from Figs. 5–7 and Table 1, the results obtained by proposed method are more accurate then those given by the classical statistical linearization method, especially for higher values of input intensity.

6. CONCLUSIONS

The method based on Rayleigh distribution is easily applicable when the analytical representation of the hysteresis loops is known. Due to the complexity of hysteresis analytical models, the classical stochastic linearization method is difficult to be applied in most cases. Another advantage of the proposed method consists in its applicability in cases when the analytical models of hysteresis loops obtained experimentally are not available.

The mean square response of a nonlinear system with hysteresis characteristic portrayed by Bouc-Wen model was compared with that of equivalent linear systems obtained by classical and Rayleigh linearization methods. For the considered case study, the results show that the method based on Rayleigh distribution is more accurate and requires less computing effort compared with the classical stochastic linearization.

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