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TRANSPORT EQUATIONS IN FRACTAL POROUS MEDIA WITHIN FRACTIONAL COMPLEX TRANSFORM METHOD

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In this paper we investigate the transport equations in fractal porous media by using the fractional complex transform method. The local fractional linear and nonlinear transport equations with local fractional time and space fractional derivatives are obtained. The proposed models adequately describe the fractal transport processes.

Key words: fractional complex transform method; transport equations; local fractional derivatives.

1. INTRODUCTION

Fractal porous materials [1] such as wool fibers [2-3], multi-scale fabric products [4], polar bear hairs [5] and air permeability [6] are described within the framework of *fractal transport phenomena* [7]. The classical transport phenomena have applications in several areas, e.g., the turbulence [8], the two-valley semiconductors [9], the QCD gluon Wigner operator [10], the clean superconductors [11], the aeronomy [12], the superconductors [13], the resting phases [14] and so on.

In recent decades the *fractional calculus* and its applications started to be one of the main directions for several researchers [15–19]. The fractional transport phenomena were reported (see [20–27] and the references therein). Based on Levy stable processes, fractional transport equations were reported [20]. During the last decade the anomalous transport with fractional operator was considered [21–22] and the hyperbolic transport equations were investigated [23]. Recently, the fractional diffusion models of nonlocal transport were addressed [24] and the transport problems in disordered semiconductors were investigated in [25]. Very recently the Walsh function method *fractional transport equation* was reported [26] and the solution of the fractional transport equation was analyzed within the generalized quadratic form [27].

As it is known the local fractional derivative and integration are set up on fractals [28–48]. Several definitions of local fractional derivatives were reported (see for example [28–48] and the references therein). The generalized local fractional derivatives were investigated in Refs. [28–29, 37–41]. Based on local fractional derivatives, the *fractional complex transform method* (also called fractal complex transform method) was proposed in [34–35]; it is a natural extension of the fractional complex transform method [41–50] originally proposed for the modified R-L fractional derivatives.

In this manuscript we will convert the conventional transport equations into the transport equations with local fractional derivatives by using the fractional complex transform method. The structure of the manuscript is given below. In Section 2, we investigate the transport equations in fractal porous media by using the fractional complex transform method. In Section 3 we briefly give our conclusions.

2. THE TRANSPORT EQUATIONS IN FRACTAL POROUS MEDIA

In this section, we derive the local fractional transport equations containing both local fractional time and space derivatives.

2.1. Local fractional linear transport equation

According to the theory of characteristics, the linear transport equation is [51]

$$\frac{\partial M(X,Y,Z,T)}{\partial T}+a(X,Y,Z,T)\left[\frac{\partial M(X,Y,Z,T)}{\partial X}+\frac{\partial M(X,Y,Z,T)}{\partial Y}+\frac{\partial M(X,Y,Z,T)}{\partial Z}\right]=0. \tag{1}$$

The fractional complex transform method was applied to switch the conventional differential equations into local fractional differential equations [34-35]. Now we can derive the fractional complex transform

$$X=\frac{x^{\alpha}}{\Gamma(1+\alpha)}\quad;\quad Y=\frac{y^{\alpha}}{\Gamma(1+\alpha)}\quad;\quad Z=\frac{z^{\alpha}}{\Gamma(1+\alpha)}\quad;\quad T=\frac{t^{\alpha}}{\Gamma(1+\alpha)}. \tag{2}$$

Using the factional complex transform (2), we have

$$\frac{\partial^{\alpha}M(x,y,z,t)}{\partial x^{\alpha}}=\frac{\partial M(X,Y,Z,T)}{\partial X}\frac{\partial^{\alpha}X}{\partial x^{\alpha}}+\frac{\partial M(X,Y,Z,T)}{\partial Y}\frac{\partial^{\alpha}Y}{\partial x^{\alpha}}+\frac{\partial M(X,Y,Z.T)}{\partial Z}\frac{\partial^{\alpha}Z}{\partial z^{\alpha}}+\frac{\partial M(X,Y,Z.T)}{\partial T}\frac{\partial^{\alpha}T}{\partial t^{\alpha}}=\frac{\partial M(X,Y,Z,T)}{\partial X}, \tag{3}$$

$$\frac{\partial^{\alpha}M(x,y,z,t)}{\partial y^{\alpha}}=\frac{\partial M(X,Y,Z,T)}{\partial X}\frac{\partial^{\alpha}X}{\partial y^{\alpha}}+\frac{\partial M(X,Y,Z,T)}{\partial Y}\frac{\partial^{\alpha}Y}{\partial y^{\alpha}}+\frac{\partial M(X,Y,Z.T)}{\partial Z}\frac{\partial^{\alpha}Z}{\partial y^{\alpha}}+\frac{\partial M(X,Y,Z.T)}{\partial T}\frac{\partial^{\alpha}T}{\partial y^{\alpha}}=\frac{\partial M(X,Y,Z,T)}{\partial Y}, \tag{4}$$

$$\frac{\partial^{\alpha}M(x,y,z,t)}{\partial z^{\alpha}}=\frac{\partial M(X,Y,Z,T)}{\partial X}\frac{\partial^{\alpha}X}{\partial z^{\alpha}}+\frac{\partial M(X,Y,Z,T)}{\partial Y}\frac{\partial^{\alpha}Y}{\partial z^{\alpha}}+\frac{\partial M(X,Y,Z.T)}{\partial Z}\frac{\partial^{\alpha}Z}{\partial z^{\alpha}}+\frac{\partial M(X,Y,Z.T)}{\partial T}\frac{\partial^{\alpha}T}{\partial z^{\alpha}}=\frac{\partial M(X,Y,Z,T)}{\partial Z}, \tag{5}$$

$$\frac{\partial^{\alpha}M(x,y,z,t)}{\partial t^{\alpha}}=\frac{\partial M(X,Y,Z,T)}{\partial X}\frac{\partial^{\alpha}X}{\partial t^{\alpha}}+\frac{\partial M(X,Y,Z,T)}{\partial Y^{2}}\frac{\partial^{\alpha}Y}{\partial t^{\alpha}}+\frac{\partial M(X,Y,Z.T)}{\partial Z}\frac{\partial^{\alpha}Z}{\partial t^{\alpha}}+\frac{\partial M(X,Y,Z.T)}{\partial T}\frac{\partial^{\alpha}T}{\partial t^{\alpha}}=\frac{\partial M(X,Y,Z,T)}{\partial T}. \tag{6}$$

Hence, we arrive at the following equation

$$\frac{\partial^{\alpha}M(x,y,z,t)}{\partial t^{\alpha}}+a(x,y,z,t)\left[\frac{\partial^{\alpha}M(x,y,z,t)}{\partial x^{\alpha}}+\frac{\partial^{\alpha}M(x,y,z,t)}{\partial y^{\alpha}}+\frac{\partial^{\alpha}M(x,y,z,t)}{\partial z^{\alpha}}\right]=0. \tag{7}$$

Remark 1. We observe that Eq. (1) is equivalent to Eq. (7) under the constraint of Eq. (2). In a similar manner, we can prove that Eq. (7) is equivalent to Eq. (1) under the constraint of Eq. (2) if the local fractional partial derivatives exist (see A1).

2.2. Local fractional nonlinear transport equation

For a given special case of Ref. [52] a parameterized quadratically nonlinear transport equation reads

$$\begin{aligned}&\frac{\partial M(X,Y,Z,T)}{\partial T}-a\left[\frac{\partial^{2}M(X,Y,Z,T)}{\partial X^{2}}+\frac{\partial^{2}M(X,Y,Z,T)}{\partial Y^{2}}+\frac{\partial^{2}M(X,Y,Z,T)}{\partial Z^{2}}\right]\\&-\frac{b}{2}M(X,Y,Z,T)\left[\frac{\partial M(X,Y,Z,T)}{\partial X}+\frac{\partial M(X,Y,Z,T)}{\partial Y}+\frac{\partial M(X,Y,Z,T)}{\partial Z}\right]=0\end{aligned} \tag{8}$$

where a and b are two real constants.

By the factional time and space complex transform (2), we have

$$\frac{\partial^{2\alpha}M(x,y,z,t)}{\partial x^{2\alpha}}=\frac{\partial^{2}M(X,Y,Z,T)}{\partial X^{2}}\frac{\partial^{\alpha}X}{\partial x^{\alpha}}+\frac{\partial^{2}M(X,Y,Z,T)}{\partial Y^{2}}\frac{\partial^{\alpha}Y}{\partial x^{\alpha}}+\frac{\partial^{2}M(X,Y,Z.T)}{\partial Z^{2}}\frac{\partial^{\alpha}Z}{\partial x^{\alpha}}+\frac{\partial^{2}M(X,Y,Z.T)}{\partial T^{2}}\frac{\partial^{\alpha}T}{\partial x^{\alpha}}=\frac{\partial^{2}M(X,Y,Z,T)}{\partial X^{2}}, \tag{9}$$

$$\frac{\partial^{\alpha}M(x,y,z,t)}{\partial y^{2\alpha}}=\frac{\partial^{2}M(X,Y,Z,T)}{\partial X^{2}}\frac{\partial^{\alpha}X}{\partial y^{\alpha}}+\frac{\partial^{2}M(X,Y,Z,T)}{\partial Y^{2}}\frac{\partial^{\alpha}Y}{\partial y^{\alpha}}+\frac{\partial^{2}M(X,Y,Z,T)}{\partial Z^{2}}\frac{\partial^{\alpha}Z}{\partial y^{\alpha}}+\frac{\partial^{2}M(X,Y,Z,T)}{\partial T^{2}}\frac{\partial^{\alpha}T}{\partial y^{\alpha}}=\frac{\partial^{2}M(X,Y,Z,T)}{\partial Y^{2}}, \tag{10}$$

$$\frac{\partial^{\alpha}M(x,y,z,t)}{\partial z^{2\alpha}}=\frac{\partial^{2}M(X,Y,Z,T)}{\partial X^{2}}\frac{\partial^{\alpha}X}{\partial z^{\alpha}}+\frac{\partial^{2}M(X,Y,Z,T)}{\partial Y^{2}}\frac{\partial^{\alpha}Y}{\partial z^{\alpha}}+\frac{\partial^{2}M(X,Y,Z,T)}{\partial Z^{2}}\frac{\partial^{\alpha}Z}{\partial z^{\alpha}}+\frac{\partial^{2}M(X,Y,Z,T)}{\partial T^{2}}\frac{\partial^{\alpha}T}{\partial z^{\alpha}}=\frac{\partial^{2}M(X,Y,Z,T)}{\partial Z^{2}} \tag{11}$$

Thus Eq. (8) becomes

$$\begin{aligned}&\frac{\partial^{\alpha}M(x,y,z,t)}{\partial t^{\alpha}}-a\left[\frac{\partial^{2\alpha}M(x,y,z,t)}{\partial x^{2\alpha}}+\frac{\partial^{2\alpha}M(x,y,z,t)}{\partial y^{2\alpha}}+\frac{\partial^{2\alpha}M(x,y,z,t)}{\partial z^{2\alpha}}\right]\\&-\frac{b}{2}M(x,y,z,t)\left[\frac{\partial^{\alpha}M(x,y,z,t)}{\partial x^{\alpha}}+\frac{\partial^{\alpha}M(x,y,z,t)}{\partial y^{\alpha}}+\frac{\partial^{\alpha}M(x,y,z,t)}{\partial z^{\alpha}}\right]=0\end{aligned}. \tag{12}$$

Remark 2. In view of Eqs. (7, 12), we find the one-dimensional *linear transport equation*

$$\frac{\partial^{\alpha}M(x,t)}{\partial t^{\alpha}}+a(x,t)\frac{\partial^{\alpha}M(x,t)}{\partial x^{\alpha}}=0 \tag{13}$$

and the one-dimensional *nonlinear transport equation*

$$\frac{\partial^{\alpha}M(x,t)}{\partial t^{\alpha}}-a\frac{\partial^{2\alpha}M(x,y,z,t)}{\partial x^{2\alpha}}-\frac{b}{2}M(x,t)\frac{\partial^{\alpha}M(x,t)}{\partial x^{\alpha}}=0. \tag{14}$$

This result was obtained with the help of the fractional complex transform [34-35]:

$$X=\frac{x^{\alpha}}{\Gamma(1+\alpha)} \quad ; \quad T=\frac{t^{\alpha}}{\Gamma(1+\alpha)}. \tag{15}$$

Remark 3. Due to Eq. (2), the phase space of Eqs. (7, 12) is a fractal space, with its dimension being α. We notice that the fractal dimensions of the phase spaces of Eqs. (7, 12) are equal to 3α and the three phase spaces are orthogonal to each other, fulfilling the condition [28]

$$\dim_H(E\times F\times G)=\dim_H E+\dim_H F+\dim_H G, \tag{16}$$

which leads to

$$\dim_H E=\dim_H F=\dim_H G=\alpha \tag{17}$$

in the three-dimensional fractal system. Here E and F are subsets of R. E and F are fractal sets, $X\in E$, $Y\in E$ and $Z\in E$.

Remark 4. Here, the fractal porous medium is described by $M(x,y,z,t)$ such that [28],

$$\left|M(x,y,z,t)-M(x_0,y_0,z_0,t)\right|<\varepsilon^{\alpha}. \tag{18}$$

Hence, the fractal dimension of the fractal porous medium is 3α. From Eq. (18), we conclude that [28]

$$C_1|x-x_0|^{\alpha}\le\left|M(x,y,z,t)-M(x_0,y,z,t)\right|\le C_2|x-x_0|^{\alpha}, \tag{19}$$

$$C_3|y-y_0|^{\alpha}\le\left|M(x,y,z,t)-M(x,y_0,z,t)\right|\le C_4|y-y_0|^{\alpha}, \tag{20}$$

$$C_5|z-z_0|^{\alpha}\le\left|M(x,y,z,t)-M(x,y,z_0,t)\right|\le C_6|z-z_0|^{\alpha}, \tag{21}$$

where $C_1,C_2,\cdots,C_6$ are constants.

3. CONCLUSIONS

In this paper we started with the classical linear and nonlinear fractional transport equations and end up with the local linear and nonlinear fractional transport equations on fractal domain. The obtained equations have local fractal property. In our work the fractal porous material is described by a local fractional continuous function. The corresponding fractal dimension is provided by the porous material. Thus, we conclude that the developed transport equations efficiently describe the local fractal behaviors of fractal porous materials.

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APPENDIX A

Local fractional partial derivatives of $f(x,y,z)$ at the point x_0 are given by [28–29]

$$\left.\frac{\partial^{\alpha} f(x,y,z)}{\partial x^{\alpha}}\right|_{x=x_0} = \lim_{x\to x_0}\frac{\Delta^{\alpha}\left[f(x,y,z)-f(x_0,y,z)\right]}{(x-x_0)^{\alpha}}, \tag{A1}$$

with $\Delta^{\alpha}\left[f(x,y,z)-f(x_0,y,z)\right]\cong\Gamma(1+\alpha)\left[f(x,y,z)-f(x_0,y,z)\right]$, and the 2α local fractional partial derivative is [28]

$$\frac{\partial^{2\alpha} f(x,y,z)}{\partial x^{2\alpha}} = \frac{\partial^{\alpha}}{\partial x^{\alpha}}\frac{\partial^{\alpha} f(x,y,z)}{\partial x^{\alpha}} = \frac{\partial^{2\alpha} f(x,y,z)}{\partial x^{\alpha}\partial x^{\alpha}} = f_{x^2}^{2\alpha}(x,y,z), \tag{A2}$$

$$\frac{\partial^{2\alpha} f(x,y,z)}{\partial y^{2\alpha}} = \frac{\partial^{\alpha}}{\partial y^{\alpha}}\frac{\partial^{\alpha} f(x,y,z)}{\partial y^{\alpha}} = \frac{\partial^{2\alpha} f(x,y,z)}{\partial y^{\alpha}\partial y^{\alpha}} = f_{y^2}^{2\alpha}(x,y,z), \tag{A3}$$

$$\frac{\partial^{2\alpha} f(x,y,z)}{\partial z^{2\alpha}} = \frac{\partial^{\alpha}}{\partial z^{\alpha}}\frac{\partial^{\alpha} f(x,y,z)}{\partial z^{\alpha}} = \frac{\partial^{2\alpha} f(x,y,z)}{\partial z^{\alpha}\partial z^{\alpha}} = f_{z^2}^{2\alpha}(x,y,z). \tag{A4}$$

The expressions of the high order local fractional partial derivatives of local fractional continuous functions can be found in Refs. [28, 29].

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