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HYSTERESIS MODELING AND FEEDFORWARD CONTROL OF A FIVE-STORY SYSTEM

Ioan URSU [1], Veturia CHIROIU [2], Ligia MUNTEANU [2]

[1] "Elie Carafoli" National Institute for Aerospace Research, B-dul Iuliu Maniu 220, 061126 Bucharest
[2] Institute of Solid Mechanics, Romanian Academy, Ctin Mille 15, 010141 Bucharest
E-mail: veturiachiroiu@yahoo.com

This paper discusses the modeling and feedforward control of hysteresis in a nonlinear five-story system using observed vibration signals. The generalized play operator is analyzed in connection with the system equations. Results show that hysteresis can be reduced to less than 35% when applying the feedforward control.

Keywords: feedforward control, hysteresis operator, five-story system, Bouc-Wen model.

1. INTRODUCTION

Nonlinear structural identification is necessary for civil engineering infrastructure if accurate simulation of response is needed. Following a literature review (Zhang *et al*. [1,2], Ibanez [3]) it has been found that the identification of dynamic parameters of linear or nonlinear structural models from experimental data is a long-standing topic of research interest. Recently, there has also been increasing interest in development of nonlinear structural identification methods [4–7].

The hysteresis is typically viewed as an undesirable effect in that it complicates the control of the relationship between input and output data in engineering systems. In the 1970s, Krasnoselskiı and Pokrovskiı studied the concept of hysteresis operator acting in spaces of time dependent functions [8]. Further researches were developed in a series of monographies dedicated to hysteresis in connection with PDEs and applicative problems [9–11]. Nonlinear semigroup theory in a Hilbert space was developed by Kōmura [12] and extended to Banach spaces by Crandal and Liggett [13] and Barbu [14].

In this paper, the generalized play operator is analyzed in connection with the governing equations of a five-story nonlinear system. When hysteresis is present along with the dynamics of this system, the overall behavior of it can be very complex. In order to control this effect, the modeling of the hysteresis has to be as precise as possible [15–20].

The scope of this paper is to model and control the hysteresis phenomenon in a five-story system with nonlinear restoring force characteristics. The control aspect is focused on feedforward or compensation technique, with no sensor requires [21]. The control problem is reduced to a system of differential inclusions and solved. This work is placed in the framework of the Visintin researches on models of hysteresis phenomena and on related PDEs [22–26].

A rate-dependent hysteresis is a hysteresis that has its shape changed when the frequency of the input is changed. Such hysteresis is called dynamic hysteresis. Contrary to a rate-dependent hysteresis, the shape of a rate-independent hysteresis does not change whatever the frequency of the input is. Such hysteresis is called static hysteresis. Usually, it is admitted the separation principle of dynamic hysteresis which states that the dynamic hysteresis can be approximated by a static hysteresis followed by a linear dynamic part [27]. In this paper, we assume this separation principle. Thus, the modeling and compensation are focused on static hysteresis [15].

2. HYSTERESIS OPERATORS

Let us consider a system whose he state is characterized by the input function $u(t)$ and the output function $w(t)$, confined to a set $L \subset \mathbb{R}^2$. $\forall t \in [0,T]$. The function $w(t)$ depends on the previous evolution of $u(t)$ and on the initial state w_0, such as

$$w(t) = A(u, w_0)(t),\ \forall t \in [0,T],\ (u(0), w_0) \in L,\ A(u, w_0)(0) = w_0, \tag{2.1}$$

where $A(u, w_0)$ is a memory operator defined in a Banach space of time-dependent functions for any fixed w_0. The memory operator is causal: for $\forall (u_1, w_0), (u_2, w_0)$ with $u_1 = u_2$ in $[0,T]$, then $A(u_1, w_0)(t) = A(u_2, w_0)(t)$. In the following we present the generalized play operator $w := A(u, w_0) : \mathbb{R}^+ \to \mathbb{R}$ defined in the sense of Visintin. Let $u(t)$ be any continuous, piecewise linear function on R^+, linear on $[t_{i-1}, t_i]$, $i = 1,2,...$ We define $w(t) = A(u, w_0)(t)$ by

$$\begin{gathered} w(t) = \min\left\{\gamma_l(u(0)), \max\left\{\gamma_r(u(0)), w_0\right\}\right\} \text{ for } t = 0 \text{ and } w_0 \in \mathbb{R}, \\ w(t) = \min\left\{\gamma_l(u(t_i)), \max\left\{\gamma_r(u(t_i)), w(t_{i-1})\right\}\right\} \text{ for } t \in (t_{i-1}, t_i),\ i = 1,2,... \end{gathered} \tag{2.2}$$

where $\gamma_l, \gamma_r : \mathbb{R} \to \mathbb{R}$ are maximal monotone, possible multivalued functions with

$$\inf \gamma_r(u) \le \sup \gamma_l(u),\ \forall u \in \mathbb{R}. \tag{2.3}$$

Note that $w(0) = w_0$ only if $\gamma_r(u(0)) \le w_0 \le \gamma_l(u(0))$. The classical play operator can be obtained from the general play operator by choosing

$$\gamma_l(u) = u + r,\ \ \gamma_r(u) = u - r\ , \tag{2.4}$$

with $r \ge 0$ a parameter, $u(t)$ a continuous input function on $[0,T]$ and $w_{r0} \in [-r,\ r]$ an initial state. Fig.2.1 presents the play operator with threshold r.

The hysteresis relationship with the PDEs can be written as [28]

$$w(x,t) = \left[A(u(x,...), w_0(x))\right](t) \text{ in } Q = \Omega \times [0,T], \tag{2.5}$$

where Ω is a bounded subset of $\mathbb{R}^n$. The generalized play operator is dissipative, in the sense that $\| (\lambda I - A)x \| \ge \lambda \| x \|$ for $\forall \lambda > 0$, where I is the identity mapping.

The PDEs with hysteresis can be transformed into systems of differential inclusions. Therefore, the generalized play operator can be defined as a solution in the Sobolev space $W^{1,1}(0,T)$, $w \in W^{1,1}(0,T)$ of a variational inclusion of the type

$$w_{,t} \in \phi(u, w) \text{ in } (0,T),\ w(0) = w_0, \tag{2.6}$$

where comma represents the differentiation with respect to the specified variable.

The rate-independent differential inclusion is given by

$$w_{,t} \in \phi(u,w) = \begin{cases} \{\infty\} & \text{if } w < \inf \gamma_r(u), \\ [0,+\infty] & \text{if } w \in \gamma_r(u) \setminus \gamma_l(u), \\ \{0\} & \text{if } \sup \gamma_r(u) < w < \inf \gamma_l(u), \\ [-\infty, 0] & \text{if } w \in \gamma_l(u) \setminus \gamma_r(u), \\ \{-\infty\} & \text{if } w > \sup \gamma_r(u), \\ [-\infty,+\infty] & \text{if } w \in \gamma_l(u) \cap \gamma_r(u). \end{cases} \tag{2.7}$$

If γ_r and γ_l are Lipschitz-continuous, then the generalized play operator transforms $(u,v)\in W^{1,1}(0,T)\times\mathbb{R}$ into the unique function $w\in W^{1,1}(0,T)$ such that $w(0)$ is the projection of v into $[\gamma_r(u(0)),\gamma_l(u(0))]$ and (2.7) is satisfied. The operator can be extended to $C^0([0,T])\times\mathbb{R}$, and it is equivalent to a variational inequality.

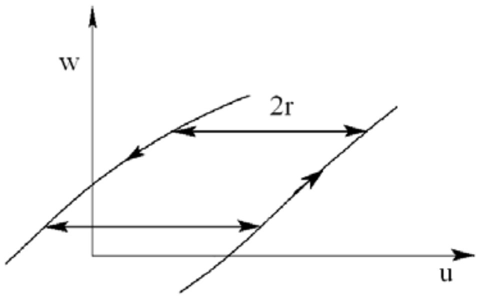

Fig. 2.1 – The play operator with threshold r.

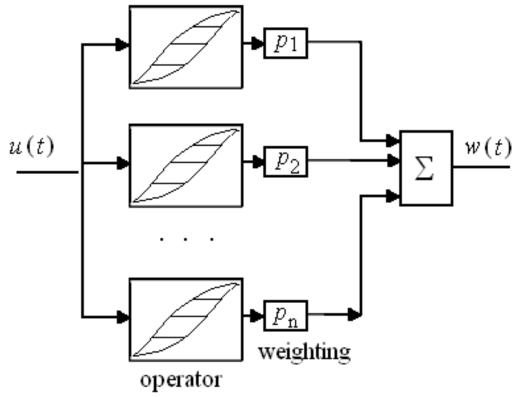

Fig. 2.2 – Arrangement in parallel of the hysteresis operators.

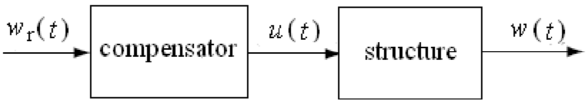

Fig. 2.3 – The scheme of the feedforward control.

It is well known that a combination in parallel of the hysteresis operators $w:=A(u,w_0):\mathbb{R}^+\to\mathbb{R}$ given by (2.2) is used in the practical problems. The block diagram of this combination is presented in Fig. 2.2. The model is the sum of the hysteresis operators with weightings p_i, $i=1,2,...,n$

$$w(t)=\sum_{i=1}^{n}p_i\min\left\{\gamma_l(u(0)),\max\left\{\gamma_r(u(0)),w_0\right\}\right\},\ t=0,\ w_0\in\mathbb{R},$$
$$w(t)=\sum_{i=1}^{n}p_i\min\left\{\gamma_l(u(t_i)),\max\left\{\gamma_r(u(t_i)),w(t_{i-1})\right\}\right\}\ t\in(t_{i-1},t_i),\ i=1,2,... \tag{2.8}$$

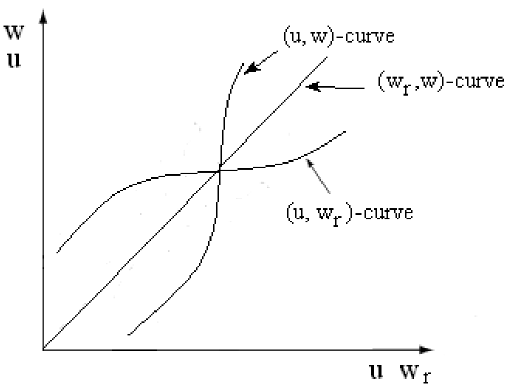

Fig. 2.4 – The scheme of compensation of the hysteresis.

Direct hysteresis can be compensated by another hysteresis put in cascade with it [22]. Such scheme is called feedforward control of the hysteresis and it is presented in Fig. 2.3. In the figure, w_r is the reference input to be tracked. Direct hysteresis and its compensator are symmetric relative to the linear curve (w_r,w), as shown in Fig. 2.4. To obtain a linear input-output (w_r,w) with a unit gain, the real system curve (u,w) and the compensator curve (w_r,u) should be symmetric [15].

This compensator is characterized by thresholds r_k' and the weightings p_k'. The calculation of these parameters follows the principle of Fig. 2.5. The thresholds r_k', $k=1,2,...,n$, are computed as follow

$$r_k'=\sum_{j=1}^{k}p_j(r_k-r_j),\ k=1,2,...,n, \tag{2.9}$$

and

$$p_1' = \frac{1}{p_1}\,,\; p_k' = \frac{-p_k}{\left(p_1 + \sum_{j=2}^{k} p_j\right)\left(p_1 + \sum_{j=2}^{k-1} p_j\right)}\,,\; k = 2,...,n\,. \tag{2.10}$$

The following approach is based on the inverse multiplicative structure scheme and gives the compensator without any additional calculation [15, 16]. The compensator is defined by

$$w(t) = \sum_{i=1}^{n} p_i \min\{\gamma_l(u(0)), \max\{\gamma_r(u(0)), w_0\}\}\,,\quad t = 0 \text{ and } w_0 \in \mathbb{R}\,,$$

$$w(t) = \sum_{i=1}^{n} p_i \min\{\gamma_l(u(t_{i-1})), \max\{\gamma_r(u(t_{i-1})), w(t_{i-2})\}\} - w_r(t)\,, t \in (t_{i-2}, t_{i-1}),\; i = 2,3,... \tag{2.11}$$

3. FIVE-STORY SYSTEM

The system under study is a five-story system with the mass, damping and stiffness of each floor, respectively, the i th floor mass $m_i = 0.125$kg, the i th floor damping coefficient $c_i = 0.03$N/m/s and the i th floor stiffness coefficient $k_i = 20.5$N/m, $i = 1,...,5$, subjected to the excitation $F(t)$. The parameters of the model match those of the structure analyzed in [2, 29]. We specify that the model was analyzed by Zhang *et al*. [2] for the case of restoring forces expressed by the Bouc–Wen model

$$\dot{r}_i = c_i \ddot{z}_i + k_i \dot{z}_i - \alpha_i \,|\, \dot{z}_i \,||\, r_i \,|^{n_i - 1} r_i - \beta_i \dot{z}_i \,|\, r_i \,|^{n_i}\,, \tag{3.1}$$

where α_i, β_i, n_i are the Bouc-Wen parameters, and $z_i = u_i - u_{i-1}$ is i th floor relative displacement. The motion equation for the system is written as

$$M\ddot{u} + r(u,\dot{u}) = F(t)\,,\; F(t) = -MI\ddot{z}_0\,, \tag{3.2}$$

where $\ddot{u}$, $\dot{u}$ and u, respectively, are the relative acceleration, velocity and displacement to the ground, $\ddot{z}_0$ is the ground motion acceleration, M is the mass matrix, I the identity matrix.

The nonlinear restoring force $r(u,\dot{u})$ is modeled in this paper by a polynomial of order five

$$r(u,\dot{u}) = c_0 u + d_0 \dot{u} + c_3 u^3 + d_3 \dot{u}^3 + c_5 u^5 + d_5 \dot{u}^5\,, \tag{3.3}$$

with unknown parameters c_0, d_0, c_3, d_3, c_5 and d_5. The input excitation of the five-story system is represented in Fig. 3.1 [30].

Now, let us consider that the hysteresis is present along with the dynamics of this system described by (3.2) and (3.3). By coupling the motion equation (3.2) with the generalized hysteresis operator (2.8) via the relation $w(x,t) = [A(u(x,...), w_0(x))](t)$ in $Q = \Omega \times [0,T]$, we obtain the following equation with hysteresis

$$M(\ddot{u} + \ddot{w}) + r(u, \dot{u} + \dot{w}) = F(t)\,,\; F(t) = -MI\ddot{z}_0\,, \tag{3.4}$$

and $w(0) = w_0$, $u(0) = u_0$.

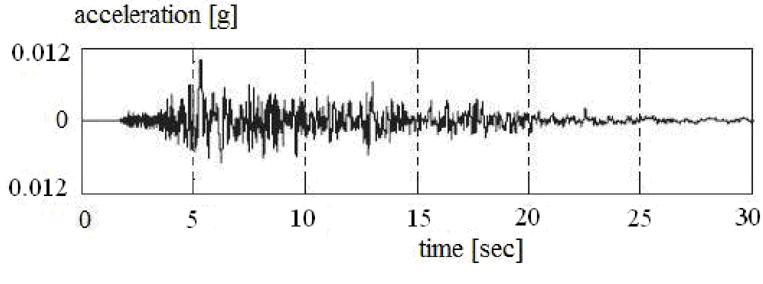

Fig. 3.1 – Ground acceleration used to excite the model.

As an assumption, the observed hysteretic response for the system described by (3.2) and (3.3), is assumed to be the hysteresis loop obtained from (3.2) with the restoring force modeled by (3.1).

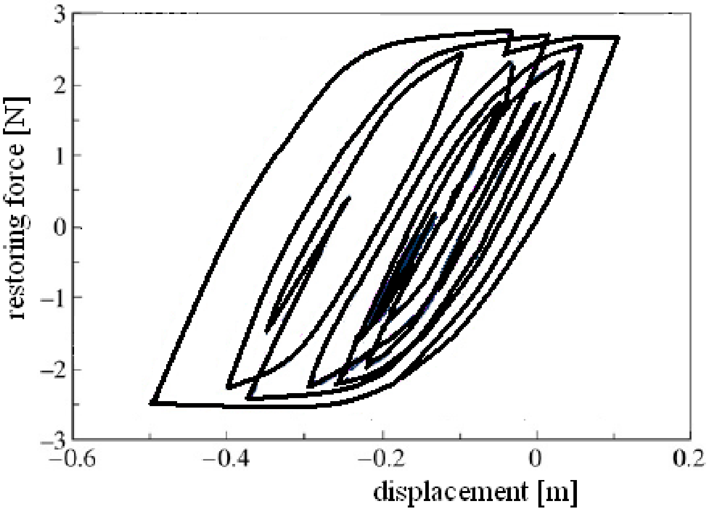

Fig. 3.2 – Observed hysteresis loop for the first floor of the system.

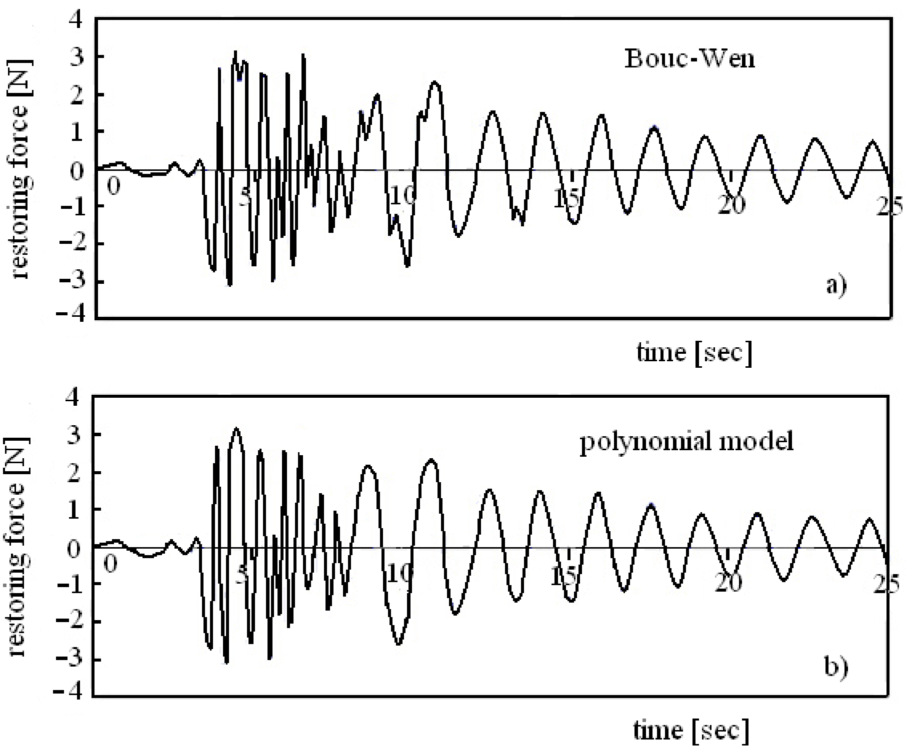

Fig. 3.3 – Profiles of the restoring force: a) Bouc-Wen model; b) polynomial model.

In this way we obtain the hysteretic response of the first floor, illustrated in Fig. 3.2, under ideal free noise conditions. Other floor hysteresis response can be constructed in a similar way. The loop is independent of frequency.

Next, in order to validate the aforementioned assumption, the evaluation of the restoring force by using both, Bouc-Wen (3.1) and the polynomial (3.2) models, respectively, is performed. Comparisons are presented in Fig. 3.3. The Bouc-Wen parameters parameters are $\alpha_i = 2$, $\beta_i = 1$ and $n_i = 2$, $i = 1,...,5$, respectively [2]. It is seen that the presented method works well and no more terms are required in (3.3) to express a good evaluation of the restoring force.

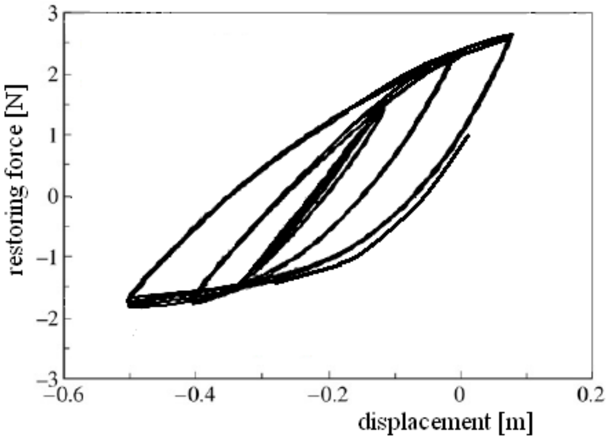

Fig. 3.4 – Restoring force *versus* the displacement when the hysteresis compensator is used.

In order to control the hysteresis effect upon the behavior of the five-story system, we apply the hysteresis compensator (2.11). The aim is to show the contribution of this compensator. The restoring force versus the displacement is illustrated in Fig. 3.4. The results show that hysteresis can be reduced to less than 35% when applying the feedforward control. Such control can improve the behavior of the system.

4. CONCLUSIONS

The subject of the paper belongs to the field of dynamics, characterization and control of structures subjected to vibrations. The generalized play operator is used to describe the hysteretic behaviour of a five-story system with nonlinear restoring force characteristics. Nonlinearities make the system lose its accuracy if not controlled. For that, the control is focused on feedforward or compensation technique with no sensor requirement. The results confirm that feedforward control can give good performances such as accuracy and speed required for this particular application.

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