THE PUBLISHING HOUSE
OF THE ROMANIAN ACADEMY

PROCEEDINGS OF THE ROMANIAN ACADEMY, Series A,
Volume 12, Number 4/2011, pp. 339–346

LOADING DEPENDENCE OF THE SITE NATURAL PERIOD

Dinu BRATOSIN

Institute of Solid Mechanics of the Romanian Academy
C-tin Mille Street, No.15, 10141 Bucharest 1, Romania
E-mail: bratosin@acad.ro

The site materials, soils or rocks, are nonlinear materials with a dynamic behavior strongly dependent of loading level and this behavior affects the whole dynamic response including the natural site period evaluation. Using a nonlinear Kelvin-Voigt model, this paper proposes a modeling method for the loading dependence of the site natural period, dependence with an important impact on site-structure resonance avoidance.

Key words: Nonlinear behavior of soils, Site natural period, Nonlinear oscillating system, Soil-structure resonance.

1. INTRODUCTION

It is very well known that the major damages arise at resonance, when the natural period of structure is equal or very close to the dominant period of the site. For this reason, a correct evaluation of the natural period of the local soil deposit is an essential parameter to estimate local site effects on ground motions.

The site materials, soils or rocks, are nonlinear materials with a dynamic behavior strongly dependent of loading level [2, 4, 14, 17]. Assuming that the geological site materials are nonlinear viscoelastic, in the previous author's papers [2–7] this nonlinear behavior was modeled with the aid of the nonlinear Kelvin-Voigt model (NKV model). This model describes the nonlinearity by the dependence on the material mechanical parameters: shear modulus G and damping ratio ζ in terms of shear strain invariant γ: $G = G(\gamma)$, $\zeta = \zeta(\gamma)$. This nonlinear behavior is met, more or less, at all site materials – more pronounced at soft materials (soils) and more reduced at rocks materials as we can see in experimental tests results from Figs. 1.1 and 1.2.

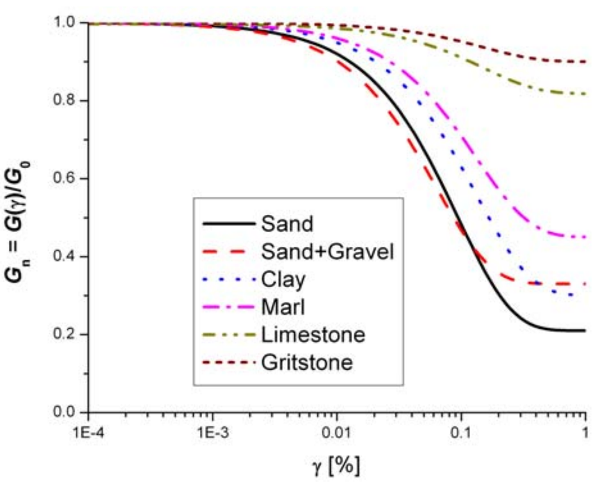

Fig. 1.1 – Modulus-functions (examples).

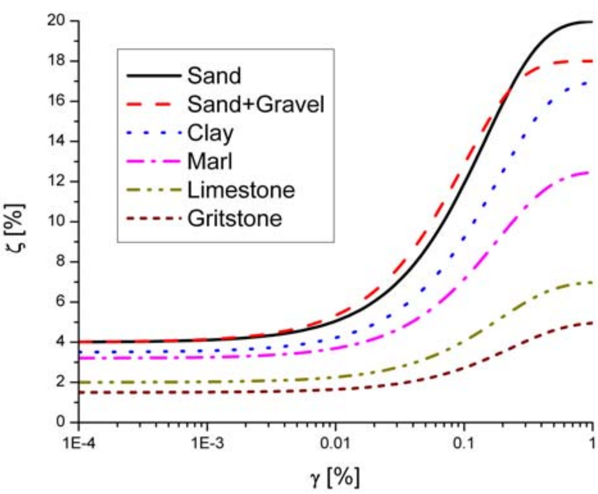

Fig.1.2 – Damping functions (examples).

Accordingly, all of the dynamic characteristics of the site oscillating system acquire strain dependence, including the rigidity $k(\gamma) = A \cdot G(\gamma)$ and natural period $T_g(\gamma) = B / k(\gamma)$ where A and B are the proportional factors.

From this reason, every site oscillating system is a nonlinear system and for every site emplacement instead of a unique natural period value T_g multiple values in terms of strain level exists, therefore for every site a function $T_g = T_g(\gamma)$ must to be evaluated [7–10].

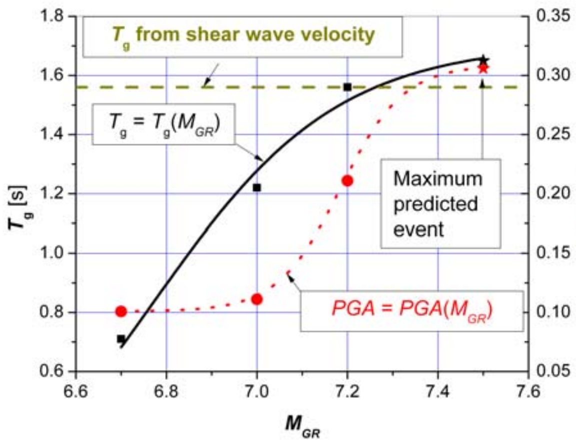

Fig. 1.3 – Seismic recorded and prediced data from INCERC station.

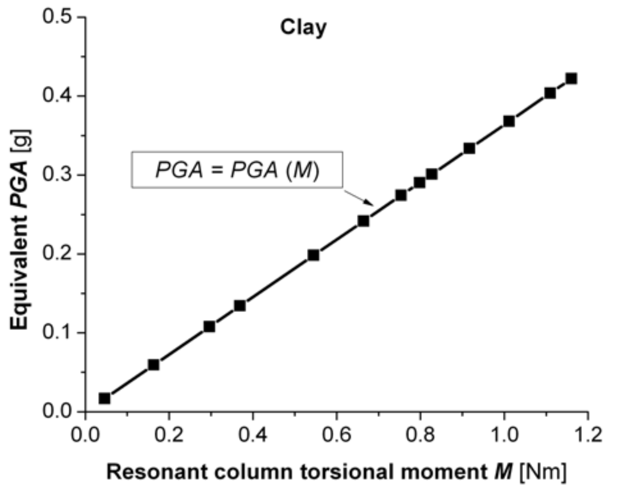

Fig. 1.4 – Specimen loading range in the resonant column device (example).

The seismic data recording during Vrancea earthquakes with different magnitudes shows a doubtless dependence of the natural periods and maximum accelerations on earthquake magnitude [13, 17–19] as we can see in the examples from Fig. 1.3, where the data recorded at INCERC seismic station is presented and where the estimation of the maximum predicted event ($\mathrm{PGA} = 0.305$ g and $T_g = 1.65$ s) [18] was added.

The usual method for site natural period determination is based on the "quarter length formula" $T_g = 4H / v_s$, where H is the site depth and v_s is the shear wave velocity. This formula treats the site as semi-infinite elastic space in contradiction with mechanical reality and gives a unique natural period value in contradiction with earthquake recording.

The present paper proposes an evaluation method of the natural period nonlinear dependence with the aid of the resonant column tests data. The resonant column device can charge the soil specimen to a loading range equivalent (Fig. 1.4) to low until strong earthquakes [4, 22] and can give the nonlinear material functions $G = G(\gamma)$ and $\zeta = \zeta(\gamma)$ as we can see in the examples from Figs. 1.1 and 1.2. Using the magnification functions of the nonlinear Kelvin-Voigt model by numerical simulation with different loading level, we can enable modeling the nonlinear dependence of the site natural period.

2. EVALUATION OF THE NATURAL PERIOD BY RESONANT COLUMN TESTS

In principle, from resonant column test under a harmonic torsional input level $M^i = M_0^i \sin \omega t$ are obtained the corresponding strain level γ_i, the modulus-function value G_i and damping value ζ^i [4, 22].

The shear-modulus value G^i is obtained using the relationship:

$$G = \rho v_s^2 = \rho \left(\frac{\omega_0 h}{\Psi} \right)^2, \qquad (2.1)$$

where ρ is the mass density of specimen, v_s is the shear wave velocity, ω_0 is the specimen natural (angular) frequency, h is the specimen height and Ψ is: $\Psi=\sqrt{R-R^2/3+4R^3/45}$, where $R=J/J_{top}$ is the ratio between torsional inertia of the specimen J and the torsional inertia of the top cap system J_{top}.

The specimen natural period for a level γ_i becomes:

$$T=\frac{2\pi}{\omega_0}=\frac{2\pi h\sqrt{\rho}}{\Psi}\cdot\frac{1}{\sqrt{G}}. \tag{2.2}$$

After several tests with different strain levels γ_i $(i=1,2...n)$ we can obtain the shear-modulus function $G=G(\gamma)$ in the normalized form:

$$G(\gamma)=G_0\cdot G_n(\gamma),\quad \text{with : }\begin{cases} G_0=G(\gamma)\big|_{\gamma=0} \\ G_n(\gamma)=G(\gamma)/G_0 \end{cases}, \tag{2.3}$$

where G_0 is the initial value and $G_n(\gamma)$ is the normalized form of the shear-modulus function and the damping function $\zeta=\zeta(\gamma)$ in the normalized form:

$$\zeta=\zeta(\gamma)=\zeta_0\cdot\zeta_n(\gamma),\quad \text{with : }\begin{cases} \zeta_0=\zeta(\gamma)\big|_{\gamma=0} \\ \zeta_n(\gamma)=\zeta(\gamma)/\zeta_0 \end{cases} \tag{2.4}$$

where ζ_0 is the initial value and $\xi_n(\gamma)$ is the normalized form of the shear damping function [4].

Also, the nonlinear natural period function of the soil specimen in becomes:

$$T_g(\gamma)=T_0\cdot T_n(\gamma)\ \text{with : }\begin{cases} T_0=T_g(\gamma)\big|_{\gamma=0}=\dfrac{2\pi h\sqrt{\rho}}{\Psi}\cdot\dfrac{1}{\sqrt{G_0}} \\ T_n(\gamma)=\dfrac{T_g(\gamma)}{T_0}=\dfrac{1}{\sqrt{G_n(\gamma)}}. \end{cases} \tag{2.5}$$

We mention that the natural periods obtained by resonant column test in the form (2.5) are the natural periods of the single degree of freedom oscillating system composed by a single mass (the vibration device) supported by a spring and a damper represented by the specimen. But, as we can see from eq. (2.5) the physical and geometrical sample properties (h, ρ, J, J_{top}) are included only in the initial value T_0. Thus, the resonant column test can offer accurate data for obtaining only the nonlinear dependence of the normalized natural period T_n [8, 9].

3. NONLINEAR NATURAL PERIOD IN TERMS OF LOADINGS

For practical applications it is necessary to determine the normalized natural period in terms of loading amplitude usually described by *peak ground acceleration* (*PGA*).

For this conversion – $T_n=T_n(\gamma)$ into $T_n=T_n(PGA)$ – we can use the numerical simulation of the resonant column specimen behavior, modeled as nonlinear Kelvin-Voigt model subjected to abutment motion $\ddot{x}_g(t)=\ddot{x}_g^0\sin\omega t$ with different acceleration amplitudes $\ddot{x}_g^0$ (Fig. 3.1). In this loading case, the motion equation reads as [3, 4, 21]:

$$\ddot{x}+c(x)\cdot\dot{x}+k(x)\cdot x=-\ddot{x}_g, \tag{3.1}$$

with the material functions $c(x)$ and $k(x)$ derived from nonlinear viscoelastic constitutive equation [4, 16]:

$$c(x) = 2J\omega_0\zeta(x) \quad ; \quad k(x) = \frac{I_p}{h}G_{re}(x) = \frac{I_p}{h}\frac{G(x)}{\sqrt{1+\zeta^2(x)}}, \tag{3.2}$$

where J is the specimen inertia moment, ω_0 is the specimen natural frequency and h is the specimen height.

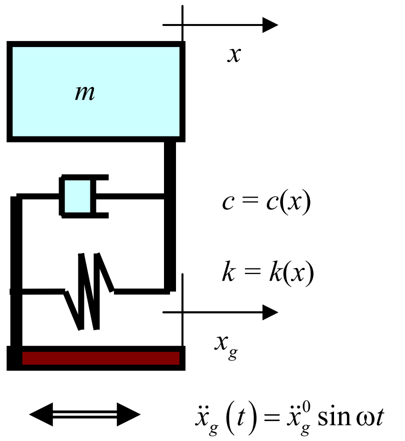

Fig. 3.1. NKV model with abutment excitation.

Using the change of variable $\tau = \omega_0 t$ and introducing the new time function $\varphi(\tau) = x(t) = x(\tau/\omega_0)$ we can obtain for eq. (3.1) another form:

$$\varphi'' + C(\varphi)\cdot\varphi' + K(\varphi)\cdot\varphi = \mu\sin\upsilon\tau, \tag{3.2}$$

where the superscript accent denotes the time derivative with respect to τ, and:

$$C(\varphi) \equiv C(x) = \frac{c(x)}{m\omega_0} = 2\zeta(x),$$
$$K(\varphi) \equiv K(x) = \frac{k(x)}{m\omega_0^2} = \frac{k(x)}{k(0)} = k_n(x), \tag{3.3}$$
$$\mu = \frac{\ddot{x}_g^0}{\omega_0^2} \quad ; \quad \upsilon = \frac{\omega}{\omega_0}.$$

The steady-state solution of the equation (3.2) can be written in the form:

$$\varphi(\tau,\upsilon,\mu) = \mu\Phi(\upsilon,\mu)\sin(\upsilon\tau - \psi), \tag{3.4}$$

where $\Phi(\upsilon,\mu)$ is *the nonlinear magnification function:*

$$\Phi(\upsilon,\mu) = \frac{\max\limits_{\tau}\left[\varphi(\tau,\upsilon,\mu)\right]}{\mu} = \frac{x_{dynamic}}{x_{static}} \tag{3.5}$$

a ratio of maximum dynamic amplitude $\varphi_{\max} \equiv x_{dynamic}$ to static displacement $\mu = x_{static}$.

By numerical simulation we can obtain some nonlinear magnification functions $\Phi(\upsilon) = \Phi(\upsilon;\mu)\big|_{\mu=\text{ct.}}$ one for each normalized loading amplitude $\mu = \ddot{x}_g^0/\omega_0^2$ (see the example from Fig. 3.2 [4, 11, 12, 15, 20]).

Then, because $\upsilon = \omega/\omega_0 = T_0/T = 1/T_n$ we can obtain the magnification functions Φ in terms of normalized period T_n (Fig. 3.3) and then a relationship $T_n = T_n(\mu)$ (Fig. 3.4). Because $\mu = \ddot{x}_g^0/\omega_0^2 = (g\cdot\text{PGA})/\omega_0^2$ from $T_n = T_n(\mu)$ a relationship $T_n = T_n(\text{PGA})$ we can be obtained (Fig. 3.5). As an example, in fig. 3.6 some functions $T_n = T_n(\text{PGA})$ for different site materials are given.

For the evaluation of the entire site normalized natural periods first we must determine from resonant column tests the nonlinear variation T_n^i for each site strata, and then we can obtain the average natural period variation for the entire site layers T_n^{av} as the average of the normalized natural period strata T_n^i weighted with the thickness h_i of each layer [10, 18, 19]:

$$T_n^{av}\big|_{\text{PGA=ct.}} = \frac{\sum T_n^i \times h_i}{\sum h_i}. \tag{3.6}$$

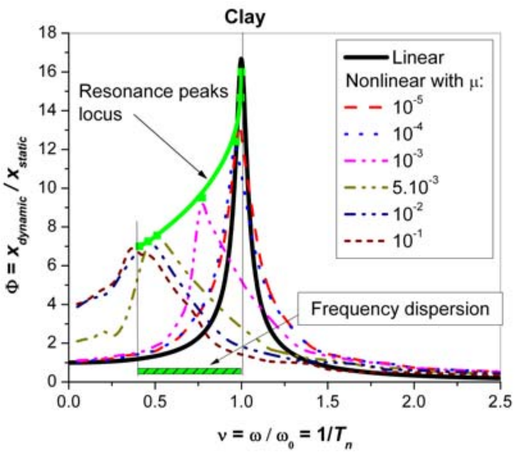

Fig. 3.2 – Nonlinear magnification functions in terms of normalized frequency (for a clay specimen).

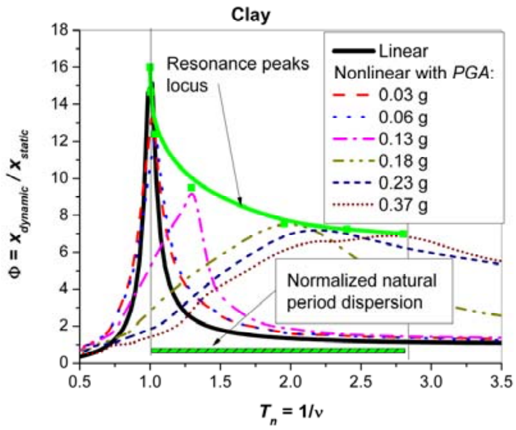

Fig. 3.3 – Nonlinear magnification functions in terms of normalized periods (for a clay specimen).

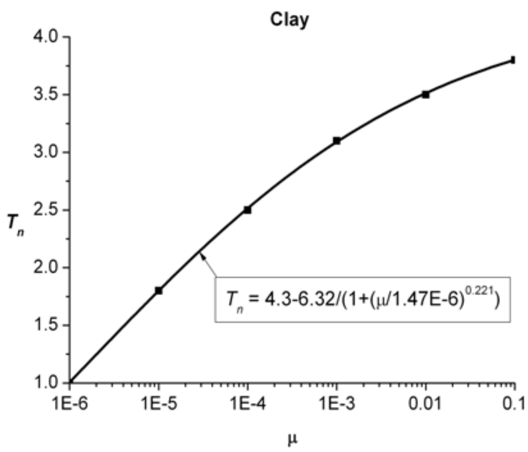

Fig. 3.4 – Relationship $T_n = T_n(\mu)$.

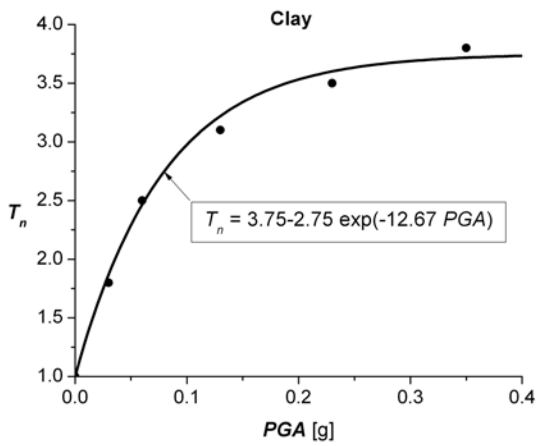

Fig. 3.5 – Relationship $T_n = T_n(PGA)$.

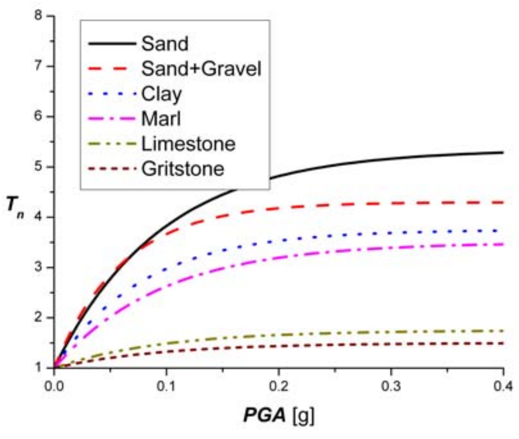

Fig.3.6 – Some functions $T_n = T_n(\mathrm{PGA})$.

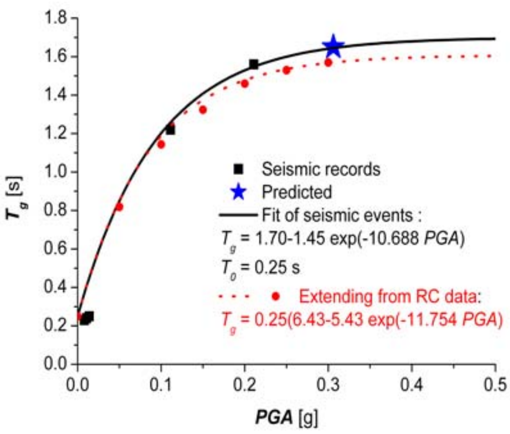

Fig. 3.7 – Dependence T_g – PGA provided by both resonant column data and seismic records.

This method was validated using the site of the seismic station INCERC with known stratification [1]. First, for each constituent layer the material functions $G = G(\gamma)$ and $\zeta = \zeta(\gamma)$ was estimated and thus, by numerical simulation, a function $T_n^i = T_n^i(\mathrm{PGA})$ for each strata *i* was obtained. Then, for some PGA values (0.05, 0.10, 0.15, 0.20, 0.25 and 0.30 g) using eq. (3.6) the site natural period averages T_n^{av} was obtained.

The results of the resonant column simulations in the form $T_g\left(\mathrm{PGA}\right)=T_0\cdot T_n^{av}\left(\mathrm{PGA}\right)$ with initial value $T_0=0.25$ s from seismic data are given in Fig. 3.7 together with seismic data recording at the same site. As e can see from this figure the differences between seismic records and resonant column simulations are acceptable.

4. PRACTICAL CONSEQUENCES OF THE SITE NATURAL PERIOD NONLINEARITY

4.1. A possibility to anticipate the strong earthquake periods starting from small and moderate events

For aforementioned validation, the geological and seismic data of the INCERC site was used because for this site there are multiple seismic recordings with different magnitudes beginning with low events until the strong March 4, 1977 earthquake.

However, for a large majority of the usual sites only seismic recording of the low and moderate events are available. In these cases, the evaluation of the dominant period for strong earthquakes using only seismic low and moderate data presumes an extrapolation procedure with inherent large errors [11, 20].

The resonant column device can charge the soil specimen to a loading range equivalent to low until strong earthquakes (Fig. 1.4) and the nonlinear natural period dependence $T_n=T_n\left(\mathrm{PGA}\right)$ can be obtained by means of interpolation statistical process with an upper accuracy [4, 22].

In this case, when only low and moderate seismic data are available, the determination in the resonant column by the interpolation of the nonlinear variations in normalized form: $T_n=T_n\left(\mathrm{PGA}\right)$ together with the determination of the normalization value T_0 from seismic recording can leads to a better approximation of the natural periods for large PGA values. As an example, in Fig. 4.1 from available records at INCERC station only low and moderate seismic records were selected as "observed data domain". With this data the extrapolation leads to inadmissible values for natural period values at large PGA values. But, using the resonant column interpolation together with initial natural period T_0 obtained from observed data domain the predicted T_g values are much closer from measurement (and predicted) values. Thus, during March 4, 1977 event the measurement value was $T_g=1.56$ s [18, 19] and from resonant column interpolation + initial low seismic value was obtained $T_g=1.48$ s .

4.2. Structural-site resonance avoidance

The dynamic response of a certain structure is strongly dependent of the ratio between the natural period of the structure and the dominant period of this emplacement. It is very well known that the major damages arise at resonance, when the natural period of structure is equal or very close to the dominant period of the site. For this reason, a correct evaluation of site dominant period has a special importance.

For linear oscillator the resonance between the excitation period and the natural period of the oscillator lead to a continuous magnification of the displacements until failure, irrespective of loading magnitude. But, in the case of nonlinear oscillators with degradable materials the resonance leads to the mechanical material degradation and thereby the rigidity decreasing, natural period growing and the oscillator gets out from resonant stage.

The site resonance due to the coincidence between earthquake period and site period it is not a continuous deformation process but rather a shock type loading. From this reason, the seismic loading level (magnitude, PGA) plays an important role in the resonance consequences because only strong events (over $M_{GR}=7$) may lead to an important structural damages which can grow until structural collapse. This assumption was validated by post-earthquake observations [18].

Thus, we can define for every site a dangerous zone corresponding to earthquakes with strong magnitudes. As an example, for seismic site INCERC the seismic recordings show that for magnitudes between $M_{GR}=7$ and maximum expected $M_{GR}=7.5$ an acceleration range $\mathrm{PGA}\simeq 0.1\to 0.3$ g and a natural period range $T_g \simeq 1.2 \rightarrow 1.65$ s correspond (Fig. 4.1). (We mention that from RC data a natural period range obtained was $T_g \simeq 1.2 \rightarrow 1.57$ s). As we can Figs. 4.1. and 4.2, these ranges delimit an avoidable domain for this site.

Treating the site as linear oscillator means a unique natural period assignment. But, even if this value is overestimated the resonance can not be avoided. In the structural strength design an overvaluation of the external loadings assure a sure structural response under inferior loadings. But, in the resonance case, the overestimations of the natural site period (as provided the wave velocity method, or else) do not assure the resonant avoidance. Thus, for example, if for INCERC site it is considered only the unique site natural period as provided by wave velocity method ($T_g = 1.56$ s and PGA = 0.30 g) there seems that the resonant danger arises only for buildings with the same natural period. Thus, for a building with a natural period $T_s = 1.30$ s , for example, the resonant is unlikely. But, if we take into account the loading dependence of the site natural period (Fig. 4.2) the natural period of $T_s = 1.30$ s can be reach under inferior loading as PGA = 0.13 g and resonant magnification can arises.

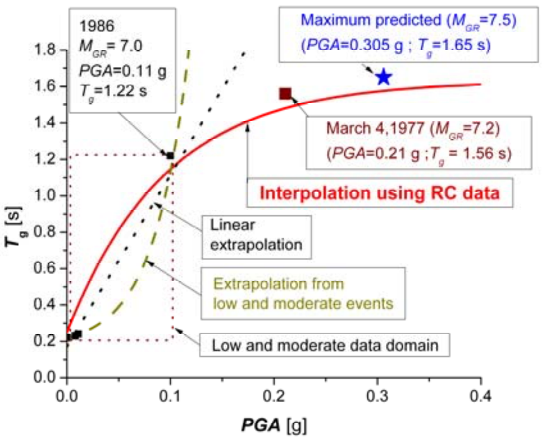

Fig. 4.1 – Predicted large natural periods of the strong earthquakes (example – site INCERC).

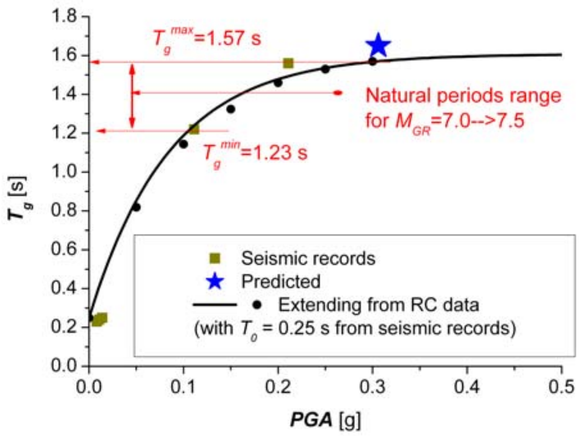

Fig. 4.2 – Natural periods for $M_{GR} = 7.0 \rightarrow 7.5$ (example – site INCERC).

5. CONCLUDING REMARKS

• The site geological materials have a degradable rigidity in terms of the strain level, and the increase excitation level leads to the rigidity decrease and the increase of the natural period values.

• The seismic data recording during Vrancea earthquakes with different magnitudes shows a doubtless dependence of the site natural periods and maximum accelerations on earthquake magnitude.

• The usual method for site natural period determination is based on the semi-infinite elastic space hypothesis and gives a unique natural period value in contradiction with earthquake recording.

• Whereas the linear oscillating systems have a unique resonance value, the nonlinear oscillating systems have multiple resonant values in terms of excitation amplitudes. The nonlinear resonance peaks occurs at different normalized frequency υ situated before the excitation frequency (frequency dispersion) and under linear resonant value.

• Because any structural-site system is a nonlinear oscillating system there are no unique “natural periods” of a certain building, irrespective of his emplacement. Thus, the natural period of the structural-site system is a function of excitation level.

• The resonant column device can charge the soil specimen to a loading range equivalent to low until strong earthquakes.

• Using the resonant column data we can evaluate the nonlinear dependence of the site degradable materials and we can quantify the material nonlinear functions for moduli, rigidity and natural period.

• The dependence of the dominant site period on the excitation level – $T_g = T_g\left(\mathrm{PGA}\right)$ – can be obtained from recorded seismic data if these data cover the entire expected PGA value range.

• The dependence of the natural period on the excitation level in normalized form – $T_n = T_n\left(\mathrm{PGA}\right)$ – can be modeled by interpolation of the resonant column data. This method described in this paper has been validated by comparison with recording earthquake data.

• When only low and moderate seismic data are available, the resonant column determination of the nonlinear variations in normalized form: $T_n = T_n\left(\mathrm{PGA}\right)$ together with the determination of the normalization value $T_0 = T_g\left(\mathrm{PGA}\right)\big|_{\mathrm{PGA}=0}$ from seismic recordings can leads to a better approximation of the dominant periods for large *PGA* values.

• Treating the site as linear oscillator with a unique natural period (even overestimated) the resonance can not be avoided.

REFERENCES

1. BĂLAN Stefan, CRISTESCU Valeriu, CORNEA Ion (Editors), *March 4 Romanian earthquakes, 1977*(in Romanian), Publishing House of the Romanian Academy, 1982.
2. BRATOSIN Dinu, A *dynamic constitutive law for soils,* Proceedings of the Romanian Academy – Series A: Mathematics, Physics, Technical Sciences, Information Science, *1–2*, pp. 37–44, 2002.
3. BRATOSIN Dinu, SIRETEANU Tudor, *Hysteretic damping modelling by nonlinear Kelvin-Voigt model,* Proceedings of the Romanian Academy – Series A: Mathematics, Physics, Technical Sciences, Information Science, *3*, pp. 99–104, 2002.
4. BRATOSIN Dinu, *Soil dynamics elements* (in Romanian), Publishing House of the Romanian Academy, 2002.
5. BRATOSIN Dinu**,** *Nonlinear aspects in soils dynamics* (Ch. 2), in *Topics in applied mechanics*, Vol. 1 (editors Veturia Chiroiu, Tudor SIRETEANU), Publishing House of the Romanian Academy, 2003, pp. 26–50.
6. BRATOSIN Dinu, *Non-linear attenuation in soils and rocks,* Proceedings of the Romanian Academy – Series A: Mathematics, Physics, Technical Sciences, Information Science, **7**, *3*, pp. 209–214, 2006.
7. BRATOSIN Dinu, BĂLAN Stefan-Florin, CIOFLAN Carmen-Ortanza, *Soils nonlinearity evaluation for site seismic microzonation,* Proceedings of the Romanian Academy – Series A: Mathematics, Physics, Technical Sciences, Information Science, **8**, *3*, pp. 235–242, 2007.
8. BRATOSIN Dinu, Florin-Stefan BĂLAN, Carmen-Ortanza CIOFLAN, *Soils nonlinearity effects on dominant site period evaluation,* Proceedings of the Romanian Academy, **10**, *3*, pp. 261–268, 2009.
9. BRATOSIN Dinu, Florin-Ştefan BĂLAN, Carmen-Ortanza CIOFLAN, *Multiple resonances of the site oscillating systems,* Proceedings of the Romanian Academy, **11**, *3*, pp. 261–268, 2010.
10. BRATOSIN Dinu, *Natural periods of the building nonlinear systems* (Ch. 2), in *Research Trends in Mechanics*, Vol. 4 (editors Ligia Munteanu, Veturia Chiroiu, Tudor Sireteanu), Publishing House of the Romanian Academy, 2010, pp. 25–50.
11. CARNAHAN, B., LUTHER, H.A., WILKES, J.O., *Applied numerical methods*, J.Willey, New York, 1969.
12. CHEN Wai-Fah (Editor), *Structural Engineering Handbook,* CRC Press LLC, 1999.
13. CIOFLAN C.O., *Investigations regarding soil local effects on assessment and reduction of the seismic risk* (in Romanian), Ph.D. Thesis, University of Bucharest, 2006.
14. ISHIHARA K., *Soil Behavior in Earthquake Geotechnics,* Clarendon Press, Oxford, 1996.
15. LEVY S., WILKINSON J.P.D., *The Component Element Method in Dynamics*, McGraw-Hill Book Company, 1976.
16. MALVERN, L.E., *Introduction to the mechanics of a continuous medium*, Prentince Hall, New Jersey, 1969.
17. MĂRMUREANU Gh., MĂRMUREANU Al., CIOFLAN C.O., BĂLAN S.F., *Assessment of Vrancea earthquake risk in a real/nonlinear seismology,* Proc.of the 3rd Conf. on Structural Control, Vienna, 12–15 July, 2004, pp. 29–32.
18. MĂRMUREANU Gheorghe, CIOFLAN Carmen-Ortanza, MĂRMUREANU Alex., *Researchers about local seismic hazard for metropolitan zone Bucharest. Seismic microzonation map* (in Romanian), Tehnopress, Iaşi, 2010.
19. MÂNDRESCU Nicolae, RADULIAN Mircea, MĂRMUREANU Gheorghe, *Geological, geophysical and seismological criteria for local response evaluation in Bucharest area,* Soil Dynamics and Earthquake Engineering, **27**, pp. 367–393, 2007.
20. PRESS W.H., FLANNERY B.P., TEUKOLSKY S.A., VETTERLING W.T., *Numerical Recipes, The art of Scientific Computing,* Cambridge University Press, 1990
21. DeSILVA, C., *Vibrations: Fundamentals and Practice*, CRC Press, 2000.
22. * * * * * *Drnevich Long-Tor Resonant Column Apparatus*, Operating Manual, Soil Dynamics Instruments Inc., 1979.

Received June 25, 2011