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PARAMETRIC METHOD FOR DYNAMIC ANALYSIS OF MECHANICAL IMPULSIVE SYSTEMS ACTIONS

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This paper presents a study on the need to correctly define impulsive type loading in order to accurately evaluate the dynamic response of mechanical systems. The theoretical evaluation of the dynamic response of a mechanical system loaded by an impulsive force is performed by developing a physical model. Based on this physical model, a mathematical model is then proposed. The solutions of this mathematical model must be as close as possible to reality, for this both disturbance forces parameters and the mechanical oscillating system characteristics must be precisely defined.

Key words: Impulsive; Loading; Vibration; Parameters; Shape; Duration; Amplitude.

1. INTRODUCTION

Impact assessment of dynamic loadings on mechanical systems has become a priority in terms of social security compliance. This evaluation is performed in situ by experimental measurements, but unfortunately this experimental evaluation is only observant. Nevertheless, the experimental evaluation may indicate the necessary measures to mitigate the negative impact of vibrations on the surroundings. For this reason, during the design phase of a system subjected to dynamic actions, it is necessary to accompany the experimental evaluation with an appropriate mathematical and dynamic modeling. By doing so, one can correctly estimate the dynamic impact that this dynamic phenomenon can have on the neighborhoods. The complete and correct definition of the impulsive type loadings can be summarized to quantifying the following three characteristic parameters: the duration of application, the form and the amplitude [5].

2. SPECIFIC PARAMETERS OF IMPULSIVE ACTIONS SIGNALS

2.1. Parametric characterization of impulsive actions signal shape

The shape of the impulsive type loadings can be estimated by analyzing the specificity of the impulse collision phenomenon. In the case of elasto-plastic collision between two metallic bodies, the following reasoning is valid: the force is gradually applied with the plastic deformation of materials; when maximum plastic deformation is attained, the loading has a maximum value and at this moment begins the period of detachment between the two bodies; due to the plastic deformation law of metallic materials which refers to the fact that any plastic deformation is always accompanied by an elastic deformation, the force that occurs during impact gradually decreases to zero. For this reason, in the literature for plastic and elastic collisions [3] are recommended several theoretical approximation functions such as: haversine, semisinusoidal, rectangular, trapezoidal, etc.

2.2. The duration of force action

This parameter can be identified by two methods as follows:

a. The first method is the analytical one. In this sense, Heinrich Hertz hypothesized that the collision duration is substantially greater than the time needed by the elastic wave to traverse bodies in contact [4]. This hypothesis was experimentally validated by researchers Hamburger and Berger. The scientific explanation of this phenomenon is that a part of the oscillations energy that propagates through the bodies, following the collision, is transformed into heat. Analyzing Hooke's law, Heinrich Hertz analytically deduced the duration of the perfectly elastic collision between two elastic bodies. Thus, the collision period is defined as in the relation (1):

$$T = k_2 \sqrt[5]{v}\,, \tag{1}$$

where v is the speed of collision of the two bodies (2):

$$k_2 = 2{,}9432 \cdot \left(\frac{5}{4c_1} \cdot \frac{m_1 \cdot m_2}{m_1 + m_2} \right)^{2/5}, \tag{2}$$

in which m_1 and m_2 are the masses of the colliding bodies and c_1 is a constant with the following expression (3):

$$c_1 = \frac{16}{3} \cdot \frac{1}{\sqrt{\frac{1}{r_1} + \frac{1}{r_2}} \cdot \left(\vartheta_1 + \vartheta_2 \right)}\,. \tag{3}$$

The expression for ϑ is given by relation (4):

$$\vartheta = \left(\frac{2}{G} \right) \cdot \left(1 - \nu \right), \tag{4}$$

where $\nu = 0.3$ is the Poisson's ratio for steel and G is the transverse elasticity modulus of steel: $G = 8.07 \cdot 10^{10}\,\text{N/m}^2$. Considering a relative speed of the bodies before collision of 8m/s, the dependence of the collision duration on the values of masses m_1 and m_2 is shown in Fig. 1.

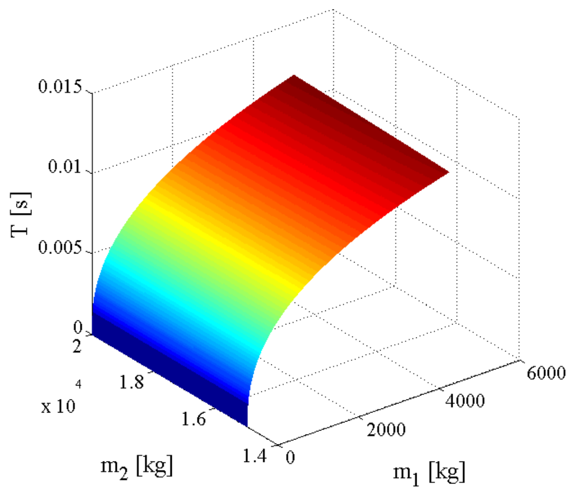

Fig. 1 – Collision duration dependence on bodies masses, v=8m/s.

b. The second method is the experimental one and refers to the use of modern video technique for measuring the duration of contact between two colliding bodies. A case in point is the determination of the contact duration between two metallic bodies both for an elastic collision and, respectively, for a plastic one. The experimental determinations concerned the collisions between ram/anvil (elastic collision) and ram/piece to be forged (plastic collision), during the technological process conducted on a free forging hammer belonging to the repair Division CFR Braila.

The experimental measurements in repair Division CFR Braila were performed on a forging hammer (Figs. 2,3) designed for free forging and with the following technical characteristics: hammer capacity 400 kgf; anvil section dimensions in horizontal plane 80×200 mm; rod diameter 300 mm; number of strokes per minute 150 strokes/minute; height of hammer above the floor 2,540 mm; pad weight 2,500 kgf; net weight 6,950 kgf.

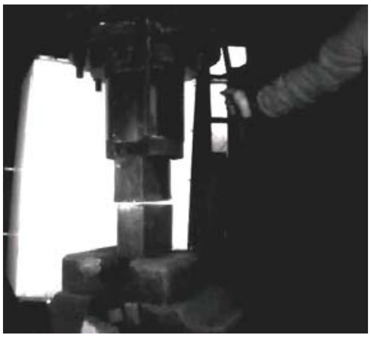

Fig. 2 – Footage shot during the elastic collision.

Fig. 3 – Footage shot during the plastic collision.

The acquisition and processing system used in this work consists of the following specific devices:

- high speed video camera Troubleshooter, produced by Fastec Imaging, with the following basic characteristics:
  - sensor resolution 640×480 pixels;
  - recording speed max. 1,000 frames/second;
  - memory installed 1,024 MB;
- power halogen lamps 2,000 W;
- laptop for data aquisition;
- Software Midas Express 4.0 - Windows.

The experimental determinations were performed for two operating regimes of the free forging hammer:

b.1. Idle operation mode of the hammer: the ram hit directly the anvil; this contact corresponds to an elastic collision.

The image processing was performed using MIDAS software and led to the following experimental determinations:

- The duration of the elastic collision between the hammer and the anvil is about 0.82 ms;
- The average ram speed before the collision is 3.8 m/s.

The variations of the two parameters previously determined are due to the fact that the service staff of the hammer can influence the hit parameters by the way of handling the control lever of the hammer.

b.2. Working in charge regime of the hammer: the ram hit the piece to be forged (previously heated to the optimum hot plastic deformation temperature); this contact corresponds to a predominantly plastic collision.

For this plastic collision between the free forging hammer and the piece to be forged, the following experimental determinations were found:

- The average duration of the plastic collision between the hammer and the anvil is 13 ms;
- The average ram speed before the collision is of 1.56 m/s.

This method can also be used to determine the duration of the loadings resulting from a vehicle passing over an obstacle.

2.3. The amplitude of impulsive actions

The amplitude of an impulsive type signal can be determined analytically based on energy considerations, thus: the kinetic energy introduced into the system through dynamic loading must be equal to mechanical work done by the theoretical force previously determined. The kinetic energy introduced into the system is given by the following expression (5):

$$E = \frac{mv^2}{2} \tag{5}$$

and the mechanical work (6) done by the theoretical force is given by the area bounded by the graph function:

$$L = \int_0^T F\mathrm{d}x\,, \tag{6}$$

where T is the loading duration.

For the haversine pulse we obtain an excitation force value of 74,770 N (Fig. 4), while for the trapezoidal pulse an excitation force value of 56,076 N is obtained (Fig. 5).

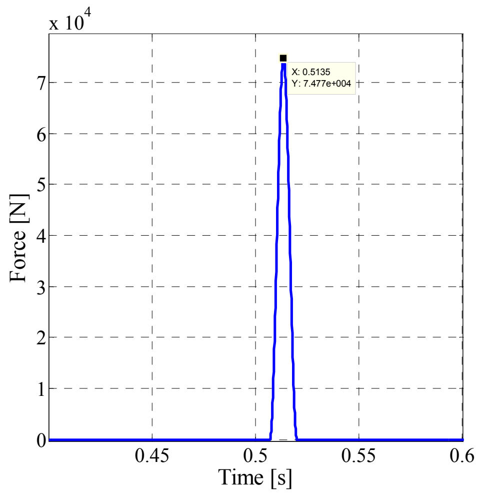

Fig. 4 –Haversine pulse – T=0.013 s.

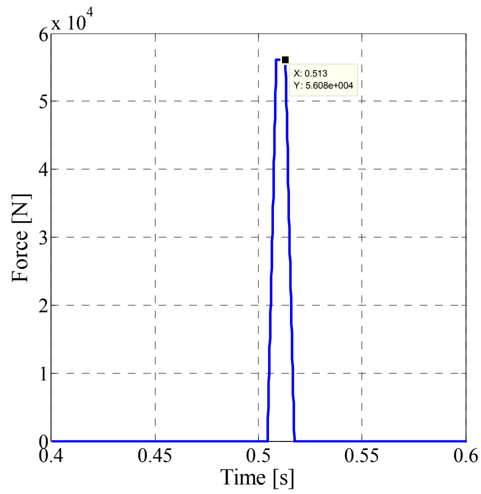

Fig. 5 – Trapezoidal pulse – T=0.013 s.

3. THE INFLUENCE OF ACTIONS SPECIFIC PARAMETERS ON THE DYNAMIC RESPONSE

3.1. Significant impulsive actions

Will be considered here the significant signals that can be analytically deterministically modeled, because they represent a majority of 70-80% in the field of human activities (industry and construction), as well as in the processes of nature (action of wind, water action, landslides and certain types of earthquakes). In the literature, the range of these signals is wide, e.g.: haversine, rectangular, trapezoidal, triangular, cycloidal, exponentially, semisinusoidal. But in this study only two of them will be analyzed, namely: haversine and trapezoidal.

3.2. Analysis of the dynamic response to significant impulsive actions

In order to point these influences, the frequency representation of two haversine and trapezoidal type signals is done, for the same amplitude but for different durations of application [1]. Thus, the characteristics in frequency of the above-mentioned functions were represented for an amplitude of 1000N and for application durations of 0.1s, 0.03s and 0.007s.

The haversine waveform is defined by the equation (7):

$$F(t) = \begin{cases} \frac{A}{2}\left(1 - \cos\frac{2\pi t}{T}\right), & 0 \le t \le T \\ 0 & , \quad -\infty < t < 0, T < t < +\infty. \end{cases} \tag{7}$$

By applying the Fourier transform to this function, it comes:

$$F(\omega)=\int_{-\infty}^{+\infty}\frac{A}{2}\left(1-\cos\frac{2\pi t}{T}\right)\mathrm{e}^{-\mathrm{i}\omega t}\,\mathrm{d}t=\frac{2\mathrm{i}A(1-\mathrm{e}^{-\mathrm{i}\omega T})\pi^2}{-4\omega\pi^2+\omega^3T^2}. \tag{8}$$

Furthermore, the shock pulse spectrum is obtained:

$$|F(\omega)|=\frac{4A\pi^2\sin\frac{\omega T}{2}}{-4\omega\pi^2+\omega^3T^2}. \tag{9}$$

The figures below show the waveform and the pulsation spectrum of a haversine function with:

- application duration T=0.1 s and amplitude A=1,000N (presented in Figs. 6,7);
- application duration T=0.03 s and amplitude A=1,000N (presented in Figs. 8,9);
- application duration T=0.007 s and amplitude A=1,000N (presented in Figs. 10,11).

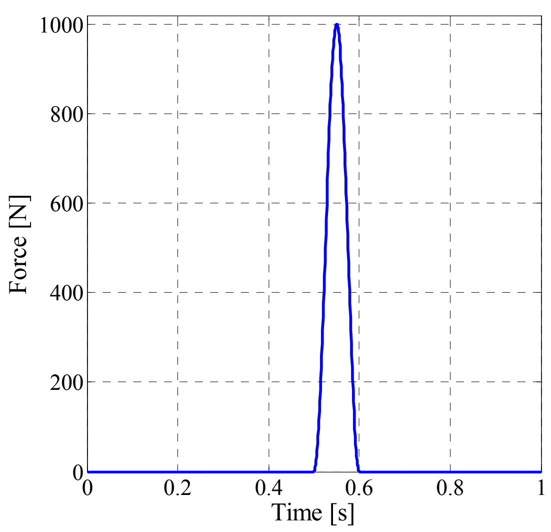

Fig. 6 – Haversine impulse – T = 0.1s.

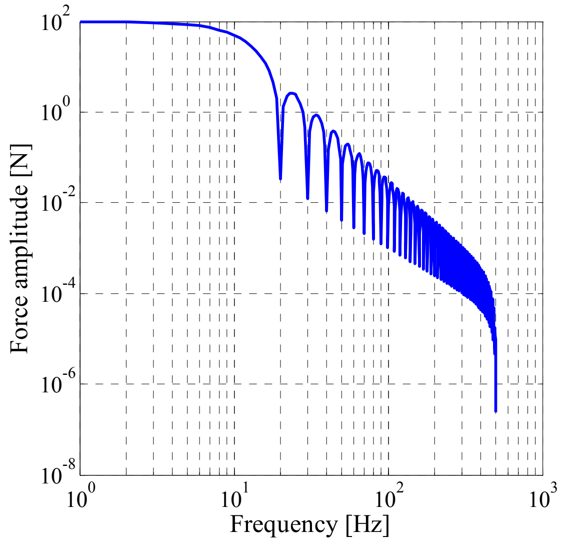

Fig. 7 – Pulsation spectrum of haversine impulse – T = 0.1s.

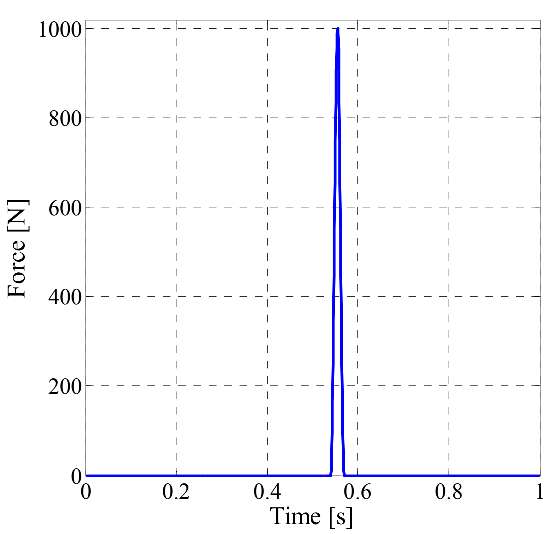

Fig. 8 – Haversine impulse – T = 0.03s.

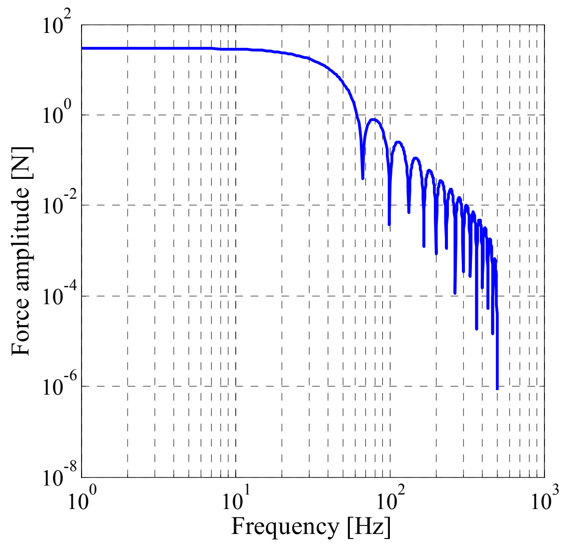

Fig. 9 – Pulsation spectrum of haversine impulse – T = 0.03s.

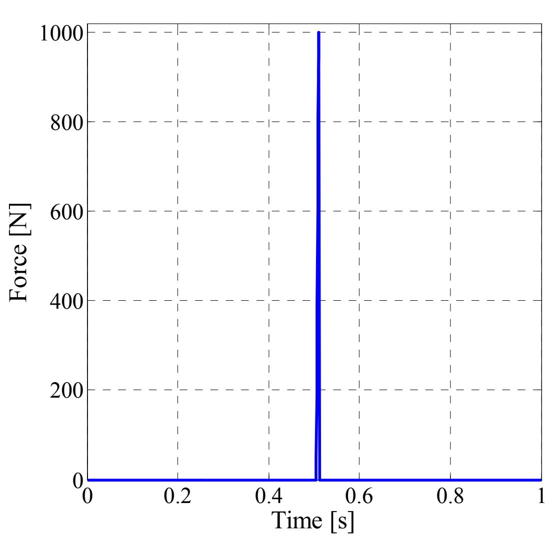

Fig. 10 – Haversine impulse – $T = 0.007$s.

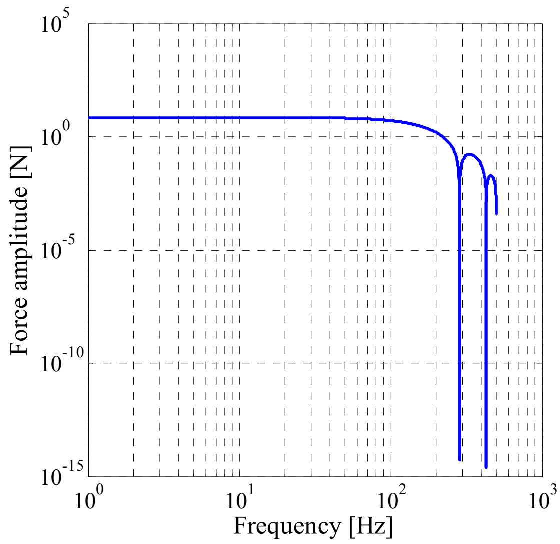

Fig. 11 – Pulsation spectrum of haversine impulse – $T = 0.007$s.

For this haversine impulse case, the significant frequency bandwidth of the shock is inversely proportional to the duration of application of the force excitation, according to the Table 1.

Table 1

Significant frequency bandwidth of the Haversine impulse

Impulse type	Application duration	Significant frequency band
Haversine impulse	T=0.1s	f=(0÷18)Hz
	T=0.03s	f=(0÷60)Hz
	T=0.007s	f=(0÷280)Hz

In what concerns the trapezoidal waveform, it is defined by the equation (10):

$$F(t) = \begin{cases} A \cdot \dfrac{t-a}{b-a}, & a \le t \le b \\ A, & b < t < c \\ A \cdot \dfrac{t-d}{c-d}, & c \le t \le d. \end{cases} \tag{10}$$

By applying the Fourier transform to this function, one obtains the relation (11):

$$\begin{aligned} F(\omega) = \int_a^b A \cdot \frac{t-a}{b-a} \mathrm{e}^{-\mathrm{i}\omega t} \mathrm{d}t + \int_b^c A \cdot \mathrm{e}^{-\mathrm{i}\omega t}\, \mathrm{d}t + \int_c^d A \frac{t-d}{c-d} \cdot \mathrm{e}^{-\mathrm{i}\omega t}\, \mathrm{d}t = -\frac{iA(\mathrm{e}^{-ib\omega} - \mathrm{e}^{-ic\omega})}{\omega} + \\ + \frac{A\mathrm{e}^{-2\mathrm{i}(a+b)\omega}(\mathrm{e}^{\mathrm{i}(a+2b)\omega} + \mathrm{i}\mathrm{e}^{\mathrm{i}(2a+b)\omega}(\mathrm{i} + a\omega - b\omega))}{(a-b)\omega^2} + \\ + \frac{A\mathrm{e}^{-2\mathrm{i}(c+d)\omega}\left(\mathrm{e}^{\mathrm{i}(2c+d)\omega} + \mathrm{e}^{\mathrm{i}(c+2d)\omega}(\mathrm{i}d\omega - \mathrm{i}c\omega - 1)\right)}{(c-d)\omega^2}. \end{aligned} \tag{11}$$

The figures below show the waveform and the pulsation spectrum of a trapezoidal function with:

- application duration T=0.1 s and amplitude A=1,000N (presented in figs. 12,13);
- application duration T=0.03 s and amplitude A=1,000N (presented in figs. 14,15);
- application duration T=0.007 s and amplitude A=1,000N (presented in figs. 16,17).

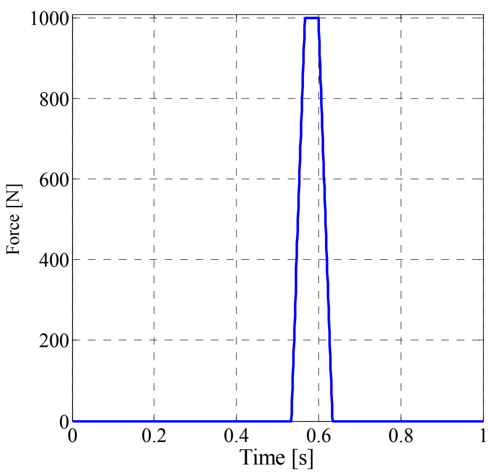

Fig. 12 – Trapezoidal impulse – $T = 0.1$s.

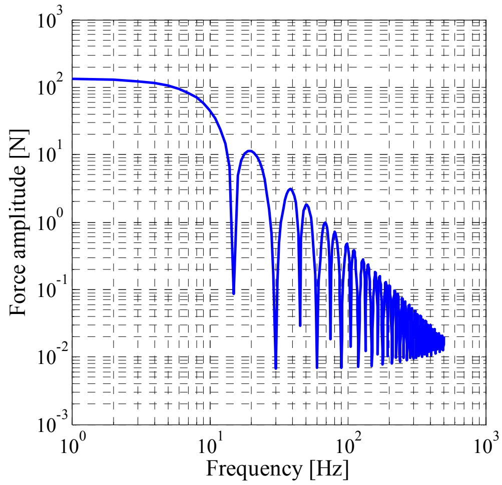

Fig. 13 – Pulsation spectrum of trapezoidal impulse – $T = 0.1$s.

The graphic representations in this paper were performed using MATLAB software [2].

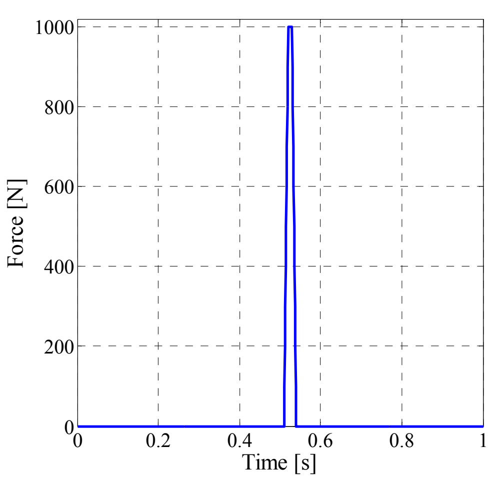

Fig. 14 – Trapezoidal impulse – $T = 0.03$s.

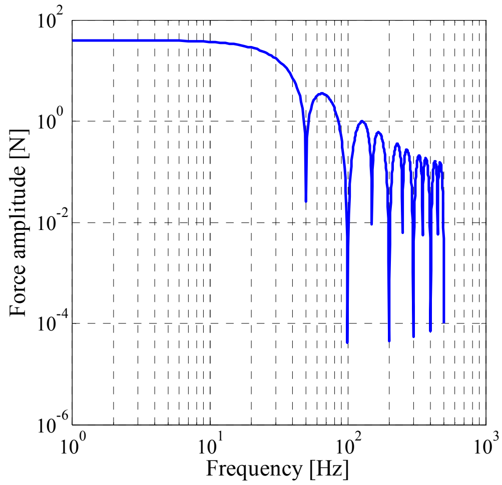

Fig. 15 – Pulsation spectrum of trapezoidal impulse – $T = 0.03$s.

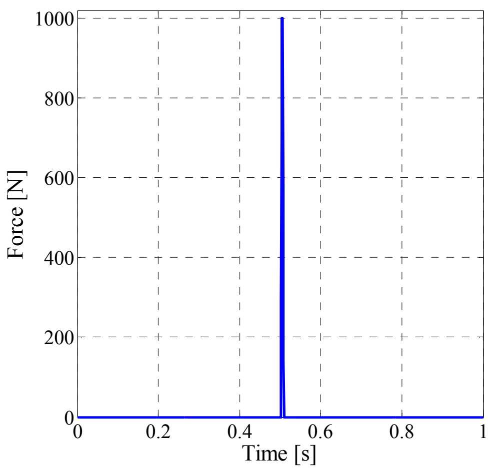

Fig. 16 – Trapezoidal impulse – $T = 0.007$s.

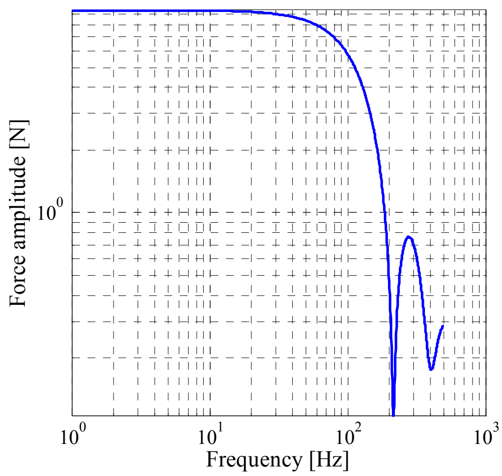

Fig. 17 – Pulsation spectrum of trapezoidal impulse –$T = 0.007$s.

For this trapezoidal impulse case, the significant frequency bandwidth of the shock depends on the duration of application of the force excitation, according to the table 2.

Table 2

Significant frequency bandwidth of the trapezoidal impulse

Impulse type	Duration of aplication	Significant frequency band
Trapezoidal impulse	T=0.1s	f=(0÷15)Hz
	T=0.03s	f=(0÷50)Hz
	T=0.007s	f=(0÷210)Hz

The comparative analysis of the frequency representations of haversine and trapezoidal functions leads to the following conclusions:

- In terms of frequency representations of the same type of function but for different application durations, significant differences of the dominant frequency bandwidth are observed. Note that in fact the dominant frequency bandwidth is inversely proportional to the duration of application of the disturbing function.
- Comparing the spectral representations of haversine and trapezoidal functions for the same duration of application, there are some differences on the dominant frequency band. Thus, in the case of trapezoidal type disturbing force, a decrease in the dominant frequency bandwidth is observed for all three cases analyzed.

4. CONCLUSIONS

This scientific study highlights the following conclusions:

- The correct definition of the specific parameters of impulsive actions arising from human activities or natural phenomena, represents an essential step in the design phase of a dynamic system, in order to obtain an accurate evaluation of its dynamic response to these impulsive actions.
- By analyzing the influence of the characteristic parameters of impulsive forces, it becomes obvious that the accuracy degree of the dynamic model is given by the correct determination of these characteristic parameters, thus making the difference between a wrong dynamic response and a good dynamic response (according to the real case).
- This methodology is based both on analytical calculation elements and on experimental results performed, showing increased adaptability to the specificity of the analyzed system. Thus, this method is applicable to the dynamic study of other mechanical systems disturbed by impulsive actions in intensive and varied regime.
- This is also the case of the correct definition of the dynamic loadings from the road traffic, in order to achieve the dynamic modeling of bridge or viaduct structures.

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