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DECOUPLING EFFECTS – A NOVEL METHOD FOR COMPOSITES CHARACTERIZATION

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The practice of mechanical characterization of composite materials demonstrates that there are at least two characteristics being very difficult to be determinate by experiments: in-plane shear modulus G_{12} and F_{12} coefficient of Tsai-Wu for failure criterion. The purpose of the study was to explore and subsequently establish experimental techniques for determination of these two mechanical characteristics by exploiting a very interesting phenomenon observed in composites mechanical answer: decoupling effects.

Key words: Composite material; Mechanical characteristic; Coupling effects.

1. INTRODUCTION

The group of composite materials that was under study here involves composites called “reinforced materials”. The basic components of these materials (sometimes referred to as “advanced composites”) are long and thin fibers possessing high strength and stiffness. The idea of combining several components to produce a new material with properties that gather in an intelligent way the interesting properties of individual components has been led to high performances materials. Correspondingly, the majority of natural materials that have emerged as a result of a prolonged evolution process can be treated as composite materials.

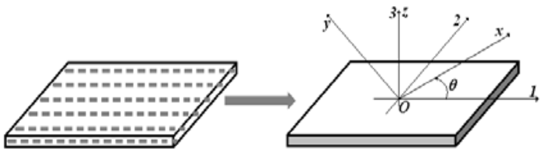

Fig. 1 – The orthotropic model for composites.

A structural study at the level of individual components of a composite, micromechanical analysis provides only qualitative prediction of the ply stiffness as well as for the ply strength.

For practical applications, the mechanical answer is modeled by considering the global answer of the material, the *macromechanical model*. The macromechanical model assumes that the reinforced composites can be considered as homogenous materials with an anisotropic mechanical answer.

2. COUPLING EFFECTS PHENOMENON

A simply analysis of the constitutive equation in the case of the plane problem for orthotropic materials demonstrates that the compliance matrix will be totally populated if it is written with respect to a coordinate system (O*xyz* in Fig. 1) that is not coincident with the principal anisotropic directions (O123 in Fig. 1), as shown in equation (1)

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_{xy} \end{Bmatrix} = \begin{bmatrix} \bar{S}_{11} & \bar{S}_{12} & \bar{S}_{16} \\ \bar{S}_{12} & \bar{S}_{22} & \bar{S}_{26} \\ \bar{S}_{16} & \bar{S}_{26} & \bar{S}_{66} \end{bmatrix} \cdot \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{Bmatrix}. \tag{1}$$

If the first and the third equations from (1) are developed, two interesting observations can be made: for a complex stress state, shearing stress will produce normal strain, and normal stress will produce shear strain. This phenomenon, characteristic for composite materials, is known as "*weak coupling effects*" and generally influences in a negative manner different mechanical tests.

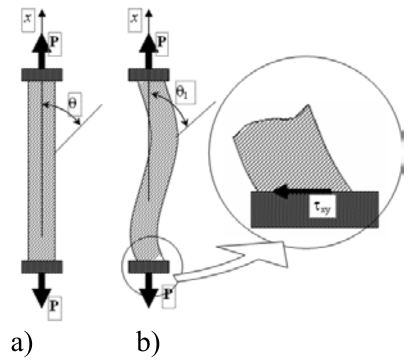

Fig. 2 – Off-axis tensile test.

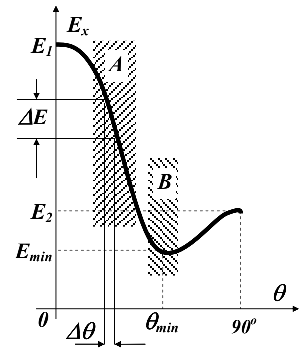

Fig. 3 – Variation of E_x modulus.

In the case of the tensile experiment shown in Fig. 2a, because of the rigid clamps of the testing machine, applied at both ends of the specimen, the shearing movement of the material in these zones is restricted. That will result in a deformation of the whole specimen as shown in Fig. 2b.

This behavior of the specimen is critical from measurements point of view. In these situations, the local deformations of the specimen will significantly affect the measurements. The deformations of the specimen take place in the test gauges area too. In this case, the value of the fiber angle, θ_1, is different of the θ value for the unloaded specimen (as shown in Fig. 2). It would be significant the study of the influence of θ angle upon the variation of conventional Young modulus, E_x.

Analyzing the curve $E_x = E_x(\theta)$ in Fig. 3, there are some important observations to be done:

- the maximum value for E_x modulus is obtained when $\theta = 0^o$ i.e., when $E_x = E_1$ (along the fibers);
- the minimum value for E_x modulus is obtained in a direction that is not always perpendicular to the fiber direction i.e. it is possible that $E_{\min} < E_2$ (and this is according to a enunciated Jones' theorem [2]);
- there is a domain where the values of Young modulus decrease very quickly (*A*-domain) and another one where these values are, practically stationary (*B*-domain).

If the tensile experiment is done in *A*-domain, small modifications of θ value (because of the specimen distortion – for example), $\Delta\theta$, will be translated in significant changes for the E_x measured value, ΔE_x.

Conclusion. It would be better if the off-axis experiment is performed for an angle, $\theta_{\min}$, where the E_x curve is stationary (*B*-domain in Fig. 3). In this case, small modifications of θ angle will be translated in small modifications of E_x modulus.

3. DECOUPLING EFFECTS

According to the previous chapter conclusion, because the evaluation of $\theta_{\min}$ angle represents the key of this study, a careful analysis of its significance has to be done.

3.1. Determination of $\theta_{\min}$ direction

For a simple tensile test on O*x* direction ($\sigma_x \neq 0$, $\sigma_y = \tau_{xy} = 0$), from (1) $\bar{S}_{11}$ is to be considered to represent $1/E_x$. In order to find out the value of $\theta_{\min}$, it is necessary to look for the stationary points of the function $E_x = E_x(\theta)$. These points characterize the stationary points of the function $1/E_x$ too, taking into account that the function E_x is differentiable and $E_x > 0$ i.e. a minimum of E_x corresponds to a maximum for $1/E_x$ as resulting from equations

$$\frac{\mathrm{d}}{\mathrm{d}\theta}\left(\frac{1}{E_x}\right) = -\frac{1}{E_x^2}\cdot\frac{\mathrm{d}E_x}{\mathrm{d}\theta} = 0 \quad \Leftrightarrow \quad \frac{\mathrm{d}\overline{S_{11}}}{\mathrm{d}\theta} = 0 \quad , \tag{2}$$

or

$$\sin\theta\cos\theta\left[-\frac{4}{E_1}\cos^2\theta + 2\left(\frac{1}{G_{12}} - \frac{2\nu_{12}}{E_1}\right)\cos 2\theta + \frac{4}{E_2}\sin^2\theta\right] = 0. \tag{3}$$

The solutions of the equation (3) within the domain [0, 90°] are

$$0^0, \quad \mathrm{arctg}\sqrt{\frac{\frac{2}{E_1}\left(1+\nu_{12}\right) - \frac{1}{G_{12}}}{\frac{2}{E_2}\left(1+\nu_{21}\right) - \frac{1}{G_{12}}}} = \mathrm{arctg}\sqrt{\lambda}, \quad 90^0, \tag{4}$$

corresponding to E_1, $E_{\min}$ and E_2, respectively.

*Observation***.** All tissue reinforced laminas as well as some long fiber reinforced composites [1] satisfy the conditions

$$\frac{1}{G_{12}} > \frac{2}{E_1}\left(1+\nu_{12}\right) \quad , \quad \frac{1}{G_{12}} > \frac{2}{E_2}\left(1+\nu_{21}\right). \tag{5}$$

In these cases the solution (4) exists and the corresponding value of $E_{\min}$, can be directly calculated:

$$\frac{1}{E_{\min}} = \frac{1}{\left(1+\lambda\right)^2}\left[\frac{1}{E_1} + \lambda\left(\frac{1}{G_{12}} - \frac{2\nu_{12}}{E_1}\right) + \frac{\lambda^2}{E_2}\right]. \tag{6}$$

3.2. Decoupling effects

A simple analysis of the equation (1') demonstrates that the avoiding of coupling effects phenomenon (decoupling effects) for a tensile loading test can be achieved if

$$\overline{S}_{16} = 0 \quad . \tag{7}$$

In this case, normal stress will cause only normal strains.

In [3] was demonstrated that

$$\overline{S}_{16} = \left[2\left(\frac{\sin^2\theta}{E_2} - \frac{\cos^2\theta}{E_1}\right) + \left(\frac{1}{G_{12}} - \frac{2\nu_{12}}{E_1}\right)\left(\cos^2\theta - \sin^2\theta\right)\right]\sin\theta\cos\theta\,. \tag{8}$$

Simple calculus demonstrates that

$$\overline{S}_{16} = -\frac{1}{2}\frac{\mathrm{d}\overline{S}_{11}}{\mathrm{d}\theta} = -\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}\theta}\left(\frac{1}{E_x}\right), \tag{9}$$

i.e. the equations (2) and (7) are equivalent. This means that the effects are not coupled if the force is applied in the following directions (with respect to fiber direction): in the fiber direction (natural elasticity direction, $\theta = 0^o$); in an in-plane orthogonal direction (natural elasticity direction, $\theta = 90^o$); in the direction corresponding to the minimum value of the Young's modulus, ($\theta = \theta_{\min}$). Or the directions where the Young modulus function is stationary (i.e., the directions where the function has points of local extreme), are coincident with those where the effect couplings phenomenon is absent, for a uniaxial load case.

These three directions can be exploited for experimental determination of different mechanical characteristics of the composite lamina, as a result of accurate stress state in the sample if the tensile or compression loading force was applied corresponding to each direction. Precisely, tensile and compression tests on O_1 and O_2 directions (Fig. 1) can be used in order to determine E_1, E_2, ν_{12}, and ν_{21} engineering constants of constitutive equation of the composite, as well as F_1, F_2, F_{11}, and F_{22} coefficients of Tsai Wu Failure Criterion. More complicated, as it will be shown later, are the experiments to determinate shearing modulus, G_{12}, and F_{12} coefficient of Tsai Wu Failure Criterion.

4. AN ITERATIVE WAY FOR SHEAR MODULUS DETERMINATION N

4.1. Generalities

In plane shear modulus determination, G_{12}, generated many studies that led to different techniques. Some of these techniques were accepted and imposed by testing regulations. Each method has its disadvantages that will influence the accuracy of the determinations as presented below:

- The $\pm 45^o$ tensile method consists in a tensile test upon a plane specimen manufactured by gluing two laminas oriented at $\pm 45^o$. According to the behavior of laminas on off axis tensile tests illustrated in fig. 2, the two opposite laminas will block the displacement tendency each other maintaining the rectilinear shape of the specimen. The main disadvantage of the method consists in the additional shearing stress at the glued faces of the sample, and as a result, a complex stress state will occur inside the sample.
- The specimen for the two rail shearing test is fixed between two metallic rails that are acted by two longitudinal forces producing the shearing of the material between the rails. The disadvantage of the method stems in the added compression forces towards the rails and bending effects that will disturb the uniaxial stress state.
- The Iosipescu shearing test seems to be more accurate. The disadvantages of the method are: the complexity of the fixture devices, the complicate shape of the sample has to be performed by machining that will alter the real structure of composite.

4.2. Shear modulus and decoupling angle determination

The existence of θ_{min} direction legitimates a tensile test similar with the $\pm 45^o$ one, but having the advantage that the effect coupling will be absent and this will assure a uniaxial stress state inside specimen. As a consequence, once known E_x, by using the relation

$$\frac{1}{E_x} = \frac{\cos^4\theta}{E_1} + \left(\frac{1}{G_{12}} - \frac{2\nu_{12}}{E_1}\right)\frac{\sin^2 2\theta}{4} + \frac{\sin^4\theta}{E_2} , \tag{10}$$

the shear modulus can be directly determined.

The main problem that appears is generated by the two unknowns in the equation (14), namely G_{12} and θ_{min}. The method that the authors propose to solve the previous problem stems in an iterative way of simultaneously determination the two unknowns.

It is assumed that the mechanical characteristics E_1, ν_{12}, E_2, ν_{21} have already been determined.

The iterative way presented below, is bound to be convergent towards the stability point θ_{min} on the E_x curve.

For the first step an arbitrary angle $\theta^{(1)}$ is selected and a specimen is cut in this direction (for example it can be chosen $\theta^{(1)} = 45^o$).

A tensile experiment is made upon this specimen and an approximate Young's modulus results: $E_x^{(1)}$. Now it is possible to calculate an approximate value for shear modulus, $G_{12}^{(1)}$ by using Equation (10), and, from equation (4) the angle $\theta^{(2)}$ for the specimen that will be used in the following steps.

Generally, for the step *i* the determinations are done in the same way.

- the angle $\theta^{(i)}$ is determined in the previous step, and a specimen is cut in this direction;

- the Young's modulus $E_x^{(i)}$ is determined in a tensile experiment made upon this specimen;
- the shear modulus is calculated by using the equation

$$\frac{1}{G_{12}^{(i)}}=\frac{4}{\sin^2\theta^{(i)}}\left[\frac{1}{E_x^{(i)}}-\frac{\cos^4\theta^{(i)}}{E_1}-\frac{\sin^4\theta^{(i)}}{E_2}+\frac{2\nu_{12}}{E_1}\frac{\sin^2 2\theta^{(i)}}{4}\right]; \tag{11}$$

- the angle $\theta^{(i+1)}$ for the following step is calculated in the same way

$$\theta^{(i+1)}=\operatorname{arctg}\sqrt{\frac{\frac{2G_{12}^{(i)}}{E_1}\left(1+\nu_{12}\right)-1}{\frac{2G_{12}^{(i)}}{E_2}\left(1+\nu_{21}\right)-1}}\;. \tag{12}$$

The process will be considered ready when the difference between two consecutive values of θ angle, is enough small

$$\left|\theta^{(i+1)}-\theta^{(i)}\right|\le\omega\,. \tag{13}$$

The error ω is considered in connection with the precision of the specimen cutting operation. By the experience of the authors the results obtained for $\omega=1^o$ are enough satisfactory. The method is very simple and can be applied in any standard laboratory of mechanics. The convergence of the method is very quick and the determinations can be as accurate as required.

4.3. Experimental results

The method was tested for several long fibers reinforced composite materials. A complete set of experiments in order to determine the engineering constants for a four layered laminate (0/0/0/0) is presented below.

Four prepreged plies tape were laid up in a 0 degree direction and processed in the autoclave. The cured panel had a nominal thickness of 0.5 mm with the fiber volume fraction of 0.61. Individual specimens were cut dry from the panel with the fibers oriented in the necessary directions. Each test specimen having nominal dimensions of 0.5 × 20 × 250 mm was then ground on the long edges to a width tolerance of ± 0.1 mm. Each specimen was instrumented with a 90^o biaxial rosette strain gauge bonded to the centre of one face of the specimen. One component of the rosette was in alignment to the loading axis of the specimen. The engineering constants were determined: E_1 = 120.0 GPa; E_2 = 10.8 GPa; ν_{12} = 0.3182; ν_{21} = 0.0288.

In order to determine the shear modulus, the iterative chain of determinations started with determination of Young's modulus for a specimen cut at a 45^o angle with respect to the fibers.

Finally, there were obtained: G_{12} = 3,810 MPa, $\theta_{min} = 61.3^o$, E_{min} = 11,172 MPa, noting the errors

$$\omega_E=\frac{E_{45^o}^{1-st}-E_{45^o}^{fin}}{E_{45^o}^{fin}}\cdot 100=\frac{12.36-11.172}{11.172}\cdot 100=10.6\%,$$
$$\omega_G=\frac{G_{12}^{1-st}-G_{12}^{fin}}{G_{12}^{fin}}\cdot 100=\frac{4.386-3.810}{3.810}\cdot 100=15.1\%, \tag{14}$$

where: ω_E – the error of E_x measurement; $E_{45^o}^{1-st}$ – the Young's modulus determined in a simple tensile test upon a specimen cut at 45 degrees with respect to the fiber direction; $E_{45^o}^{fin}$ – the final (correct) value of Young modulus for the same direction determined by using the authors test method. And, similar for shearing modulus

The results demonstrate that even with the relative length of the specimen enough great (25/2) the error for shear modulus determination is quite significant, if a simple tensile experiment is used. The presented arguments demonstrated that the method offers the possibility to determine this mechanical characteristic with a precision as great as it is desired. The method is a very simple one. Its potential disadvantage is connected to the number of experiments necessary to be done in order to obtain accurate results.

The experience of the authors in this problem demonstrated that, generally, the convergence of the method is quite fast. Fig. 4 shows the measurement convergence rate.

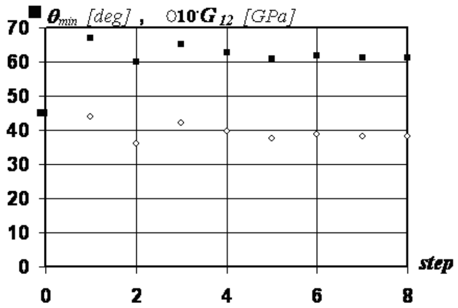

Fig. 4 – Measurement rate convergence in the iterative way.

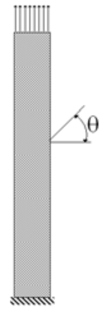

Fig. 5 – Specimen model.

4.4. Numerical Simulation

The numerical simulation was done by using FEM code ANSYS 10 (Swanson Analysis Systems Inc.).

Because of the unidirectional orientation of the fibers inside the specimen there were considered isoparametric finite elements: orthotropic plates, SHELL63, with both bending and membrane capabilities with four nodes and six degrees of freedom on each node.

According with equations (1), (9), and the boundary conditions of the model, the shearing strain expression

$$\varepsilon_{xy} = -\frac{1}{2}\cdot\frac{\mathrm{d}}{\mathrm{d}\theta}\left(\frac{1}{E_x}\right)\cdot\sigma_x = \frac{1}{2E_x^2}\cdot\frac{\mathrm{d}E_x}{\mathrm{d}\theta}\cdot\sigma_x \tag{15}$$

will produce displacements of the whole specimen towards left or right, depending on the sign of the derivative (slope) in equation (9) ($0^{\circ} < \theta_1 < \theta_{\min}$, the displacement of the specimen will take place towards left side and for $\theta_{\min} < \theta_2 < 90^{\circ}$, towards right side). Finally, for $\theta = \theta_{\min}$, no displacements will take place except the transversal contractions. In figure 6 the deformed shape for the three considered cases (45°, 61.3° = θ_{min}, and 80°) are presented. The ***x***-displacements of the top line nodes situated on a part and the other of this middle node are negative and, respective, positive because of the specimen transversal contraction, in the tensile loading (the Poisson's phenomenon).

All physical & numerical results verify the theoretical considerations and validate in good terms the experimental method proposed by the authors presented for determination in-plane shear modulus.

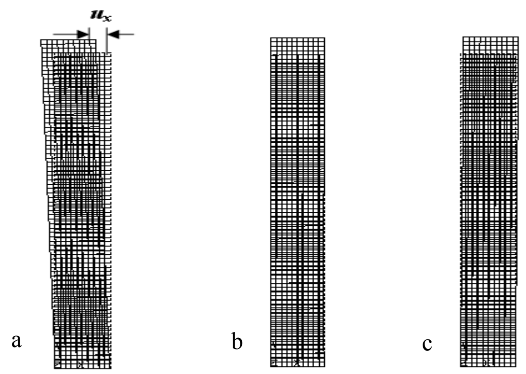

Fig. 6 – Numerical simulation of the specimen answer in different tensile tests:
a) $\theta = 45^{o} < \theta_{\min}$; b) $\theta = 61.3^{o} = \theta_{\min}$; c) $\theta = 80^{o} > \theta_{\min}$.

5. TSAI WU FAILURE CRITERION F12 COEFFICIENT DETERMINATION

5.1. Generalities

Tsai Wu failure criterion is more common in composite structure calculus. All professional codes for structure analysis (ANSYS, NASTRAN, ABAQUS, COSMOSM, etc.) have this criterion implemented in its simplest (but efficient) form for plane problem:

$$F_1\sigma_1 + F_2\sigma_2 + F_{11}\sigma_1^2 + 2F_{12}\sigma_1\sigma_2 + F_{22}\sigma_2^2 + F_{66}\sigma_6^2 = 1\,. \tag{16}$$

If the experimental characterization of F_1, F_2, F_{11}, F_{22}, F_{66} coefficients is quite simple (tensile, compression and shearing tests), the coefficient F_{12} generated many studies that led to different techniques because of the requirement to obtain a controlled complex stress state inside the sample. Each method has its disadvantages that will influence the accuracy of the determinations as presented below:

- The biaxial tensile test [4] applied upon a “cross sample” can achieve a complex stress state, but the reciprocal influence at joining corners will generate an uncontrollable stress state. The experience demonstrates that the fracture starts from these points that will influence the accuracy of the failure estimation via F_{12}.
- The uniaxial tensile and compression test (Fig. 7) proposed by Evans & Zhang [5] represents an experiment quite difficult to be performed. The frictions near the wall are difficult to be estimated and this will influence the accuracy of determinations.
- The 45^o tensile loading test [3] induces major errors grace of the coupling effects phenomenon.

The complex stress state (σ_1 and σ_2) achieved by using a cylindrical specimen loaded in tensile (or compression) and with interior pressure [6] seems to offer accurate results.

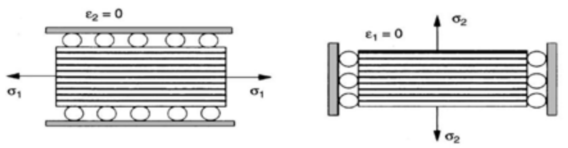

Fig. 7 $\sigma_1 - \sigma_2$ complex stress state in a uniaxial compression test.

Fig. 8 – Cylindrical specimens.

The disadvantages of the method consist in the difficulty to perform such experiments and in the technical observation that because the technologies used in manufacturing the composite cylinder and the composite structure are different. That can lead to different materials with different characteristics (volume fracture, orientation of fibers etc.).

As a natural conclusion results that the decoupling effects direction can be a simple and efficient way. This conclusion is highlighted by the observation that θ_{min} direction was already determined together with the shearing modulus determination.

5.2. Determination of F_{12} failure coefficient

Considering that the decoupling direction had already been determined, the sample was cut with the fibers at θ_{min} orientation. The test will be continued until the fracture of the sample, noting with Q the value of the strength. Under the hypothesis that the composite behavior is linear until the fracture, a uniaxial stress state will be present inside the sample during the whole test and, more important, the shearing stress is absent.

For the axis systems as presented in Fig. 1 the relation for the stress tensors can be written

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_{12} \end{Bmatrix} = \left[T(\theta)\right]\cdot\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{Bmatrix}, \tag{17}$$

where $[T(\theta)]$ was the rotating matrix. Under the mentioned hypothesis, for the ultimate loading the equation (17) becomes

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_{12} \end{Bmatrix} = \left[T\left(\theta_{\min}\right)\right] \cdot \begin{Bmatrix} Q \\ 0 \\ 0 \end{Bmatrix}. \tag{17'}$$

Noting $c = \cos\theta_{\min}$, and $s = \sin\theta_{\min}$, and introducing the stress components from (17') in equation (16), the final the expression of F_{12} is obtained

$$F_{12} = \frac{1}{2c^2s^2Q^2}\left(1 - F_1c^2Q - F_2s^2Q - F_{11}c^4Q^2 - F_{22}s^4Q^2 + F_{66}s^2c^2Q^2\right). \tag{18}$$

Noting the ultimate stress values obtained in simple tensile, compression, and shearing tests for samples with $\theta = 0^o$, and $\theta = 90^o$ as follows: X_t - tensile strength on direction 1, X_c - compression strength on direction 1, Y_t - tensile strength on direction 2, Y_c - compression strength on direction 2, S – shearing strength in 1-2 plane, the F_1, F_2, F_{11}, F_{22}, F_{66} Tsai Wu coefficients can be easy [2, 3, 4] determined as

$$F_1 = \frac{1}{X_t} - \frac{1}{X_c}, \quad F_2 = \frac{1}{Y_t} - \frac{1}{Y_c}, \quad F_{11} = \frac{1}{X_tX_c}, \quad F_{22} = \frac{1}{Y_tY_c}, \quad F = \frac{1}{S^2}. \tag{20}$$

6. CONCLUSIONS

- The decoupling effects direction can be considered an additional mechanical characteristic for plane reinforced composites. Despite the quite complicate determining method (iterative) the advantages offered by $\theta_{\min}$ direction can be considered compensatory in accuracy terms.
- The G_{12} and F_{12} experimental determination by using $\theta_{\min}$ direction offers simple and precise methods using simple facilities present in any mechanical testing lab.
- Both numerical and experimental results validate the method for G_{12} determination. Simple calculations in this case demonstrate that the results obtained by using off axis 45^o tensile test can lead to errors.
- The experimental validation of F_{12} coefficient is conditioned by the accepted degree of accuracy of Tsai Wu failure criterion itself.

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