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# ANALYSIS OF THE NATURAL ANGULAR FREQUENCIES VARIATION UPON SOIL STIFFNESS, DURING DYNAMIC SOIL COMPACTING

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This paper studies the influence of soil stiffness upon the natural angular frequencies of soil compacting machines (called also vibrator-rollers). Two different Romanian types of vibrator-rollers, CVA 4-5 and CVA 10, were considered in this study. The soil stiffness has been experimentally determined for different types of soils.

*Key words:* Vibrator-roller, Influence analysis, Soil stiffness, Natural angular frequency.

## 1. INTRODUCTION

The self-propelled soil compacting machines, called also self-propelled vibrator-rollers, are aimed to perform vibratory soil compactions for road systems, canals, barrages, dams, special foundations, etc. These self-propelled vibrator-rollers must be designed to simultaneously accomplish two opposite goals: to generate and maintain the desired working vibrations of the dynamic roller-compactor, on one hand, and to isolate the undesired vibrations transmitted to the compacting machine chassis, on the other hand.

ICECON Company has designed such self-propelled vibrator-rollers, with two stages of vibration isolation elements (based on rubber dampers). Using two vibration isolation stages is obviously much more advantageous than using a single vibration isolation stage.

In order to obtain the appropriate levels of the roller working vibrations and to reduce the vibrations at the other subassemblies, a study of the soil stiffness influence upon the machine natural angular frequencies is required. In this respect, this paper studies the variation of the natural angular frequencies of two self-propelled vibrator-rollers (CVA 4-5 and CVA 10), upon the soil class [20, 21].

## 2. DYNAMIC MODEL

Figure 1 shows the real model of a self-propelled vibrator-roller, composed of the following elements: 1 – roller-compactor; 2 – front chassis; 3 – first elastic vibration isolation stage; 4 – second elastic vibration isolation stage; 5 – rear chassis; 6 – rear roller (used for the rear drive).

Taking into account the symmetry of the vibrator-roller with respect to the longitudinal median plane, the symmetrical construction of the front chassis with respect to the roller-compactor axis and the fact that the rubber damping elements are qualitatively identical, the vibrator-roller can be modelled as shown in Fig. 2. This system has four degrees of freedom. The two vibration isolation stages, as well as the compacted soil, have been modelled by using only elastic elements. This simplification is possible due to the fact that the error, in terms of the natural angular frequencies, between the simplified elastic modelling and the more realistic Voigt-Kelvin viscoelastic model, is only about 5%, so it can be neglected.

The mass elements of the vibrator-roller, described by the dynamic real model shown in Fig. 2, are as follows: $m_1$ – mass of the roller-compactor; $m_2$ – mass of the front chassis; $m$, $J$ – mass and, respectively, moment of inertia of the rear chassis with respect to the mass center C ; $m_s$ – mass of the static drive roller. In what concerns the elastic vibration isolation elements, $k_2$ and $k_3$ denote the equivalent stiffness coefficients corresponding to the two vibration isolation stages, while $k_1$ and $k_4$ denote the stiffness coefficients of the soil to be compacted. More precisely, $k_1$ corresponds to the contact surface of the front rollers, while $k_4$ is the stiffness coefficient of the soil at the contact surface with the rear roller.

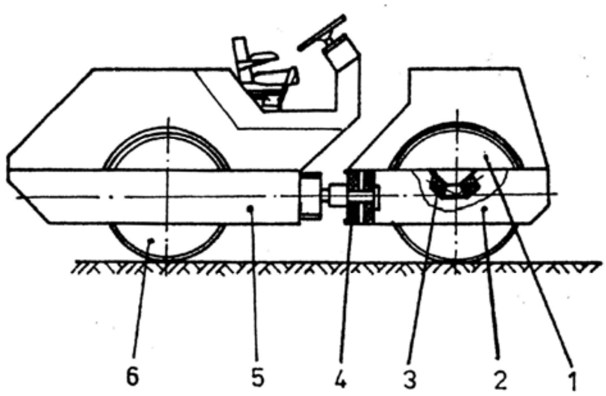


Fig. 1 – Self-propelled vibrator-roller.

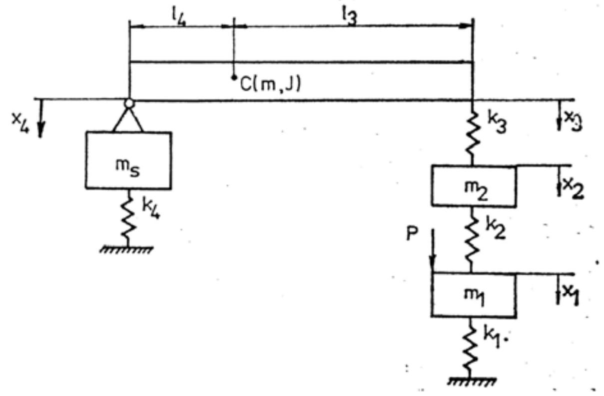


Fig. 2 – 2D simulation model of the 3elf-propelled vibrator-roller.

## 3. MOTION DIFFERENTIAL EQUATIONS

The kinetic energy of the mechanical system described by the dynamic model in Fig. 2 is:

$$2T = m_1\dot{x}_1^2 + m_2\dot{x}_2^2 + m_3\dot{x}_3^2 + m_4\dot{x}_4^2 + 2m_{34}\dot{x}_3\dot{x}_4\,, \tag{1}$$

where:

$m_3 = \dfrac{ml_4^2 + J}{l^2}$ – reduced mass of the rear chassis at supporting point 3;

$m_4 = \dfrac{ml_3^2 + J}{l^2} + m_s$ – reduced mass of the rear chassis at supporting point 4, to which is added the mass $m_s$ of the static drive roller;

$m_{34} = \dfrac{ml_3 l_4 - J}{l^2}$ – reduced mass of the rear chassis.

Using the matrix notation and the inner product in Hilbert spaces, the quadratic form (1) can be summarized as:

$$2T = \langle \dot{\mathbf{x}}, \mathbf{M}\dot{\mathbf{x}} \rangle\,, \tag{2}$$

where: $\dot{\mathbf{x}}$ – velocities vector, defined as $\dot{\mathbf{x}}^{\mathrm{T}} = \left[x_1, x_2, x_3, x_4\right]$; $\mathbf{M}$ – inertia matrix.

The inertia matrix $\mathbf{M}$ is positive definite, symmetric and nonsingular:

$$\mathbf{M} = \begin{bmatrix} m_1 & 0 & 0 & 0 \\ 0 & m_2 & 0 & 0 \\ 0 & 0 & m_3 & m_{34} \\ 0 & 0 & m_{34} & m_4 \end{bmatrix}. \tag{3}$$

The deformation potential energy of the elastic mechanical system, with respect to the static equilibrium position, is:

$$2\Pi = k_1 x_1^2 + k_2\left(x_1 - x_2\right)^2 + k_3\left(x_2 - x_3\right)^2 + k_4 x_4^2 . \tag{4}$$

The quadratic form (4) can be written in matrix notation as:

$$2\Pi = \left\langle \mathbf{u}, \mathbf{K}_0 \mathbf{u} \right\rangle , \tag{5}$$

where: $\mathbf{K}_0 = \mathrm{DIAG}\{k_1, k_2, k_3, k_4\}$ – the stiffness matrix; $\mathbf{u}$ – the elastic deformation vector, with $\mathbf{u}^{\mathrm{T}} = \left[u_1, u_2, u_3, u_4\right]$.

The components of the elastic deformation vector $\mathbf{u}$ are determined from the displacements $x_j$, $j = 1,\ldots,4$, as follows:

$$\begin{aligned} u_1 &= x_1, \\ u_2 &= x_1 - x_2, \\ u_3 &= x_2 - x_3, \\ u_4 &= x_4. \end{aligned}$$

The relation between displacements and deformations is linear:

$$\mathbf{u} = \mathbf{A}\mathbf{x} , \tag{6}$$

where: $\mathbf{A}$ – influence matrix, characterizing the influence of the displacements on deformations; $\mathbf{x}$ – displacements vector, with $\mathbf{x}^{\mathrm{T}} = \left[x_1, x_2, x_3, x_4\right]$.

The influence matrix $\mathbf{A}$ is defined as:

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{7}$$

Using relation (6), the quadratic form (5) can be expressed in terms of the displacements vector $\mathbf{x}$ as follows:

$$2\Pi = \left\langle \mathbf{A}\mathbf{x}, \mathbf{K}_0 \mathbf{u} \right\rangle . \tag{8}$$

Let us consider a Hilbert space $\mathcal{H}$, then for every continuous linear operator $V$, there is only one continuous linear operator $V^*$, called adjoint operator, which fulfils the identity:

$$\left\langle Vx, y \right\rangle = \left\langle x, V^* y \right\rangle \quad \text{for any } x, y \in \mathcal{H} ,$$

and also the following equality:

$$\left| V \right| = \left| V^* \right| .$$

In our case, the influence matrix $\mathbf{A} = \left(a_{ij}\right), i, j = 1,\ldots,n$ can be considered as a continuous linear operator on the Hilbert space $\mathcal{H}$, having as adjoint operator its transposed matrix $\mathbf{A}^{\mathrm{T}} = \left(a_{ji}\right)$. Based on the above definitions, relation (8) can be written as:

$$2\Pi = \left\langle \mathbf{x}, \mathbf{A}^{\mathrm{T}}\mathbf{K}_0\mathbf{u} \right\rangle, \tag{9}$$

and, by taking into account again relation (6), it comes:

$$2\Pi = \left\langle \mathbf{x}, \mathbf{A}^{\mathrm{T}}\mathbf{K}_0\mathbf{A}\mathbf{x} \right\rangle. \tag{10}$$

Denoting $\mathbf{K} = \mathbf{A}^{\mathrm{T}}\mathbf{K}_0\mathbf{A}$, it finally results:

$$2\Pi = \left\langle \mathbf{x}, \mathbf{K}\mathbf{x} \right\rangle. \tag{11}$$

For the pure elastic case and without considering any disturbing forces, the differential equations of motion are given by the Lagrange equations of second kind, as follows:

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{K}\mathbf{x} = 0. \tag{12}$$

The solutions of the differential equation (12) are of the following form:

$$\mathbf{x} = \mathrm{Re}(\mathbf{a}\mathrm{e}^{\mathrm{i}pt}), \tag{13}$$

where: $\mathbf{a}$ – vector of the unknown amplitudes, with $\mathbf{a}^{\mathrm{T}} = \left[a_1, a_2, a_3, a_4\right]$; $p$ – natural angular frequency.

By replacing the searched solution (13) into (12), one obtains the following algebraic equations system:

$$\left(\mathbf{K} - p^2\mathbf{M}\right)\mathbf{a} = 0. \tag{14}$$

The algebraic system (14) represents an eigenvalues and eigenvectors problem, involving the given $4\times 4$ square matrices **K** and **M**. The unknowns to be determined are vector **a** and scalar $p$.

The determination of the natural angular frequencies (eigenvalues $p_1$, $p_2$, $p_3$, and $p_4$) and of the corresponding eigenvectors was performed using a FORTRAN code. All these computations were performed for different soil classes, characterized by different soil stiffness coefficients, and for two Romanian types of vibrator-rollers (CVA 4-5 and CVA 10) [20, 21].

## 4. SOIL STIFFNESS COEFFICIENT

Several mechanical models are available in order to describe as accurately as possible the soil influence upon the vibration parameters of the compacting machine. These soil models must take into account the main soil properties, such as: elasticity, plasticity, viscous damping and dry friction. The most commonly used soil models are: Voigt-Kelvin, Bathelt P-E, Ephremides, Dvorak and Peter, Hartman, Maxwell. These more complex models are derived from the basic models: Hooke, Newton, Saint-Venant and Bethelt P [2–5].

For soil compacting machines, the effective working pressure is generally less than 1,5 daN/cm$^2$ (in order to avoid blockage into soil) and the working vibrations amplitudes are small (0.25–1.5 mm). At these levels of pressure and amplitudes, one can consider only the elasticity property for the soil models listed above, and neglect the others properties [1, 6, 14].

Let us also remark that the working regime of the vibrator-rollers is a post-resonance regime, where the amplitude of the roller-compactor remains constant, while the amplitudes of the vibrations induced to the other machine subassemblies are small.

So, only the reversible, elastic deformations will be taken into account in this paper. In this case, the soil stiffness coefficient is given by [8, 13, 15]:

$$k = C_z \cdot S, \tag{15}$$

where: $k$ – soil stiffness coefficient; $C_z$ – coefficient of uniform elastic contraction of the contact surface of area $S$; $S$ – area of the contact surface between the roller-compactor and the soil.

The equivalent contact surface is defined as for the case of a rectangular plate, as follows [8, 12, 15]:

$$C_z = \chi_z \frac{E}{1-\mu^2} \cdot \frac{1}{\sqrt{S}}, \tag{16}$$

where: $\chi_z$ – coefficient taking into account the dimensions of the equivalent plate; $E$ – soil elastic modulus; $\mu$ – Poisson's ratio.

The experimental soil stiffness coefficient $k^*$, obtained using a test plate with rectangular contact surface of area $S^*$, is given by:

$$k^* = \chi_z \frac{E}{1-\mu^2} \cdot \sqrt{S^*}, \tag{17}$$

and the soil stiffness coefficient $k$ for the real contact surface of area $S$ has the following expression:

$$k = \chi_z \frac{E}{1-\mu^2} \cdot \sqrt{S}. \tag{18}$$

From the similar equations (17) and (18), the value of the real soil stiffness coefficient $k$ depends on the experimental coefficient $k^*$, as follows:

$$k = k^* \sqrt{\frac{S}{S^*}}. \tag{19}$$

*Table 1*

Soil stiffness coefficients $k$ for different types of soil and vibrator-rollers

| Soil type | Stiffness coefficient $10^6 k$ [N/m] | |
|---|---|---|
| | CVA 4-5 | CVA 10 |
| Sandy soil with loose gravel of size (3-7) mm | 20.0 | 44.0 |
| Loose gravel of size (7-15) mm; Loose loamy sand. | 30.0 | 67.5 |
| Loose and slightly loamy medium granulation sand | 40.0 | 90.0 |
| Medium to large granulation sand and pre-compacted gravel of size (7-15) mm; Compacted clay and gravel. | 52.0 | 120.0 |

ICECON Company has performed soil stiffness experiments using a test plate with contact surface of area $S^* = 900\,\text{cm}^2$, attached to an electrodynamic shaker, which is controlled by a variable frequency generator [20]. The experimental results obtained by ICECON have been compared with results available in the literature [9, 15]. Table 1 presents the values of stiffness coefficients determined in dynamic regime for four different soil types and for two types of vibrator-rollers, i.e., CVA 4-5 and CVA 10.

## 5. NATURAL ANGULAR FREQUENCIES OF SOIL-COMPACTING MACHINE SYSTEM

Table 2 presents the natural angular frequencies of the self-propelled vibrator-roller CVA 4-5, computed for the different soil types (with the corresponding stiffness coefficients) described in Table 1. The following equivalent stiffness coefficients corresponding to the two isolation stages were considered: $k_2 = 2.132 \cdot 10^6$ N/m and $k_3 = 2.00 \cdot 10^6$ N/m.

*Table 2*

Natural angular frequencies of CVA 4-5 for different soil types

| Soil stiffness coefficient $10^6$ $k$ [N/m] | Natural angular frequencies [$s^{-1}$] | | | |
|---|---|---|---|---|
| | $p_1$ | $p_2$ | $p_3$ | $p_4$ |
| 20 | 24.13 | 62.22 | 132.76 | 161.89 |
| 30 | 24.45 | 62.66 | 162.29 | 194.71 |
| 40 | 24.61 | 62.87 | 187.23 | 222.82 |
| 50 | 24.72 | 63.01 | 213.36 | 352.48 |
| Equivalent stiffness coefficient of the first isolation stage $k_2 = 2.132 \cdot 10^6$ N/m ; Equivalent stiffness coefficient of the second isolation stage $k_3 = 2.00 \cdot 10^6$ N/m . | | | | |

The natural angular frequencies were also computed for the self-propelled vibrator-roller CVA 10. Table 3 shows the results obtained for $k_3 = 3.04 \cdot 10^6$ N/m , while the natural angular frequencies presented in Table 4 are obtained considering $k_3 = 8.70 \cdot 10^6$ N/m . In both tables concerning CVA 10, the equivalent stiffness coefficient of the first isolation stage was considered $k_2 = 2.42 \cdot 10^6$ N/m .

*Table 3*

Natural angular frequencies of CVA 10 for different soil types and for $k_3 = 3.04 \cdot 10^6$ N/m

| Soil stiffness coefficient $10^6$ $k$ [N/m] | Natural angular frequencies [$s^{-1}$] | | | |
|---|---|---|---|---|
| | $p_1$ | $p_2$ | $p_3$ | $p_4$ |
| 44.0 | 13.21 | 85.32 | 109.67 | 165.95 |
| 67.5 | 13.30 | 86.36 | 134.91 | 203.14 |
| 90.0 | 13.34 | 86.72 | 155.40 | 233.37 |
| 120.0 | 13.37 | 86.95 | 179.30 | 268.48 |
| Equivalent stiffness coefficient of the first isolation stage $k_2 = 2.42 \cdot 10^6$ N/m ; Equivalent stiffness coefficient of the second isolation stage $k_3 = 3.04 \cdot 10^6$ N/m . | | | | |

*Table 4*

Natural angular frequencies of CVA 10 for different soil types and for $k_3 = 8.70 \cdot 10^6$ N/m

| Soil stiffness coefficient $10^6$ $k$ [N/m] | Natural angular frequencies [$s^{-1}$] | | | |
|---|---|---|---|---|
| | $p_1$ | $p_2$ | $p_3$ | $p_4$ |
| 44.0 | 15.32 | 121.70 | 129.95 | 166.45 |
| 67.5 | 15.46 | 121.98 | 141.01 | 203.25 |
| 90.0 | 15.52 | 122.75 | 158.31 | 233.42 |
| 120.0 | 15.57 | 122.07 | 181.07 | 268.50 |
| Equivalent stiffness coefficient of the first isolation stage $k_2 = 2.42 \cdot 10^6$ N/m ; Equivalent stiffness coefficient of the second isolation stage $k_3 = 8.70 \cdot 10^6$ N/m . | | | | |

## 6. CONCLUSIONS

This study concerns the vibratory soil compacting machines with two rollers, the front one being the vibratory roller-compactor, while the rear roller is used just for the rear drive. The real dynamic model is described in Fig. 2. This type of soil compacting machines are working in post-resonance regime, i.e., $\omega > p_4$ . This paper analyses the working regimes of these compacting machines as a function of the soil stiffness. The conclusions are:

a) the soil stiffness coefficients have to be determined experimentally, by taking into account the contact surface between the roller-compactor and the soil;

b) the consideration of different soil types (listed in Table 1) has almost no influence on the first two natural angular frequencies $p_1$ and $p_2$, but induces important changes in what concerns the 3rd and the 4th natural angular frequencies, i.e., $p_3$ and $p_4$.

Knowing that the angular frequency ω of the disturbing force takes values in the interval (180-314) $s^{-1}$, then the forced vibrations regime must be chosen so that, for every type of soil to be compacted, the system works in post-resonance regime, i.e., $\omega > p_4$.

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